SIGRAV SCHOOL
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BINARY SYSTEMS AS TEST-BED OF GRAVITY THEORIES

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LECTURE N° 2

TIMING OF BINARY PULSARS IN GENERAL RELATIVITY

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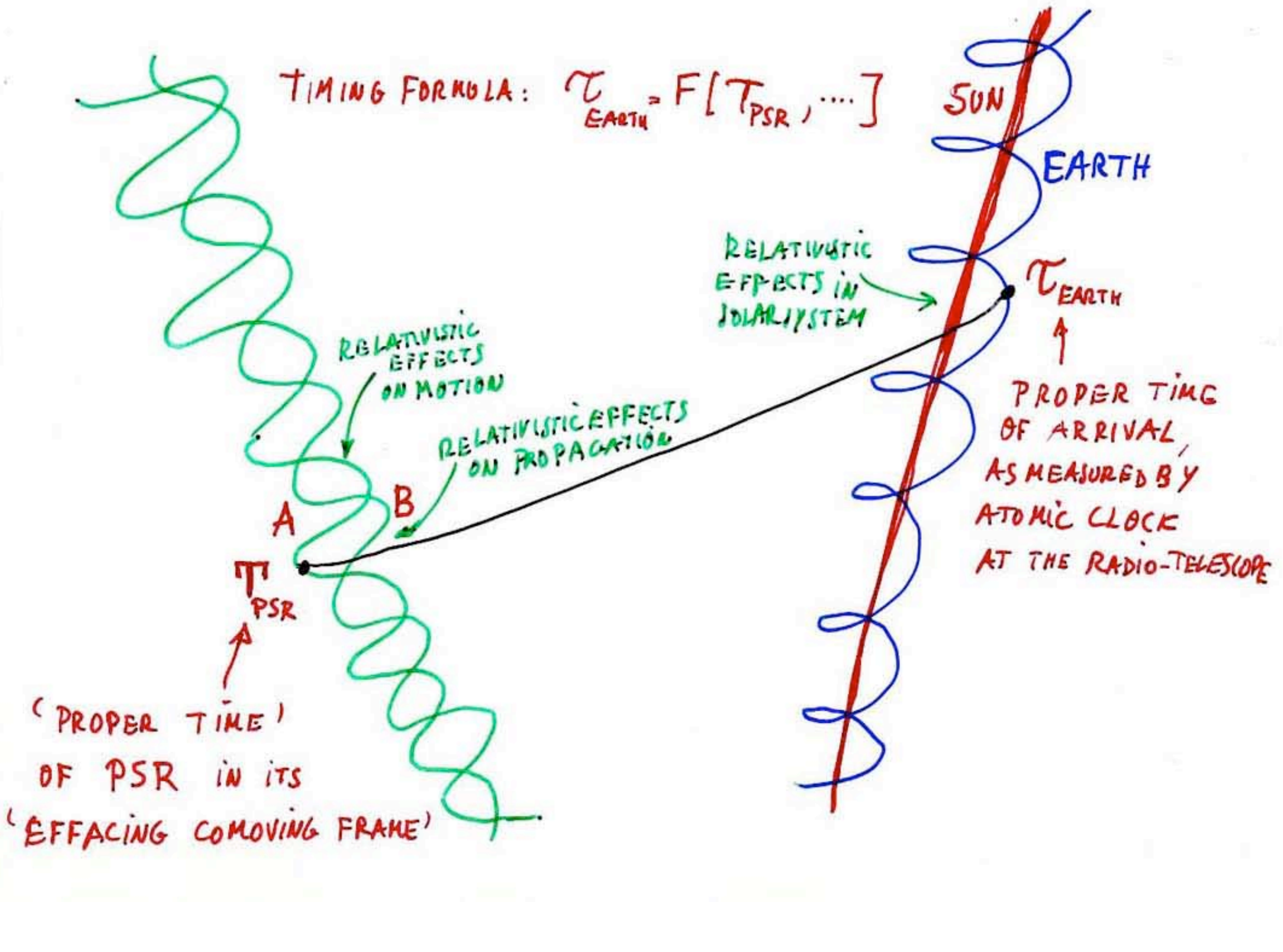
INTERPRETATION OF GENERAL RELATIVITY

Einstein had been confused for several years by "general covariance", meaning of coordinates, ...

G.R. DEFINES ITS OWN INTERPRETATION

TIMING OF BINARY PULSARS IS A PRIME EXAMPLE

'PB OF MOTION' MARRED BY 'COORDINATE AMBIGUITIES' FOR MANY YEARS BUT:



THE TIMING FORMULA

Blandford Teukolsky '76, Damour Deruelle '86

CHAIN OF TRANSFORMATIONS BETWEEN $T_{PSR} \rightarrow \tau_{EARTH}$

WE FOCUS HERE ON THE EFFECTS LINKED TO RELATIVISTIC BINARY MOTION

I.E. WE ASSUME THAT ONE HAS ALREADY SUBTRACTED FROM τ_{EARTH}

- DISPERSION EFFECTS $\tau_{EARTH} - D/f^2$ « INFINITE FREQUENCY TOA »
- EARTH → BARYCENTER OF SOLAR SYSTEM

BEST DERIVED BY USING MULTI-CHART APPROACH IN SOLAR SYSTEM

- TRAVEL TIME IN SOLAR SYSTEM
- GRAVITATIONAL DELAY BY GRAVITATIONAL FIELD OF SUN (AND PLANETS)
- VARIABLE REDSHIFT BETWEEN PROPER TIME OF EARTH AND PROPER TIME AT BARYCENTER

- VARIATION OF DOPPLER FACTOR BETWEEN CoM WORLDLINE OF BINARY AND BARYCENTER WORLDLINE:

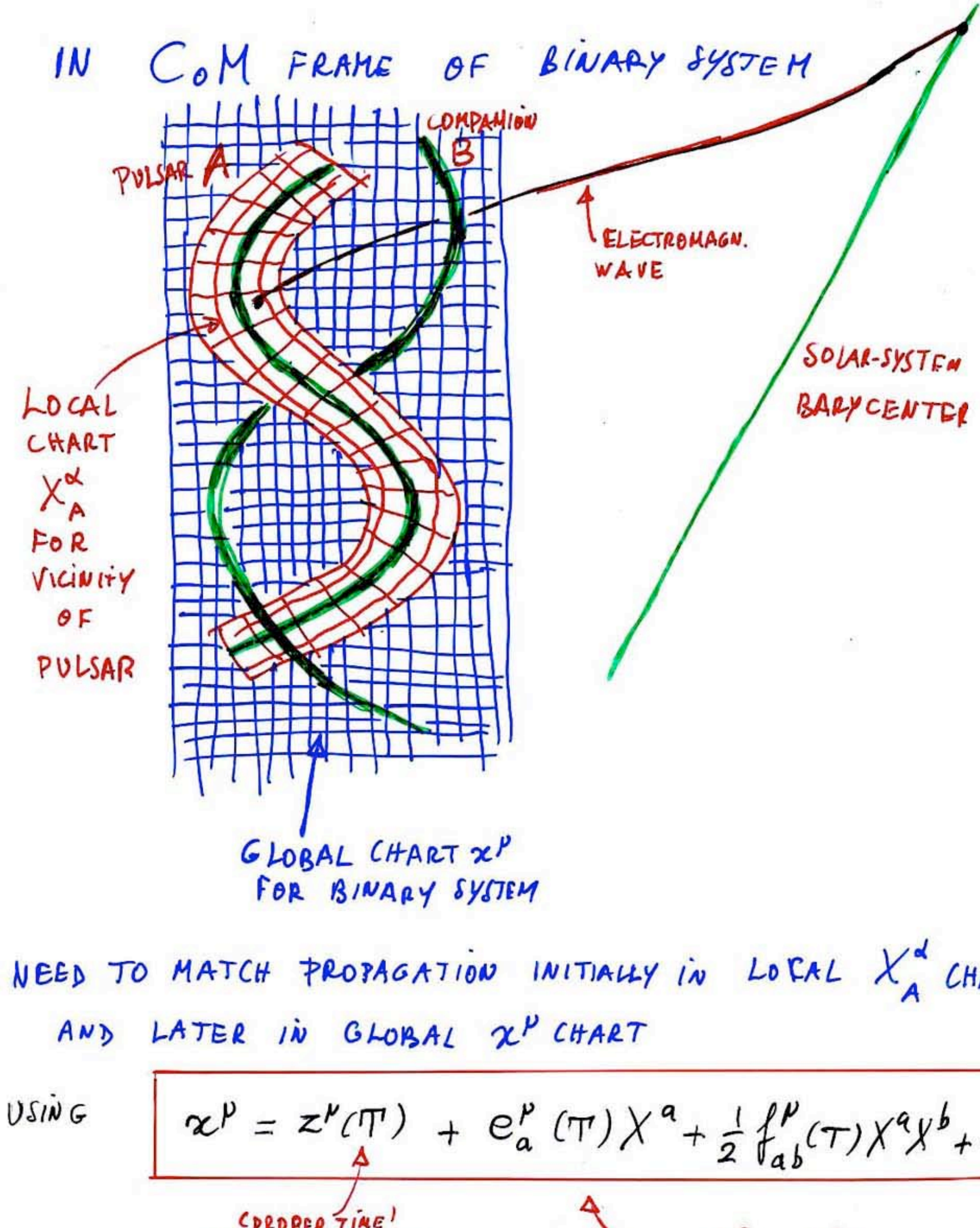
$$\frac{d}{dt}\left(1+\frac{v_R}{c}\right)=\frac{d}{dt}\frac{\vec{n}_{10}\cdot(\vec{v}_1-\vec{v}_0)}{c}=\frac{1}{c}\,\vec{n}_{10}\cdot(\vec{a}_1-\vec{a}_0)+\frac{v_T^2}{c\,d}$$

GALACTIC ACCELERATION OF BINARY CoM

GAL ACC OF SUN

LINKED TO PROPER MOTION AND DISTANCE

TIMING EFFECTS LINKED TO RELATIVISTIC BINARY MOTION



- NEED TO MATCH PROPAGATION INITIALLY IN LOCAL X_A^α CHART AND LATER IN GLOBAL x^μ CHART

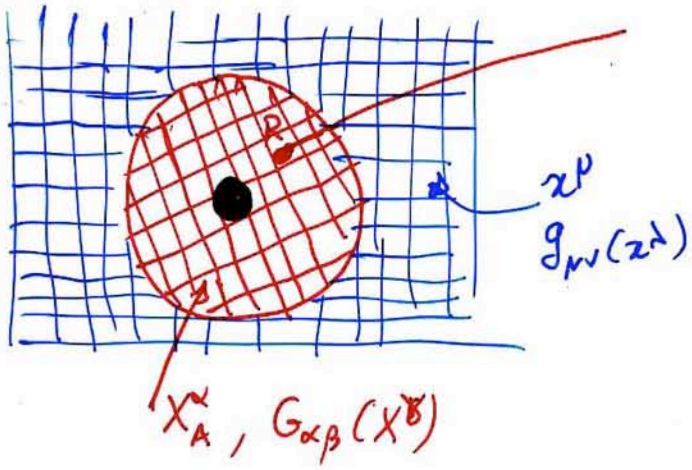
USING
$$\boxed{x^\mu = z^\mu(T) + e_a^\mu(T) X^a + \frac{1}{2} f_{ab}^\mu(T) X^a X^b + \ldots}$$

(PROPER TIME)
OF PULSAR FRAME

$$ds^2 = -\left(1 - \frac{2GM}{c^2 R} + \ldots\right) c^2 dT^2 + \left(1 + \frac{2GM}{c^2 R} + \ldots\right) \delta_{ij} dX^i dX^j$$

$$e_a^0 \sim v_A^a / c$$
$$e_a^i \sim \delta_a^i + \frac{v_A^2}{c^2} + \frac{G m_B}{c^2 d}$$

• IN COMBINING THE TWO PARTS OF THE PROPAGATION



ONE CHECKS THAT THERE ARE NO DELICATE COUPLING BETWEEN INTERNAL (STRONG-FIELD) AND EXTERNAL (WEAK-FIELD) EFFECTS

ALL STRONG-FIELD EFFECTS ENTER AS UNIMPORTANT ADDITIVE CONSTANTS

THE ONLY OBSERVATIONALLY SIGNIFICANT EFFECTS (AT LEADING ORDER) ARE:

- The components $e^0_a = \frac{v_a}{c}\left[1 + \frac{1}{2}\vec{v}^2 + 3\frac{Gm_B}{d} + \cdots\right]$ in $\underbrace{e^\mu_a}_{\text{LINEAR PART}} X^a$

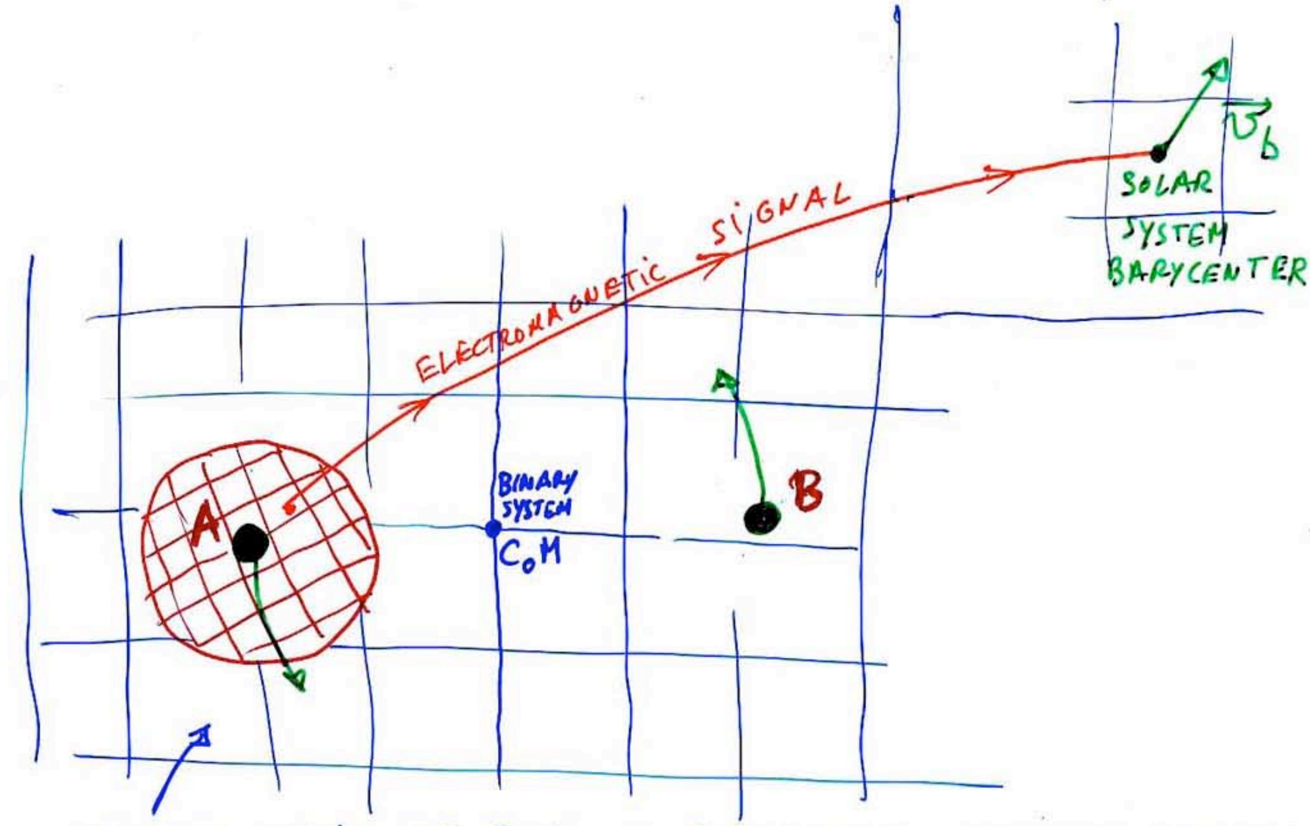
- The link $-c^2 dT^2 = \underbrace{\tilde{g}}_{\text{REGULARIZED}}{}_{\mu\nu}(z^\lambda_A)\, dz^\mu_A dz^\nu_A$ (E.G. $d = 3+\varepsilon$)

  between the pulsar 'asymptotic proper time' and $g_{\mu\nu} = \eta_{\mu\nu} + h^A_{\mu\nu} + h^B_{\mu\nu} + \cdots$

- $e^0_a \rightarrow$ 'ABERRATION EFFECT': $\left[\delta^a_i + O\left(\frac{v^2}{c^2}\right)\right] \underbrace{n^i}_{\text{DIRECTION IN GLOBAL CHART}} \simeq \underbrace{N^a}_{\text{DIRECTION IN LOCAL}}(T) + \frac{v^a}{c} - N^a \frac{\vec{N}\cdot\vec{v}}{c}$

- $T = \frac{1}{c}\int\sqrt{-\tilde{g}_{\mu\nu}(z^\lambda_A)dz^\mu dz^\nu} \simeq \int dt\sqrt{1 - 2\frac{Gm_B}{c^2 r_{AB}} - \frac{\vec{v}_A^{\,2}}{c^2}} \simeq \int dt\left(1 - \frac{Gm_B}{c^2 r_{AB}} - \frac{\vec{v}_A^{\,2}}{2c^2}\right)$

  i.e. 'COMBINED TIME-DILATION AND GRAVIT. REDSHIFT FROM B'



DESCRIBE MOTION OF PSR, ELM SIGNAL AND SOLAR BARYCENTER IN GLOBAL CHART: x^μ

$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu$$

ELM SIGNAL: $$0 = ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu \simeq -\left(1-\frac{2U}{c^2}\right)c^2dt^2 + \left(1+\frac{2U}{c^2}\right)d\vec{x}^2$$

$$U = \frac{Gm_A}{|\vec{x}-\vec{z}_A|} + \frac{Gm_B}{|\vec{x}-\vec{z}_B|} \qquad \delta_{ij}\, dx^i dx^j$$

$$c^2dt^2 \simeq \frac{1+2U/c^2}{1-2U/c^2}\, d\vec{x}^2 \simeq \left(1+4U/c^2\right) d\vec{x}^2$$

$$dt \simeq \frac{1}{c}\left(1+\frac{2U}{c^2}\right)|d\vec{x}|$$

TO SECOND ORDER, CAN BE $\int$ ALONG STRAIGHT LINE

$$\int_{t_{emission}}^{t_{arrival}} dt \equiv t_a - t_e \simeq \frac{1}{c}\int_{t_e}^{t_a} |d\vec{x}| + \frac{2}{c^3}\int_{t_e}^{t_a}\left(\frac{Gm_A}{|x-z_A|} + \frac{Gm_B}{|x-z_B|}\right)|d\vec{x}|$$

EUCLIDEAN DISTANCE

MATCHED TO LOCAL ⊞

GRAVITATIONAL DELAY OF B "SHAPIRO DELAY"

$$t_a = t_e + \frac{1}{c}\left|\vec{z}_b(t_a) - \vec{z}_A(t_e)\right| + \Delta_S$$

(GLOBAL) COORDINATE TIME OF ARRIVAL AT SOLAR BARYCENTER

(GLOBAL) COORDINATE TIME OF EMISSION

TIME OF PROPAGATION

'SHAPIRO DELAY' DUE TO m_B

PROPER TIME OF ARRIVAL

$$d\tau_a = \frac{1}{c}\sqrt{-g_{\mu\nu}(z_b^\lambda)\,dz_b^\mu dz_b^\nu}$$

TIME DILATION DUE TO $\vec{v}_b$ (AND $[\Delta U]^b_{COM}$)

EXPAND

$$\vec{z}_b(t_a) = \vec{z}_b(0) + \vec{v}_b t_a$$

$$\frac{1}{c}\left|\vec{z}_b(0) + \vec{v}_b t_a - \vec{z}_A(t_e)\right|$$

$$\simeq \frac{1}{c}|\vec{z}_b(0)| + \frac{1}{c}\vec{n}\cdot(\vec{v}_b t_a - \vec{z}_A(t_e)) + \mathcal{O}\left(\frac{v^2}{|z_b|}\right)$$

$\vec{n} \equiv \frac{\vec{z}_b}{|\vec{z}_b|}$ = UNIT VECTOR FROM BINARY SYSTEM TOWART SOLAR BARYCENTER

$= -\ \overrightarrow{\text{LINE OF SIGHT}}$

$$\boxed{D\cdot\tau_a = t_e + \Delta_R(t_e) + \Delta_S(t_e) + \begin{cases} \text{CST} \\ +\mathcal{O}(v^2/|z_b|) \\ +\text{ACCELERATION EFFECTS}\end{cases}}$$

DOPPLER FACTOR

SHAPIRO DELAY

$$\boxed{D \equiv \frac{1 - \vec{n}\cdot\vec{v}_b/c}{\sqrt{1 - \vec{v}_b^2/c^2}}}$$

$$D - 1 \sim \mathcal{O}\left(\frac{v}{c}\right) \sim 10^{-3}$$

NOT AT ALL NEGLIGIBLE

ROEMER TIME DELAY
≡ TIME OF FLIGHT ACROSS THE ORBIT OF A

$$\boxed{\Delta_R \equiv -\frac{1}{c}\vec{n}\cdot\vec{z}_A(t_e)}$$

LINK BETWEEN t_e AND T_e.

t_e: GLOBAL COORDINATE TIME OF EMISSION

T_e: LOCAL-FRAME TIME OF EMISSION

TWO EFFECTS

$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu$$

$$ds^2 = G_{\alpha\beta}(X^\gamma)\, dX^\alpha dX^\beta$$

$$G_{\alpha\beta}(X^\gamma) \simeq G^{\text{isolated}}_{\alpha\beta}(X^\gamma)$$

$$dT_e = \frac{1}{c}\sqrt{-\underbrace{\tilde{g}_{\mu\nu}(z^\lambda_A)}_{\text{REGULARIZED (e.g. } d=3+\varepsilon)}\, dz^\mu_A dz^\nu_A}$$

$$\simeq \sqrt{1 - \frac{2Gm_B}{c^2 r_{AB}} - \frac{\vec{v}_A^{\,2}}{c^2}}\; dt_e$$

$$\simeq \left(1 - \frac{Gm_B}{c^2 r_{AB}} - \frac{1}{2}\frac{\vec{v}_A^{\,2}}{c^2}\right) dt_e$$

$$\Downarrow$$

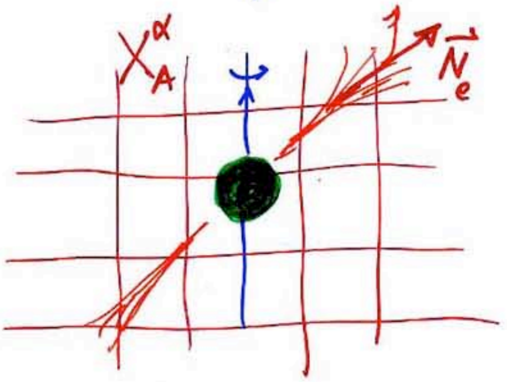
$$\boxed{t_e = T_e + \Delta_E}$$

EINSTEIN TIME DELAY
= SR TIME-DILATION ($\frac{1}{2}v^2/c^2$)
+ GR 'EINSTEIN EFFECT' ($Gm_B/c^2 r_{AB}$)

PULSAR PHASE

$$\frac{\Phi}{2\pi} = N_0 + \nu T + \frac{1}{2}\dot{\nu} T^2 + \frac{1}{6}\ddot{\nu} T^3$$

$$\nu = \frac{1}{P_p} = \text{PULSAR FREQUENCY}$$



$$x^\mu = z^\mu(T) + e^\mu_a(T) X^a + \cdots$$

ABERRATION

$$n^i_e(\delta^a_i + \cdots) \simeq N^a_e + \frac{v^a_A}{c} - N^a\frac{(\vec{N}\vec{v}_A)}{c}$$

$$\Phi(T_e) = \Phi_0 + 2\pi N + \delta_A\Phi(T_e)$$

N: INTEGER: N^{th} TURN

$$\delta_A\Phi = + \frac{\vec{v}_A . (\vec{n} \times \vec{e}_3)}{c\,(\vec{n} \times \vec{e}_3)^2}$$

$\vec{e}_3$: SPIN AXIS

DEFINE $$\boxed{\Delta_A \equiv \frac{\delta_A \Phi}{2\pi\nu}}$$

ABERRATION TIME DELAY

AND $$\boxed{T \equiv T_e - \Delta_A}$$

TIMING FORMULA

WITH DEFINITION $T \equiv T_e - \Delta_A$

$$\boxed{T_N \text{ DEFINED BY: } N = N_0 + \nu T + \frac{1}{2}\dot{\nu} T^2 + \frac{1}{6}\ddot{\nu} T^3}$$

'PROPER TIME OF EMISSION OF N^{TH} PULSE' CORRECTED FOR ABERRATION

THEN

$$\boxed{D \cdot \tau_a = T + \Delta_R(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T)}$$

DOPPLER FACTOR

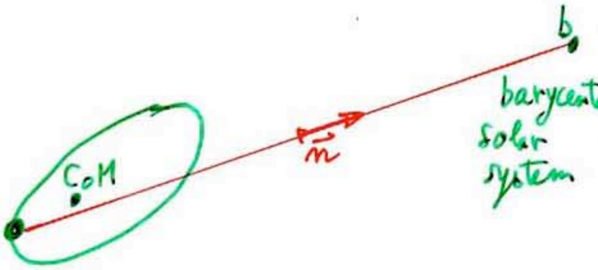
PROPER TIME OF ARRIVAL AT SOLAR BARYCENTER (CORRECTED FOR: DISPERSION, POSITION OF ⊕, EINSTEIN + SHAPIRO SOLAR-SYSTEM EFFECTS)

ROEMER TIME DELAY

EINSTEIN TIME DELAY

SHAPIRO TIME DELAY

ABERRATION TIME DELAY



$$D = \frac{1 - \vec{n} \cdot \vec{v}_b / c}{\sqrt{1 - v_b^2/c^2}}$$

$$\Delta_R = -\frac{1}{c}\, \vec{n} \cdot \vec{z}_A$$

TIME OF FLIGHT ACROSS PSR ORBIT

IF $a \sim 300\,000$ km $\rightarrow \Delta_R \sim 1$ s

$$\Delta_E = +\frac{1}{c^2}\int dt \left(\frac{G m_B}{r_{AB}} + \frac{1}{2}\vec{v}_A^{\,2} - \left\langle \frac{G m_B}{r_{AB}} + \frac{1}{2}\vec{v}_A^{\,2} \right\rangle \right)$$

AFTER RESCALING T BY MEAN GR + SR EFFECTS IN BINARY $\rightarrow$ MODIFIES D

$$\Delta_S = +\frac{1}{c^3}\int_{t_e}^{t_a} \frac{2 G m_B}{|\vec{x} - \vec{z}_B|}\, d|\vec{x}|$$

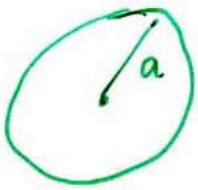
ELM SIGNAL

$$\Delta_A = \frac{1}{2\pi\nu} \, \frac{\vec{v}_A \cdot (\vec{n} \times \vec{e}_3)}{c\, (\vec{n} \times \vec{e}_3)^2}$$

ORDERS OF MAGNITUDE

$$D = 1 + \mathcal{O}\left(\frac{v_b^{SOLAR} - v_{CoM}^{BINARY}}{c}\right) = 1 + \mathcal{O}(10^{-3})$$

? EFFECT IN VIEW OF HIGH-ACCURACY PSR MEASUREMENTS ?



$$\frac{GM}{a} \sim v^2$$

$$v \sim n a \sim \frac{2\pi}{P_b} a$$

typically $\frac{v}{c} \sim (\text{few})\, 10^{-3}$

$a/c \sim s$

$$\Delta_R \sim \frac{a}{c}\left[1 + \frac{v^2}{c^2} + \frac{v^4}{c^4} \cdots\right] \sim \frac{v}{c}\frac{P_b}{2\pi}\left[1 + \frac{v^2}{c^2} + \frac{v^4}{c^4} + \cdots\right]$$

$$\sim \underset{\sim s}{\frac{a}{c}} + \underset{\sim \mu s}{\frac{GM}{c^3}} + \underset{\sim 10^{-12}\, s}{\frac{v^2}{c^2}\frac{GM}{c^3}}$$

$$\boxed{\frac{GM_\odot}{c^3} = 4.92549\ldots\ \mu s}$$

$$\Delta_E \sim \frac{v^2}{c^2}\frac{P_b}{2\pi} \sim \frac{v}{c}\Delta_R\left[1 + \frac{v^2}{c^2} + \cdots\right] \sim ms + 10^{-9} s$$

$$\Delta_S \sim \frac{v^3}{c^3}\frac{P_b}{2\pi} \sim \frac{v^2}{c^2}\Delta_R \sim \frac{GM}{c^3} \sim \mu s + 10^{-9} s$$

10^{-9} s ← FROM $\mathcal{O}(v/c)$ CORRECTIONS

$$\Delta_A \sim \frac{v}{c}\frac{P_p}{2\pi} \sim 10^{-3}\frac{P_p}{2\pi}$$

$\Rightarrow$ 1PK TIMING FORMULA SEEMS ACCURATE ENOUGH

EINSTEIN TIME DELAY

$$\Delta_E = +\frac{1}{c^2}\int dt\left(\underbrace{\frac{Gm_B}{r_{AB}} + \frac{1}{2}\vec{v}_A^{\,2} - \left\langle\frac{Gm_B}{r_{AB}} + \frac{1}{2}\vec{v}_A^{\,2}\right\rangle}_{\text{SUFFICIENT TO USE NEWTONIAN APPROX:}}\right)$$

$$\vec{V} \equiv \vec{v}_A - \vec{v}_B\,, \quad \vec{R} \equiv \vec{z}_A - \vec{z}_B \qquad m_A\vec{v}_A = -m_B\vec{v}_B = \frac{\vec{v}_A}{\frac{1}{m_A}} = \frac{-\vec{v}_B}{\frac{1}{m_B}} = \frac{\vec{v}_A - \vec{v}_B}{\frac{1}{m_A} + \frac{1}{m_B}}$$

$$M \equiv m_A + m_B;\ \frac{1}{\mu} \equiv \frac{1}{m_A} + \frac{1}{m_B}$$

$$\mu = \frac{m_A m_B}{M}$$

$$\frac{1}{2}V^2 - \frac{GM}{R} = -\frac{GM}{2a_R} = \text{cst}$$

$$m_A\vec{v}_A = \mu\vec{V} = \frac{m_A m_B}{M}\vec{V}$$

$$\vec{v}_A = \frac{m_B}{M}\vec{V}$$

$$\frac{1}{2}\vec{v}_A^{\,2} = \frac{1}{2}\left(\frac{m_B}{M}\right)^2 V^2 = \left(\frac{m_B}{M}\right)^2\left[\frac{GM}{R} + \text{cst}\right]$$

$$\Delta_E = \frac{1}{c^2}\int dt\left[\frac{Gm_B}{R} + \left(\frac{m_B}{M}\right)^2\frac{GM}{R} + \text{cst}\right]$$

$$= \frac{Gm_B}{c^2}\left(1 + \frac{m_B}{M}\right)\left[\int\frac{dt}{R} - \text{cst}\times t\right]$$

$$n(t - t_0) = u - e_t\sin u \Rightarrow n\,dt = (1 - e_t\cos u)\,du$$

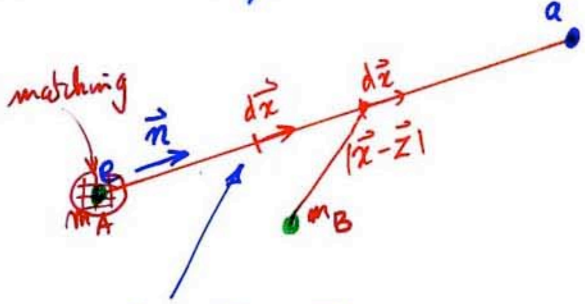
$$R = a_R(1 - e_R\cos u)$$

$$\int\frac{dt}{R} = \frac{1}{na_R}\int\frac{1 - e_t\cos u}{1 - e_R\cos u}\,du \simeq \frac{1}{na_R}u = \frac{1}{na_R}\Big[\underbrace{n(t - t_0)}_{\text{RESCALED AWAY}} + \underbrace{e\sin u}_{\text{KEEP}}\Big]$$

$$\boxed{\Delta_E \simeq \frac{e}{n}\,\frac{Gm_B(1 + m_B/M)}{c^2\,a_R}\,\sin u}$$

SHAPIRO TIME DELAY

$$\Delta_S = \frac{2 G m_B}{c^3} \int_{\text{emission}}^{\text{arrival}} \frac{|d\vec{x}|}{|\vec{x}-\vec{z}_B|}$$



$$\vec{x} = \vec{x}_e + \vec{n}\, s$$

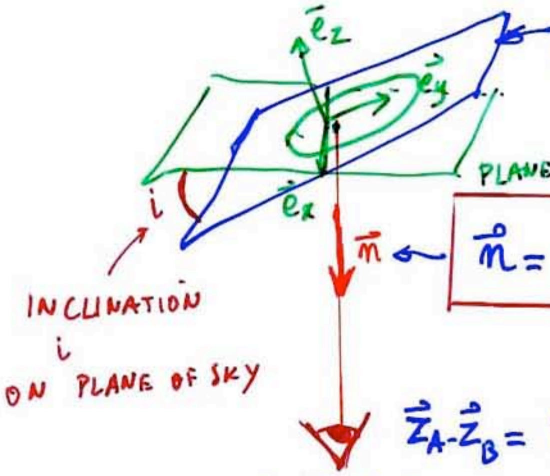
$$\int_{\vec{x}_e}^{\vec{x}_a} \frac{|d\vec{x}|}{|\vec{x}-\vec{z}_B|} = \ln\left(\frac{\vec{n}\cdot(\vec{x}_a-\vec{z}_B)+|\vec{x}_a-\vec{z}_B|}{\vec{n}\cdot(\vec{x}_e-\vec{z}_B)+|\vec{x}_e-\vec{z}_B|}\right)$$

$$\simeq -\ln\left(\vec{n}\cdot(\vec{x}_e-\vec{z}_B)+|\vec{x}_e-\vec{z}_B|\right) + \text{cst}$$

$\exists$ large effect $\Delta_S \simeq \frac{2GM}{c^3} \log 2|\vec{z}_{\text{barycenter}} - \vec{z}_{\text{COM}}| = \text{const}$

$$\Delta_S \simeq -\frac{2 G m_B}{c^3} \ln\left(\vec{n}\cdot(\vec{z}_A-\vec{z}_B) + |\vec{z}_A-\vec{z}_B|\right)$$

$|\vec{z}_A-\vec{z}_B| = R = a_R(1-e\cos u)$



INCLINATION ANGLE

$$\vec{n} \leftarrow \boxed{\vec{n} = -\sin i\, \vec{e}_y - \cos i\, \vec{e}_z}$$

$\perp$ TO ORBITAL PLANE

$$\vec{z}_A-\vec{z}_B = R\left[\cos\theta\, \vec{e}_x + \sin\theta\, \vec{e}_y\right]$$

$$\vec{n}\cdot(\vec{z}_A-\vec{z}_B) = -R \sin i \sin\theta$$

keep this secular relativistic effect

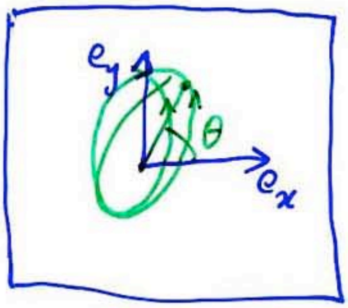
$R = a_R(1-e\cos u)$ $\qquad \theta = \omega_0 + (1+k)\, 2\arctan\left[\sqrt{\frac{1+e_\theta}{1-e_\theta}} \tan\frac{u}{2}\right]$

ROEMER TIME DELAY

$$\vec{n} = -\sin i\, \vec{e}_y - \cos i\, \vec{e}_z$$

$$\Delta_R = -\frac{1}{c}\vec{n}\cdot\vec{z}_A = +\frac{1}{c}\sin i\, \vec{e}_y\cdot\vec{z}_A = +\frac{1}{c}\sin i\, r_A \sin\theta$$

$$\vec{z}_A = \vec{z}_A - \vec{z}_{COM} = r_A\cos\theta\,\vec{e}_x + r_A\sin\theta\,\vec{e}_y$$



AS $\Delta_R \sim \frac{a}{c} \sim \frac{v}{c}\frac{P_b}{2\pi}$ IS THE LEADING TERM IN TIMING FORMULA

$\Rightarrow$ NEED TO CAREFULLY TAKE INTO ACCOUNT $\frac{v^2}{c^2}$ CORRECTIONS TO KEPLER MOTION. THEY WILL GIVE $\Delta_R \sim \frac{a}{c} + \frac{GM}{c^3}$ ← COMPARABLE TO SHAPIRO

USE QUASI-KEPLERIAN FORM

$$n(t-t_0) = u - e_t \sin u$$

$$\begin{cases} \theta = \omega_0 + (1+k)\, 2\arctan\left[\sqrt{\frac{1+e_\theta}{1-e_\theta}}\tan\frac{u}{2}\right] \\ r_A = a_r\,(1 - e_r\cos u) \end{cases}$$

$$a_r = \frac{m_B}{M}\, a_R$$

VANISHING $O\left(\frac{v^2}{c^2}\right)$ CORRECTION HERE!

YIELDS $r_A\sin\theta$ IN TERMS OF ECCENTRIC ANOMALY u

USE OF THE PROPER TIME OF THE PULSAR

NEED TO EXPRESS $\Delta_R, \Delta_E, \ldots$ IN TERMS OF T RATHER THAN t

USE: $$t = T + \Delta_E$$

$x_A \equiv \frac{m_A}{M}$ $x_A + x_B \equiv 1$

$x_B \equiv \frac{m_B}{M}$

$$\Rightarrow T = t - \Delta_E = t - \frac{e}{n} \frac{G m_B (1 + x_B)}{c^2 a_R} \sin u$$

REPLACE $n(t - t_0) = u - e_t \sin u$

REMARKABLY OF SAME FORM

'PROPER TIME KEPLER EQUATION'

$$n(T - T_0) = u - \overset{\circ}{e}_T \sin u$$

COMES FROM Δ_E

$$\overset{\circ}{e}_T = e_t \left(1 + \frac{G m_B (1 + x_B)}{c^2 a_R}\right)$$

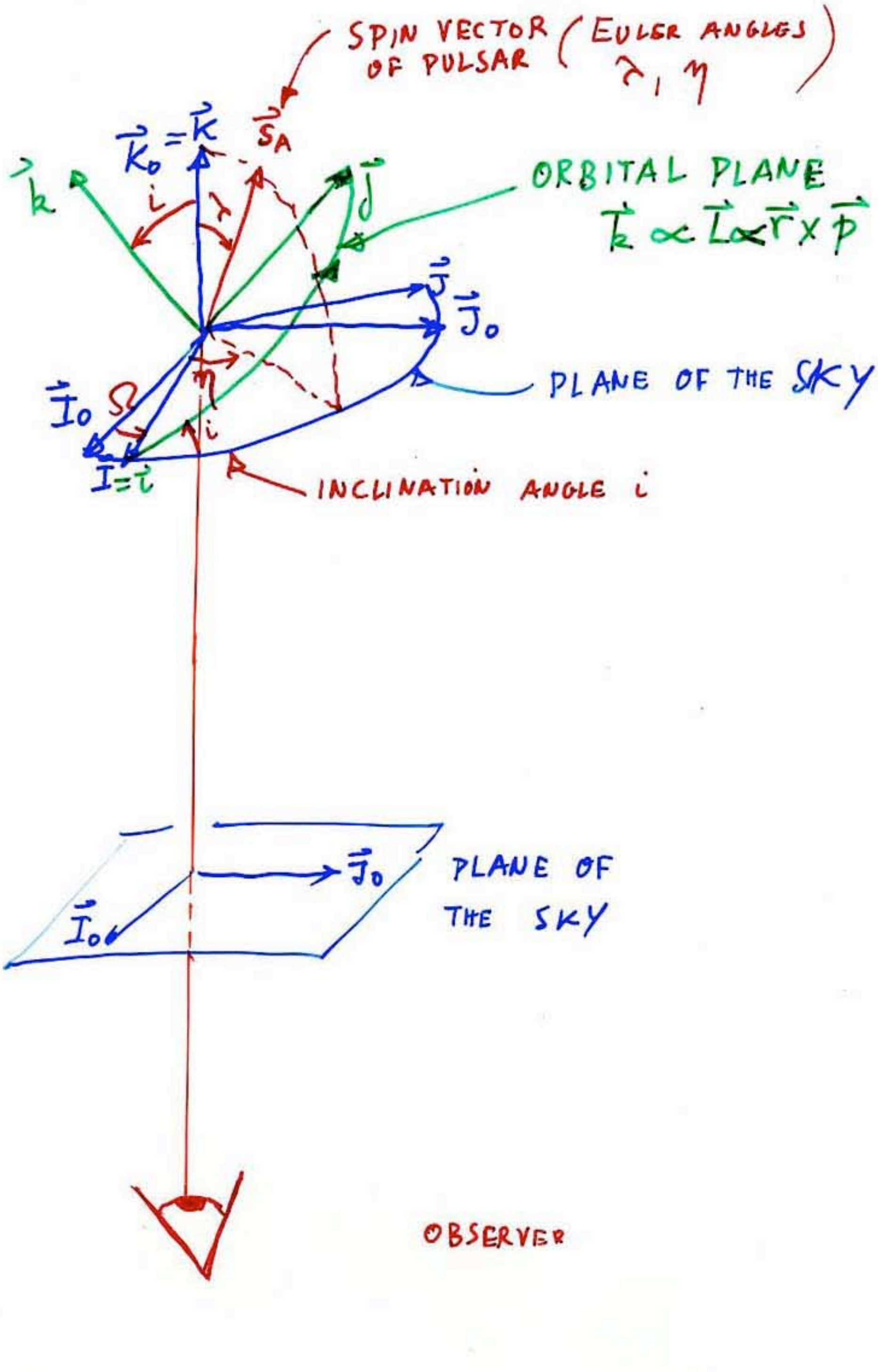
HOWEVER, THIS LEADS TO SLIGHTLY UNPLEASANT $(1 - e_r \cos u)/(1 - e_\theta \cos u)$

REMARKABLY: IT IS POSSIBLE TO FURTHER SIMPLIFY THE TIMING FORMULA BY DEFINING

$$\boxed{n(T - T_0) = u^{new} - e_T \sin u^{new} \qquad e_T \equiv \overset{\circ}{e}_T + e_\theta - e_r}$$

ORIENTATION CONVENTIONS

(Damour, Taylor 1992)



FINAL DD TIMING FORMULA

(Damour, Deruelle '86)

PROPER TIME OF ARRIVAL (INFINITE-FREQUENCY) AT SOLAR-SYSTEM BARYCENTER OF N^{TH} PULSE, $N = 0, 1, 2, 3, \dots$

$$N = N_0 + \nu_p T + \frac{1}{2}\dot{\nu}_p T^2 + \frac{1}{6}\ddot{\nu}_p T^3$$

DEFINES T_N

$$D\, \tau_{\substack{\text{arrival}\\ \text{barycenter}}} = T + \underbrace{\Delta_R(T)}_{\text{Roemer}} + \underbrace{\Delta_E(T)}_{\text{Einstein}} + \underbrace{\Delta_S(T)}_{\text{Shapiro}} + \underbrace{\Delta_A(T)}_{\text{Aberration}}$$

$$\Delta_R(T) = x \sin\omega\left[\cos u - e(1+\delta_r)\right] + x\sqrt{1-e^2(1+\delta_\theta)^2}\,\cos\omega \sin u$$

$$\Delta_E(T) = \gamma \sin u$$

$$\Delta_S(T) = -2r \ln\left\{1 - e\cos u - s\left[\sin\omega(\cos u - e) + \sqrt{1-e^2}\cos\omega\sin u\right]\right\}$$

$$\Delta_A(T) = A\left\{\sin\left[\omega + A_e(u)\right] + e\sin\omega\right\} + B\left\{\cos\left[\omega + A_e(u)\right] + e\cos\omega\right\}$$

WITH

$$x = x_0 + \dot{x}(T - T_0)$$

$$e = e_0 + \dot{e}(T - T_0) \qquad A_e(u) \equiv 2\arctan\left[\sqrt{\frac{1+e}{1-e}}\tan\frac{u}{2}\right]$$

$$\omega = \omega_0 + k\,A_e(u)$$

$$u - e\sin u = 2\pi\left[\frac{T - T_0}{P_b} - \frac{1}{2}\dot{P}_b\left(\frac{T-T_0}{P_b}\right)^2\right]$$

ORBITAL PARAMETERS

KEPLERIAN PARAMETERS

$x = a_A \sin i / c$

$$\{p^K\} = \{P_b, T_0, e_0, \omega_0, x_0\}$$

SEPARATELY MEASURABLE POST-KEPLERIAN PARAMETERS

$$\{p^{PK}\} = \{k, \gamma, \dot{P}_b, \underbrace{r, s}, \delta_\theta, \underbrace{\dot{e}, \dot{x}}\}$$

- k: FRACTIONAL PERIASTRON ADVANCE
- γ: EINSTEIN DELAY
- $\dot{P}_b$: SECULAR CHANGE OF ORBITAL PERIOD
- r, s: RANGE AND SHAPE OF SHAPIRO DELAY
- δ_θ: RELATIVISTIC DEFORMATION OF ELLIPTIC MOTION
- $\dot{e}, \dot{x}$: SECULAR VARIATIONS OF $x \simeq a_A \sin i$ AND eccentricity

NOT SEPARATELY MEASURABLE POST-KEPLERIAN PARAMETERS

$$\{q^{PK}\} = \{\delta_r, \underbrace{A, B}, D\}$$

- δ_r: RELATIVISTIC DEFORMATION OF ELLIPTIC MOTION
- A, B: ABERRATION
- D: DOPPLER FACTOR

USING A TIMING FORMULA WITH $D \to 1$

$\Leftrightarrow$ MEASURING

$$\left\{ \begin{aligned} a_R^{WRONG} &= a_R^{TRUE}/D \\ m^{WRONG} &= m^{TRUE}/D \\ P_b^{WRONG} &= P_b^{TRUE}/D \end{aligned} \right.$$

DOES NOT AFFECT TESTS OF GR. IMPORTANT ONLY FOR P_b WHEN D VARIES (Damour, Taylor '91)

DDGR TIMING FORMULA

IN GENERAL RELATIVITY ALL POST-KEPLERIAN (PK) PARAMETERS ARE FUNCTIONS OF THE KEPLERIAN ONES AND THE TWO MASSES m_A, m_B (+ THE SPIN PARAMETERS λ, η)

E.G. IF ONE DEFINES

$M \equiv m_A + m_B$

$x_A \equiv m_A/M$

$x_B \equiv m_B/M \equiv 1 - x_A$

$x_A x_B \equiv \frac{m_A m_B}{(m_A+m_B)^2} \equiv \nu$

$n \equiv 2\pi/P_b$

$$\boxed{\beta_0(M) \equiv \left(\frac{G M n}{c^3}\right)^{1/3} \sim \frac{v_{orbital}}{c}}$$

ACTUALLY ONLY $\frac{GM}{c^3} = \frac{M}{M_\odot}\frac{GM_\odot}{c^3} \equiv \frac{M}{M_\odot} 4.925490947 \mu s$

$$k^{GR}(M) = 3\,\frac{\beta_0^2(M)}{1-e^2}$$

$$\frac{n}{e}\gamma^{GR}(m_A,m_B) \equiv \delta^{GR}(m_A,m_B) = x_B(1+x_B)\,\beta_0^2(M)$$

$$r^{GR}(m_B) = \frac{G m_B}{c^3}$$

$$s^{GR}(m_A,m_B) = \frac{x\, n}{x_B\,\beta_0(M)} \quad (= \sin i)$$

$$\dot{P}_b^{GR}(m_A,m_B) = -\frac{192\pi}{5}\, x_A x_B\, \beta_0^5(M)\,\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}}$$

$$\delta_\theta^{GR}(m_A,m_B) = \left(\frac{7}{2}x_A^2 + 6\,x_A x_B + 2\,x_B^2\right)\beta_0^2(M)$$

PULSE-STRUCTURE PARAMETRIZED FORMULA

(Damour, Taylor 1992)

EFFECT OF PULSAR ORBITAL MOTION ON DIRECTIONAL SPECTRAL LUMINOSITY RECEIVED BY OBSERVER

$$\ell(\nu_{\rm OBS}, \vec{n}, T) = \underbrace{(1+\vec{n}\cdot\vec{\beta}_A)^3}_{\text{MODULATES FLUX DENSITY}} L_{\rm PROPER}\Big(\underbrace{\frac{\nu_{\rm OBS}}{1+\vec{n}\cdot\vec{\beta}_A}}_{\text{Doppler shift of obs. frequency}}, \underbrace{\vec{n}(1+\vec{n}\cdot\vec{\beta}_A)-\vec{\beta}_A}_{\text{Aberration of direction of emission}}, T\Big)$$

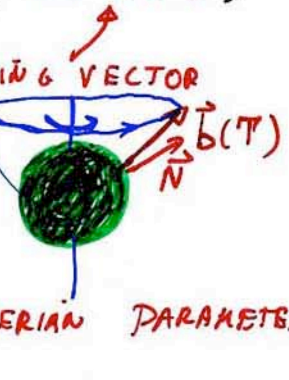
$$\vec{\beta}_A \equiv \frac{\vec{v}_A}{c}$$

NEED A MODEL OF PULSAR EMISSION

E.G. ROTATING VECTOR MODEL:

$$L_{\rm PROPER}(\nu, \vec{N}, T) = F(\nu, \vec{N}\cdot\vec{b}(T))$$

ROTATING VECTOR



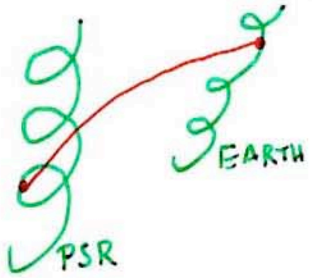
$\Rightarrow$ NEW SET OF 'PULSE-STRUCTURE' "POST-KEPLERIAN PARAMETERS"

$$\{\tilde{p}^{PK}\} = \{\lambda, \dot{\lambda}, \kappa, \dot{\kappa}, \sigma, \dot{\sigma}, \varphi_0, \kappa', \dot{\kappa}', \sigma', \dot{\sigma}'\}$$

$\kappa = \dfrac{\cos\eta}{\sin i}$ $\quad \sigma = \cot i \sin\eta$ $\quad \kappa' = \cot\lambda \cot i \cos\eta$ $\quad \sigma' = \cot\lambda \dfrac{\sin\eta}{\sin i}$

SUMMARIZING

- TIMING FORMULA = A PRIME EXAMPLE OF A COORDINATE-INDEPENDENT PHYSICAL OBSERVABLE:



$$\tau_{\text{ARRIVAL}} = F\left[T_{\text{PSR}} \right]$$

τ_{ARRIVAL}: PROPER TIME; T_{PSR}: PROPER TIME

- $\exists$ REMARKABLY SIMPLE UNIVERSAL TIMING FORMULA (1PK ACCURACY) WHICH IS AN EXPLICIT FUNCTION OF

  5 KEPLERIAN PARAMETERS: P_b, T_0, e_0, ω_0, $x_0 = (a_A \sin i)_0$

  8 POST-KEPLERIAN PAR.: k, γ, $\dot{P}_b$, r, s, δ_θ, $\dot{e}$, $\dot{x}$

  [+ 4 NOT SEPARATELY MEASURABLE ONES: δ_r, A, B, D]

- IN GENERAL RELATIVITY STRONG-FIELD EFFECTS ARE EFFACED AND

$$p_i^{PK} = p_i^{GR}\left[m_A, m_B; \lambda, \eta; p^K \right]$$

- IF ONE ASSUMES SOME GENERAL MODEL OF PULSAR EMISSION ONE CAN ALSO WRITE A PARAMETRIZED FORMULA FOR PULSE STRUCTURE. IT INVOLVES UP TO 11 PHENOMENOLOGICAL PK PARAMETERS $\tilde{p}^{PK}$

- IN GENERAL RELATIVITY

$$\tilde{p}_i^{PK} = \tilde{p}_i^{GR}\left[m_A, m_B; \lambda, \eta; p^K \right]$$

E.G. $$\dot{\lambda} = -\sin i \cos\eta \, x_A x_B \left(2 + \frac{3 m_B}{2 m_A} \right) \frac{\beta_0^2(M)\, n}{1-e^2}$$ DUE TO GR SPIN-ORBIT COUPLING

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