Villa Olmo
May 17-21, 2005

BINARY SYSTEMS AS TEST-BED OF GRAVITY THEORIES

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IHES

LECTURE N° 3

PHENOMENOLOGICAL APPROACH TO TESTING RELATIVISTIC GRAVITY WITH BINARY PULSAR DATA

TESTING RELATIVISTIC GRAVITY WITH BINARY PULSARS

WHY TEST RELATIVISTIC GRAVITY ?

- GENERAL RELATIVITY = ONE OF THE PILLARS OF MODERN PHYSICS
- CHALLENGE : RECONCILE GR WITH QUANTUM THEORY

$\Rightarrow$ POSSIBLE LONG-RANGE MODIFICATIONS OF GR.

E.G. EXTRA-DIMENSIONS $\rightarrow$ KALUZA-KLEIN PARTNERS OF $g_{\mu\nu}$

STRING THEORY $\rightarrow$ RICHER GRAVITATIONAL SECTOR

$$g_{\mu\nu} + B_{\mu\nu} + \Phi + KK \text{ MODULI ETC...}$$

- $\exists$ 'TWO DARK CLOUDS' IN COSMOLOGY :

"DARK MATTER" + "DARK ENERGY"

? SIGNALS OF MODIFICATIONS OF GR IN THE INFRA RED ?

- USEFUL TO CONTRAST OBSERVATIONAL PREDICTIONS

$\rightarrow$ DEEPER UNDERSTANDING OF WHAT IS REALLY TESTED IN GR

WHY USE BINARY PULSARS TO TEST GRAVITY?

- NEW ARENA $\neq$ SOLAR SYSTEM OR COSMOLOGY
- UNIQUE IN TESTING STRONG-FIELD REGIME OF RELATIVISTIC GRAVITY
- ALLOWS TO PROBE RADIATIVE ASPECTS OF GRAVITY
- HIGH ACCURACY OF PULSAR MEASUREMENTS

  $\longrightarrow$ HIGH-PRECISION TESTS OF GRAVITY

HOW TO USE BINARY PULSARS TO TEST GRAVITY?

TWO APPROACHES

- 'PHENOMENOLOGICAL': 'COMPARE'

$$P_i^{OBS} \stackrel{?}{=} P_i^{THEORY}$$

  - PARAMETRIZED POST-KEPLERIAN FORMALISM
  - OTHER PARAMETRIZED POST-EINSTEINIAN APPROACHES

- 'THEORY-SPACE': 'CONTRAST'

GOODNESS OF FIT

$$\chi^2(\beta_a) = \sum_{n \in \text{DATA SET}} \left(\frac{x_n^{OBS} - x_n^{THEORY}(\beta_a)}{\sigma_n^{OBS}} \right)^2$$

  - PARAMETERS BEYOND USUAL 'POST-NEWTONIAN' ONES
  - CONTINUOUS FAMILIES OF ALTERNATIVE THEORIES OF GRAVITY

METHODOLOGICAL REMARKS

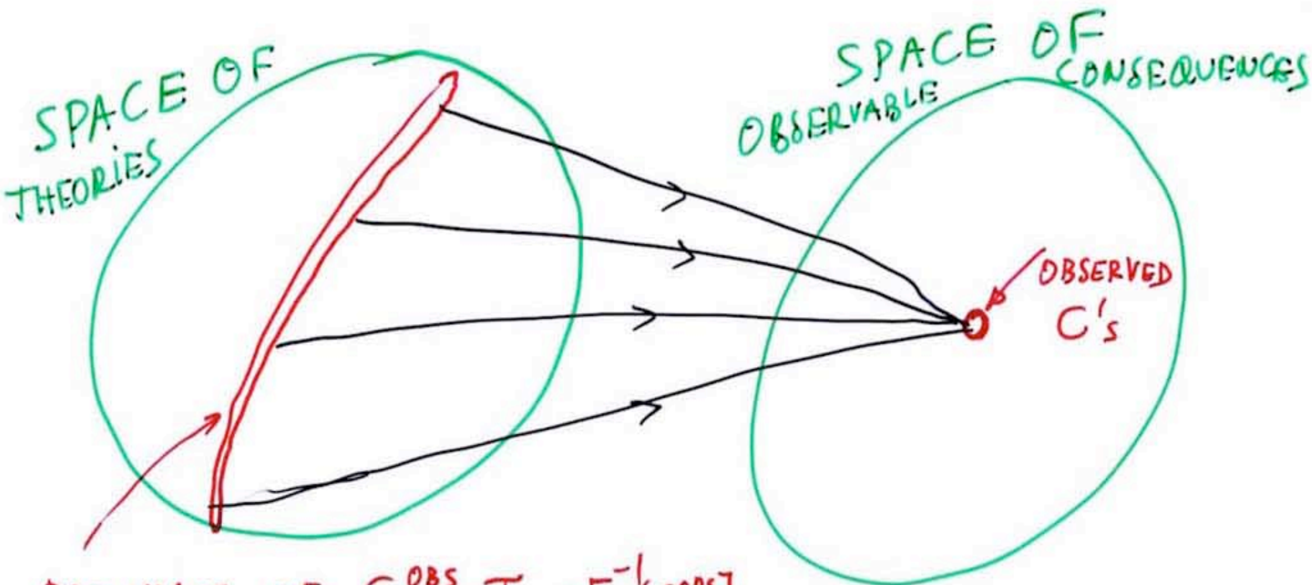
IF $\mathbb{T} \equiv$ A THEORY
$C \equiv$ SOME CONSEQUENCE

LOGICALLY

$$(\mathbb{T} \Rightarrow C) \Leftrightarrow (\text{NON } C \Rightarrow \text{NON } \mathbb{T})$$

THIS IS THE RATIONALE FOR SAYING THAT EXPERIMENTS CAN ONLY
FALSIFY A THEORY, BUT NOT VERIFY IT

HOWEVER, A SCIENTIFIC RATIONALE FOR EXPLORING WHAT STRUCTURES OF A THEORY ARE VERIFIED BY EXPERIMENT IS:



PRE-IMAGE OF C^{OBS}: $T_{C^{OBS}} \equiv F^{-1}[C^{OBS}]$

WE CAN CONCLUDE THAT THE COMMON FEATURES (IF ANY) OT $\mathbb{T}_{C^{OBS}} \equiv F^{-1}[C^{OBS}]$ ARE "TRUE"

PHENOMENOLOGICAL APPROACH TO TESTING RELATIVISTIC GRAVITY

'COMPARE' $\boxed{P_i^{OBS} \stackrel{?}{=} P_i^{THEORY}}$

EXAMPLE: $\dot{\omega}^{OBS}_{MERCURY} \stackrel{?}{=} \dot{\omega}^{NEWTON}_{MERCURY}$

TWO REMARKS

- $\dot{\omega}$ IS A PHENOMENOLOGICAL PARAMETER WHICH HAS MEANING IN A LARGER FRAMEWORK THAN NEWTON'S THEORY:

  'KINEMATICAL PARAMETRIZATION': $\omega_{PLANET}(t) = \omega_0 + t\dot{\omega}_0 + \frac{1}{2}t^2\ddot{\omega}_0 + \dots$

  + PERIODIC EFFECTS

- $\dot{\omega}^{NEWTON}_{MERCURY} = F$ [UNDERLYING MODEL AND THEORY PARAMETERS]

  E.G. m_i^{PLANET}

1859 LEVERRIER FINDS (ROUGHLY) [AFTER SUBTRACTION PRECESSION OF EQUINOXIA $\sim 5000''/cy$]

$$\dot{\omega}^{OBS}_{MERCURY} \simeq 573''$$

$$\dot{\omega}^{NEWTON}_{MERCURY} \simeq \underbrace{280''\frac{m_{VENUS}}{m_{VENUS}^{GLOBAL\ FIT}} + 90''\frac{m_{EARTH}}{m_{EARTH}^{GL.\ FIT}} + 155''\frac{m_{JUPITER}}{m_{JUP}^{G\ FIT}} + 3''\frac{m_{MARS}}{m_{MARS}^{FIT}} + 7''\frac{m_{SATURN}}{m_{SATURN}^{FIT}}}_{\simeq 535'' = 573'' - 38''}$$

93% OF $\dot{\omega}^{OBS}_{MERCURY}$ IS EXPLAINED BY NEWTON, COULD EXPLAIN ALL BY CHANGING m_{VENUS} BY $\simeq +13.5\%$

BUT LEVERRIER WAS CONFIDENT IN HIS GLOBAL FIT OF SOLAR SYSTEM; $\Rightarrow$ HE CONJECTURED $\exists$ INTRA-MERCURIAL PLANET, 'VULCAN'

DURING THE 19^{TH} CENTURY:

- NO EVIDENCE FOR ANY INTRA-MERCURIAL PLANET
- OTHER NEWTONIAN EXPLANATIONS EXPLORED:
  RING OF MATTER, ZODIACAL LIGHT, ...
- SOME 'THEORY SPACE' EXPLORED:

EG: $\frac{1}{r^2} \longrightarrow \frac{1}{r^n}$ BUT MOON?

'$\frac{v^2}{c^2}$' CORRECTIONS TO NEWTON: $\frac{1}{r^2}\left[1+\mathcal{O}\left(\frac{v^2}{c^2}\right)\right]$

E.G. LORENTZ 1900
POINCARÉ 1906: CLASSES OF SPECIAL RELATIVISTIC THEORIES OF GRAVITATION

COULD HAVE SUGGESTED A GOOD DIRECTION

EVEN: 1898, GERBER BY SOME $\frac{1}{r^2}\left[1+\mathcal{O}\left(\frac{v^2}{c^2}\right)\right]$
GOT THE 'EXACT GR ANSWER'

$$k = \left(\frac{\Delta\omega}{2\pi}\right)_{\text{PER REVOLUTION}} = \frac{3\,GM}{c^2 a(1-e^2)} \quad !$$

- GR CONVINCED MOST PEOPLE AFTER 1919, IE.
  WHEN BOTH $\dot{\omega}_{\text{MERCURY}}$ AND $(\Delta\theta)^{\text{LIGHT}}_{\text{DEFLECTION}}$
  AGREED WITH EXPERIMENT

PHENOMENOLOGICAL ANALYSIS OF BINARY PULSAR DATA

'PARAMETRIZED POST-KEPLERIAN FORMALISM'

PPK Blandford, Teukolsky '76, Damour, Deruelle '86, Damour '88, Damour, Taylor '92

BASED ON THE THEORY-INDEPENDENT (OR 'KINEMATICAL') STRUCTURE OF FINAL TIMING FORMULA

RAW PULSAR DATA $\{t_N^{OBS}\}$

OR FLUX: $\{\mathcal{F}(t)\}$

$5\ p^K$ $\quad 8\ p^{PK}$

$$t_N^{PPK} = F[N, p_i^K, p_i^{PK}]$$

$$\text{OR } \mathcal{F}^{PPK}(t) = F'[t, p_i^K, \tilde{p}_i^{PK}]$$

$11\ \tilde{p}^{PK}$

FIT

$$p_i^{K\,OBS} = \cdots \pm \cdots$$

$$p_i^{PK\,OBS} = \cdots \pm \cdots$$

PHENOMENOLOGICAL ANALYSIS OF TIMING DATA OF 'SINGLE-LINE' BINARY PSR

IN PRINCIPLE CAN MEASURE 5 KEPLERIAN PARAMETERS

$$\{p^K\} = \{P_b, T_0, e_0, \omega_0, x_0\}$$

AND UP TO 8 POST-KEPLERIAN PARAMETERS

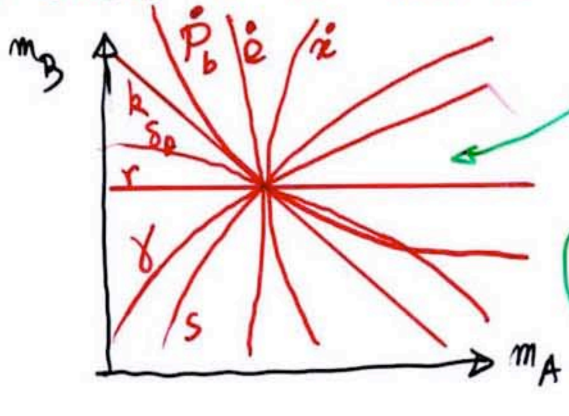
$$\{p^{PK}\} = \{k, \gamma, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\}$$

IN GR

$$\boxed{\underset{i=1,\dots,8}{p_i^{PK}} = f_i^{GR}(m_A, m_B (, \lambda, \eta); p^K)}$$

TIMED PSR

$\Rightarrow$ 8 CURVES IN THE MASS PLANE (m_A, m_B)



CAN USE TWO OF THEM

$$\begin{cases} p_1^{PK} = f_1^{GR}(m_A, m_B) \\ p_2^{PK} = f_2^{GR}(m_A, m_B) \end{cases} \rightarrow \begin{cases} m_A = m_A^{GR}(p_1^{PK}, p_2^{PK}, p^K) \\ m_B = m_B^{GR}(p_1^{PK}, p_2^{PK}, p^K) \end{cases}$$

$8 - 2 = 6$ TESTS OF GENERAL RELATIVITY

$$\boxed{\underset{i'=3,4,\dots,8}{p_{i'}^{PK\,\mathrm{OBS}}} \overset{?}{=} f_{i'}^{GR}\left(m_A^{GR}(p_1^{PK}, p_2^{PK}), m_B^{GR}(p_1^{PK}, p_2^{PK})\right)}$$

RELATIVISTIC TIMING FORMULA

Damour and Deruelle [36, 47] proved that it is possible to describe all of the independent $O(v^2/c^2)$ timing effects in a simple mathematical way common to a wide class of alternative theories. This made it possible to revert to a theory-independent analysis of timing data, and led to the possibility of working within a strong-field analog of the PPN formalism, the so-called [37] "parametrized post-Keplerian" approach. The part of the Damour–Deruelle phenomenological timing model describing orbital effects reads

$$\boxed{t_b - t_0 = F[T; \{p^{\mathrm{K}}\}; \{p^{\mathrm{PK}}\}; \{q^{\mathrm{PK}}\}]\ ,} \tag{2.1a}$$

where t_b denotes the solar-system barycentric (infinite frequency) arrival time, T the pulsar proper time (corrected for aberration, see below),

$$\{p^{\mathrm{K}}\} = \{P_b, T_0, e_0, \omega_0, x_0\} \tag{2.1b}$$

is the set of Keplerian parameters,

$$\boxed{\{p^{\mathrm{PK}}\} = \{k, \gamma, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\}} \tag{2.1c}$$

the set of *separately measurable* post-Keplerian parameters, and

$$\{q^{\mathrm{PK}}\} = \{\delta_r, A, B, D\} \tag{2.1d}$$

the set of *not separately measurable* post-Keplerian parameters. The right hand side of Eq. (2.1a) is given by

$$F(T) = D^{-1}[T + \Delta_R(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T)]\ , \tag{2.2a}$$

$$\Delta_R = x \sin\omega[\cos u - e(1+\delta_r)] + x[1 - e^2(1+\delta_\theta)^2]^{1/2} \cos\omega \sin u\ , \tag{2.2b}$$

$$\Delta_E = \gamma \sin u\ , \tag{2.2c}$$

$$\Delta_S = -2r \ln\{1 - e\cos u - s[\sin\omega(\cos u - e) + (1-e^2)^{1/2}\cos\omega \sin u]\}\ , \tag{2.2d}$$

$$\Delta_A = A\{\sin[\omega + A_e(u)] + e\sin\omega\} + B\{\cos[\omega + A_e(u)] + e\cos\omega\}\ , \tag{2.2e}$$

where

$$x = x_0 + \dot{x}(T - T_0)\ , \tag{2.3a}$$

$$e = e_0 + \dot{e}(T - T_0)\ , \tag{2.3b}$$

and where $A_e(u)$ and ω are the following functions of u,

$$A_e(u) = 2\arctan\left[\left(\frac{1+e}{1-e}\right)^{1/2} \tan\frac{u}{2}\right]\ , \tag{2.3c}$$

$$\omega = \omega_0 + k\, A_e(u)\ , \tag{2.3d}$$

and u is the function of T defined by solving the Kepler equation

$$u - e\sin u = 2\pi\left[\left(\frac{T-T_0}{P_b}\right) - \frac{1}{2}\dot{P}_b\left(\frac{T-T_0}{P_b}\right)^2\right]\ . \tag{2.3e}$$

EACH SUCH TEST IS SIMILAR TO

$$\dot{\omega}^{OBS}_{MERCURY} \overset{?}{=} \dot{\omega}^{THEORY}_{MERCURY}\left(m_V^{BEST\,FIT}, m_J^{BEST\,FIT}, \dots\right)$$

AND IS A POTENTIAL KILLER OF GR

+ PHENOMENOLOGICAL ANALYSIS OF PULSE-STRUCTURE DATA OF (SINGLE) BINARY PSR

IN PRINCIPLE CAN MEASURE UP TO 11 'POST-KEPLERIAN PARAMETERS'

$$\{\tilde{p}^{PK}\} = \{\lambda, \dot{\lambda}, \kappa, \dot{\kappa}, \sigma, \dot{\sigma}, \dot{\psi}_0, \kappa', \dot{\kappa}', \sigma', \dot{\sigma}'\}$$

$$\{\underset{\substack{\uparrow \\ 8 \\ \text{TIMING}}}{p^{PK}}, \underset{\substack{\uparrow \\ 11 \\ \text{PULSE STRUCTURE}}}{\tilde{p}^{PK}}\} = \underset{j=1,\dots,19}{P_j^{PK}} = f_j^{GR}(m_A, m_B, \lambda, \eta; PK)$$

$$19 - 4 = 15 \quad \text{TESTS OF GR}$$

$$\underset{j'=5\dots19}{P_{j'}^{PK\,OBS}} \overset{?}{=} f_{j'}^{GR}\left(m_A(P_1^{PK} P_2^{PK} P_3^{PK} P_4^{PK}), m_B(P_1^{PK} \cdots P_4^{PK}), \lambda(P_1^{PK} \cdots), \eta(P_1^{PK} \cdots)\right)$$

PHENOMENOLOGICAL ANALYSIS OF TIMING DATA OF 'DOUBLE-LINE' BINARY PULSAR

SUCH AS PSR J 0737−3039A+B
(Burgay et al '03, Lyne et al '04)

IN ADDITION TO PK PARAMETERS (ONLY MEASURABLE FOR A IN 0737)
CAN ALSO USE KEPLERIAN PARAMETERS

NOTABLY

$$x_A = a_A \sin i / c$$
$$x_B = a_B \sin i / c$$

$$\Rightarrow \quad \boxed{R^{OBS} \equiv \frac{x_B}{x_A}} \quad \Rightarrow \quad \boxed{R^{GR} = \frac{m_A}{m_B} + \mathcal{O}\left(\frac{v^3}{c^3} \text{ or } \frac{v^4}{c^4}\right)}$$

(Lyne et al '04)

FROM DD'86
$\nexists$ v^2/c^2 CORRECTIONS

IN PRINCIPLE ONE CAN ALSO GET A NEW TEST OF GR BY USING THE RATIO OF DD-TIMING ECCENTRICITIES:

$$\boxed{E^{OBS} \equiv \frac{e_B^{OBS}}{e_A^{OBS}}} \qquad \boxed{E^{GR} = 1 + c^{GR}\left(\frac{GMn}{c^3}\right)^{\frac{2}{3}}\frac{m_A - m_B}{M}}$$

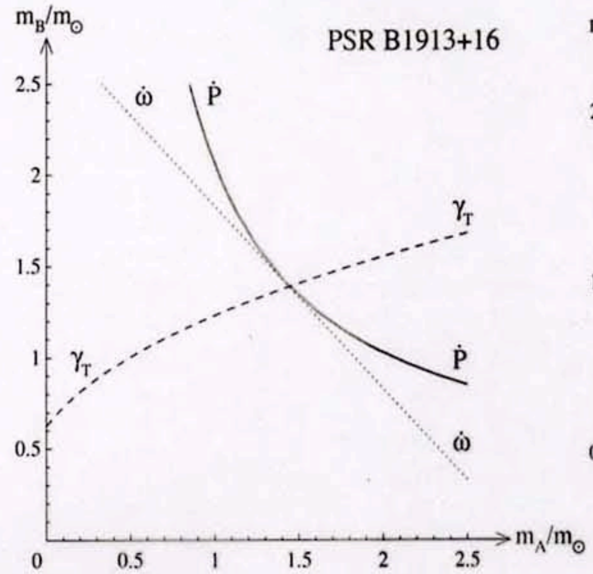
(Damour, Esposito-Farèse '05)

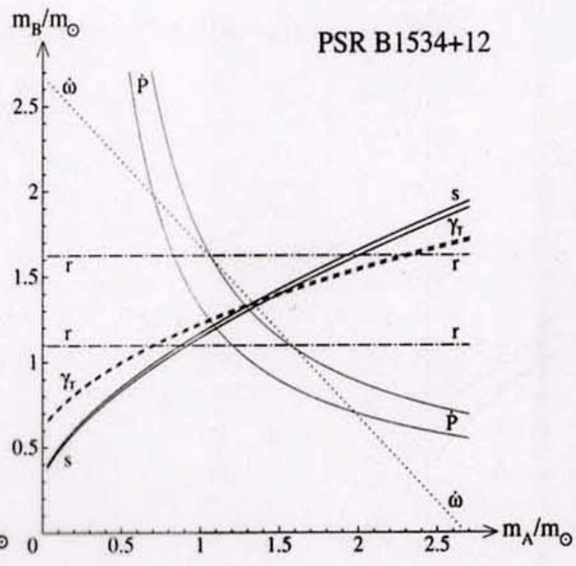
HOWEVER QUITE SMALL $\sim \frac{\Delta m}{m}\left(\frac{v}{c}\right)^2$
$\sim 2 \times 10^{-7}$ FOR 0737

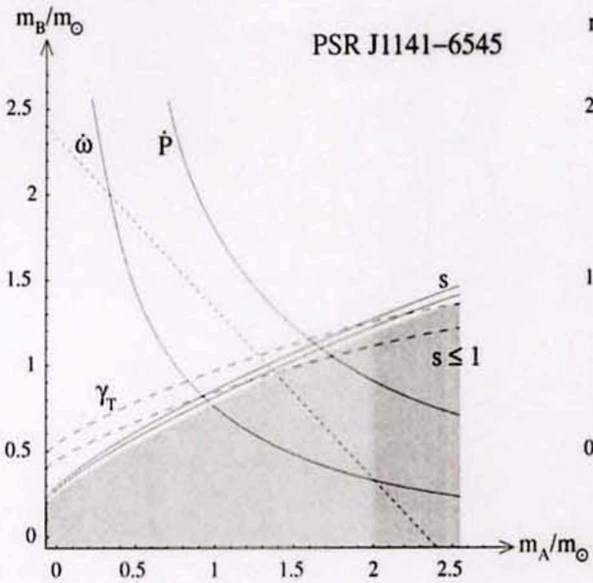
T6

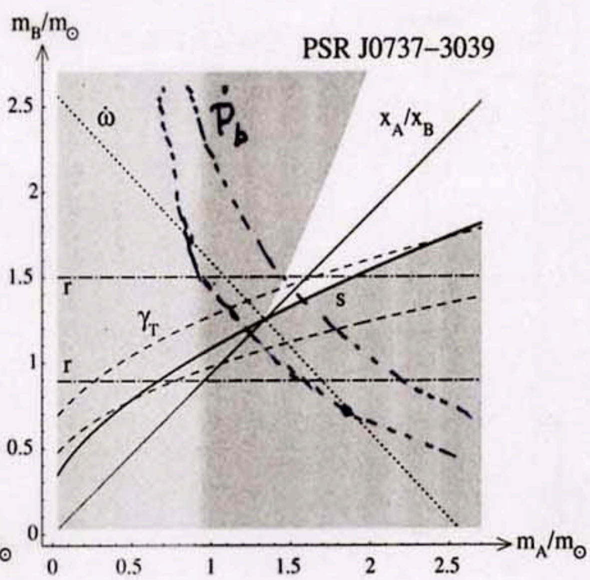
3-2 = 1

5-2 = 3







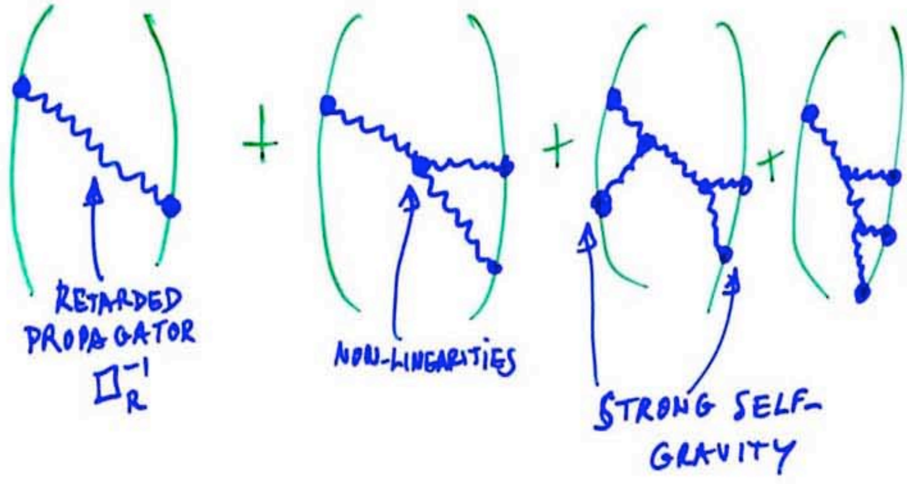


4-2 = 2

6-2 = 4

REMEMBERING THAT IN GR

$\dot{P}_b \Leftarrow \left(\frac{v}{c}\right)^5$:



- A TEST INVOLVING $\dot{P}_b$ PROBES, IN A COMBINED MANNER, RADIATIVE + STRONG SELF-GRAVITY + NON-LINEAR FEATURES OF GR

$\exists$ 4 SUCH SUCCESFUL TESTS $\dot{P}_b^{OBS} = \dot{P}_b^{GR}$

NOTE IN PASSING THE NEED TO CORRECT $\dot{P}_b^{OBS}$ 1913+16 FOR GALACTIC ACCELERATION EFFECTS (Damour Taylor '91)

$$\dot{P}_b^{OBS}{}_{CORRECTED} = -2.4211(14)\times 10^{-12} + 0.0125(50)\times 10^{-12} = -2.4086(52)\times 10^{-12}$$

$$\dot{P}_b^{GR}(k^{OBS}, \gamma^{OBS}) = -2.40247(2)\times 10^{-12}$$ IF NOT 13σ DISCREPANCY !

(Weisberg Taylor '02)

- THE TESTS INVOLVING THE OTHER PK (and K) PARAMETERS PROBE THE QUASI-STATIC STRONG-FIELD REGIME OF GR

FIRST SUCH TESTS : r, s IN PSR 1534+12

$r^{OBS} = 6.2(1.3)\,\mu s \simeq r^{GR} = 6.58\,\mu s$ (Taylor, Wolszczan, Damour, Weisberg '92)

$s^{OBS} = 0.986(7) \simeq s^{GR} = 0.982$

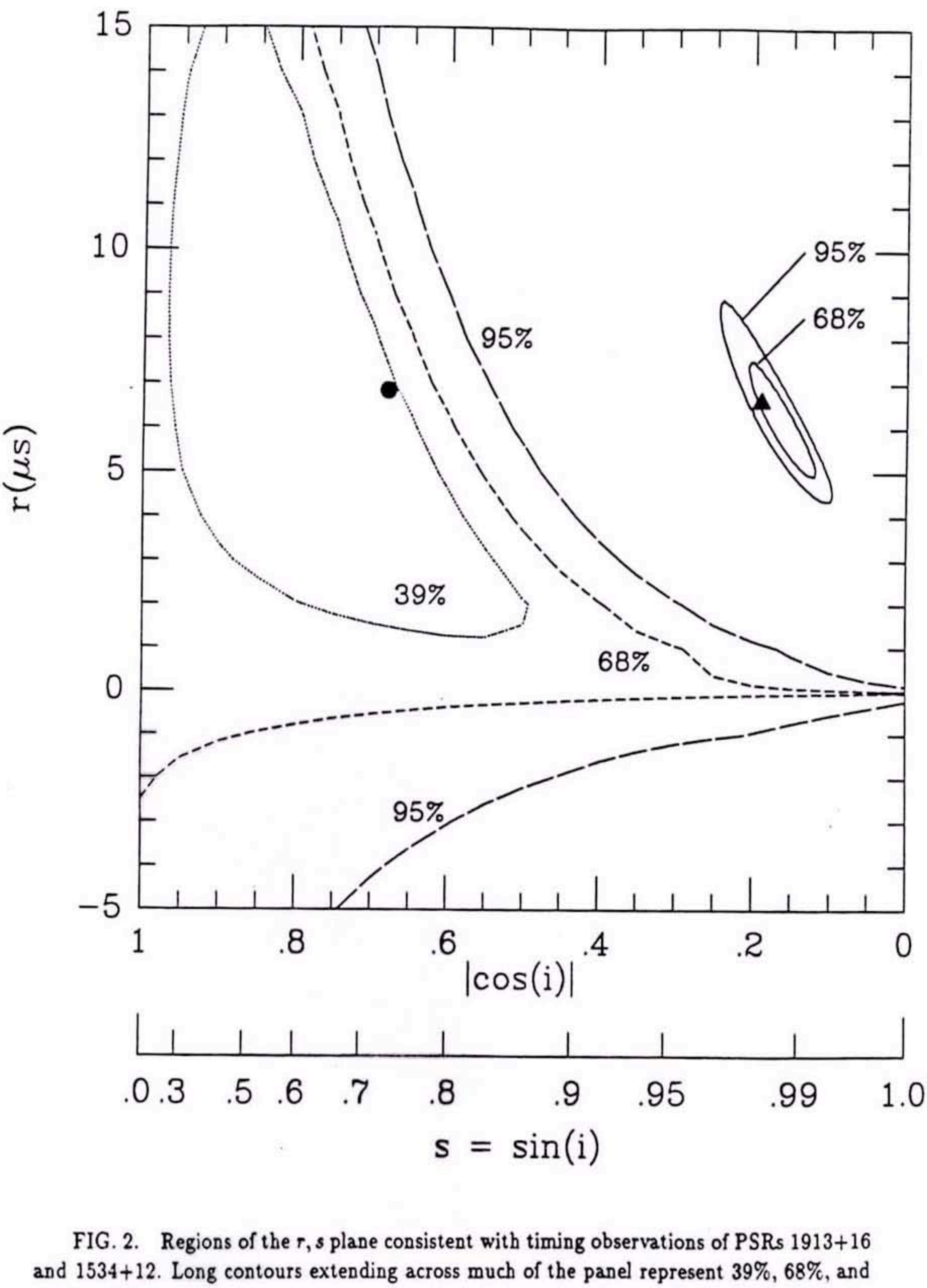


FIG. 2. Regions of the r, s plane consistent with timing observations of PSRs 1913+16 and 1534+12. Long contours extending across much of the panel represent 39%, 68%, and 95% confidence regions for the r and s parameters of PSR 1913+16. The values consistent with general relativity are indicated by a filled circle at $r = 6.83\,\mu s$, $s = 0.734$. Closed contours surrounding the triangle at the right represent 68% and 95% confidence intervals for r and s in the PSR 1534+12 system. The triangle marks the general relativistic prediction at $r = 6.58\,\mu s$, $s = 0.982$.

(Taylor, Wolszczan, Damour, Weisberg '92)

PULSE-STRUCTURE DATA

NO THEORY-INDEPENDENT (PHENOMENOLOGICAL) TESTS YET,

BUT CONFIRMATION OF REALITY OF SPIN-ORBIT COUPLING IN GR

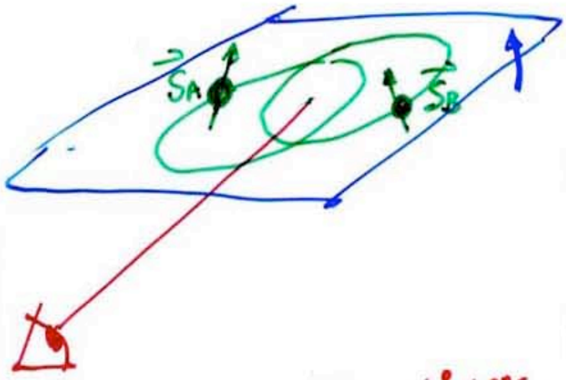
PREDICTION: Damour, Ruffini '74

FULL CALCULATION: Barker, O'Connel '75

OBSERVATION IN 1913+16: Kramer '98 [Weisberg, Romani, Taylor '89]; Weisberg Taylor '02

IN 1141-6545 Hotan, Bailes, Ord '05

NOTE: ∃ SEVERAL OBSERVABLE EFFECTS LINKED TO GR SPIN-ORBIT COUPLING



- SPIN PRECESSION ⇒ EFFECT ON PULSE STRUCTURE
- ORBITAL EFFECTS: δk Barker O'Connel '75; δ $\sin i$ Damour Schäfer '88

⇓

SEVERAL INDIRECT EFFECTS ON PK OBSERVABLES

E.G. $\left(\frac{\dot{e}}{e}\right)^{OBS} \ni \frac{d}{dt}\varepsilon_A$ ABERRATION — Damour, Taylor '92

$$\left(\frac{\dot{x}}{x}\right)^{OBS} \ni \cot i \left(\frac{di}{dt}\right)^{SO} + \frac{d\,\varepsilon_A}{dt}$$

T2

RELATIVITÉ. — *Sur certaines vérifications nouvelles de la Relativité générale rendues possibles par la découverte d'un pulsar membre d'un système binaire.* Note (*) de

MM. **Thibaut Damour** et **Remo Ruffini**, présentée par M. André Lichnerowicz.

Cette Note montre comment la récente découverte d'un pulsar membre d'un système binaire pourrait fournir des informations très importantes pour la vérification de la Relativité générale ainsi que pour la connaissance de la structure et des processus d'émission des pulsars. On présente des estimations des principaux effets relativistes que l'on peut espérer observer dans un tel système.

Récemment J.H. Taylor [1] a découvert une nouvelle radiosource pulsante (*cf.* tableau pour les paramètres) membre d'un système binaire (comme l'atteste la variation régulière de la période de pulsation P sur des temps $\tau \sim 8$ h). Il est important de vérifier (*a*) que cet objet est bien un pulsar [2], c'est-à-dire que sa période P doit croître lentement avec le temps et satisfaire de plus l'inégalité $(dE/dt)_{em} \leqq I\,(4\,\pi^2/P^3)\,(dP/dt)$, où I est le moment d'inertie de l'étoile à neutron et $(dE/dt)_{em}$ la puissance émise par le pulsar, (*b*) et ensuite que le compagnon de cet objet est une étoile compacte : naine blanche, étoile à neutron ou black-hole. Si le compagnon n'est pas une étoile compacte les « marées » induites le doteraient d'un moment quadrupolaire qui se signalerait par une rapide précession du périastre (360°/an) [3].

Si ces deux conditions sont remplies, alors on disposera dans cet objet du meilleur outil depuis l'observation des sources de rayons X membres d'un système binaire pour l'étude de la structure des corps compacts plongés dans des champs gravitationnels intenses.

où le deuxième membre est évalué au point d'émission. Le terme $(1 + v^i n_i)$ donne l'effet Doppler ordinaire qui est ici modifié entre autres par l'effet Einstein $(-g_{00})$ et par l'effet transverse $(-g_{ik}\, v^i\, v^k)$. Le dernier terme enfin, usuellement plus faible que les autres peut devenir important si le signal lumineux passe près du compagnon de masse M_2 et de vitesse v_2 (il faut que l'inclinaison $i \simeq 90°$) on peut alors l'estimer à $u^0_{em}\,(4\,M_2/b_0)\,(v^b_2 - v^b_{em})$ où b_0 est le paramètre d'impact mesuré sur un axe b abaissé du compagnon sur le rayon.

Le périastre précesse avec la vitesse angulaire

$$\Omega_p \simeq 3\,\Omega_L\,(M_1 + M_2)/a\,(1 - e^2).$$

où $\Omega_L \simeq (M_1 + M_2)^{1/2}\, a^{-3/2}$ est la vitesse angulaire moyenne sur l'orbite, M_1 est la masse du pulsar, M_2 celle du compagnon et a le demi-grand axe de l'orbite relative. Le tableau montre que cet effet est appréciable. Appréciable aussi (tableau) sera la précession du moment cinétique propre du pulsar induite par le couplage spin-orbite :

$$\Omega_{SL} \simeq 3\,\Omega_L\,M_2/\{2\,a\,(1 - e^2)\}.$$

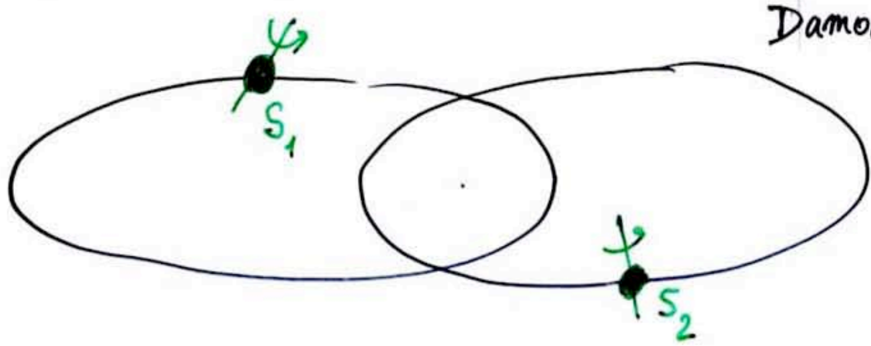
C'est précisément cet effet (ainsi que celui dont on parle plus bas) que cherche à mesurer une expérience terrestre dont la conception est due à Schiff. Cet effet est du plus grand intérêt non seulement pour la vérification de la Relativité générale mais encore pour la connaissance de la structure des étoiles à neutron. En effet pour la première fois il sera possible d'observer les processus d'émission d'un pulsar à des angles variables. Si comme cela a été théoriquement postulé, la radiation est émise dans des cônes issus des pôles magnétiques l'intensité observée sera modulée par la fréquence Ω_{SL}. Elle pourra même s'annuler périodiquement si le faisceau émis est assez étroit.

TABLEAU

Paramètres du pulsar binaire correspondant à une masse de l'étoile à neutron de 0,7 $M_\odot$ en fonction de l'inclinaison i de l'orbite

SPIN-ORBIT COUPLING AND MOMENTS OF INERTIA OF NEUTRON STARS

Damour, Schäfer '88



$$\vec{a} \sim \frac{GM\vec{n}}{r^2}\left[1 + \frac{v^2}{c^2} + \frac{v^4}{c^4}\right] + \text{SPIN-ORBIT TERMS} + \text{SPIN-SPIN ONES}$$

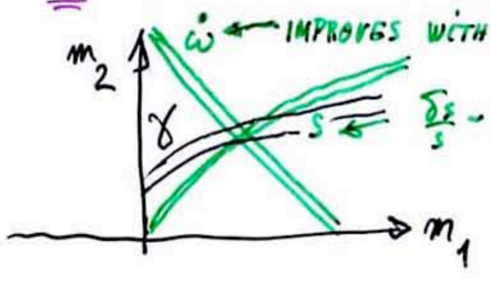
PERIASTRON ADVANCE

$$\frac{\dot{\omega} P_b}{2\pi} = \frac{3\,(GMn)^{2/3}}{c^2(1-e_T^2)}\left[1 + \underbrace{\frac{(GMn)^{2/3}}{c^2(1-e_T^2)}\left(\frac{39}{4}x_1^2 + \frac{27}{4}x_2^2 + 15x_1x_2\right) - \frac{(GMn)^{2/3}}{c^2}\left(\frac{13}{4}x_1^2 + \frac{1}{4}x_2^2 + \frac{13}{3}x_1x_2\right) - \frac{\left(\frac{4}{3}x_1^2 + x_1x_2\right)(GMn)^{\frac{1}{3}}}{\sqrt{1-e_T^2}}\frac{I_1\omega_1}{Gm_1^2}\cdots}_{\text{a few } 10^{-5}}\right]$$

⇒ POSSIBILITY TO MEASURE I_1, AND TEST EQ OF STATE OF N.S.

Damour Schäfer '88, Lattimer Schutz '04, Lyne et al '04, Morrison et al '04

HOWEVER: NEED $\underline{\underline{3}}$ OBSERVABLES WITH $\sim 10^{-5}$ ACCURACY



$\dot{\omega}$ ← IMPROVES WITH TIME $\left(\frac{\delta\dot{\omega}}{\dot{\omega}}\right)_{\text{NOW}} < 10^{-4}$

s ← $\frac{\delta s}{s} \sim 2\times 10^{-5}$ FROM SCINTILLATION

Coles et al '04

? WILL A THIRD OBSERVABLE EVER BE MEASURED TO 10^{-5} ?

? IF SO NEED TO WORRY ABOUT $\mathcal{O}(v^2/c^2)$ - ACCURATE DEFINITION OF ALL OBSERVABLES INVOLVED IN THE PROCESS

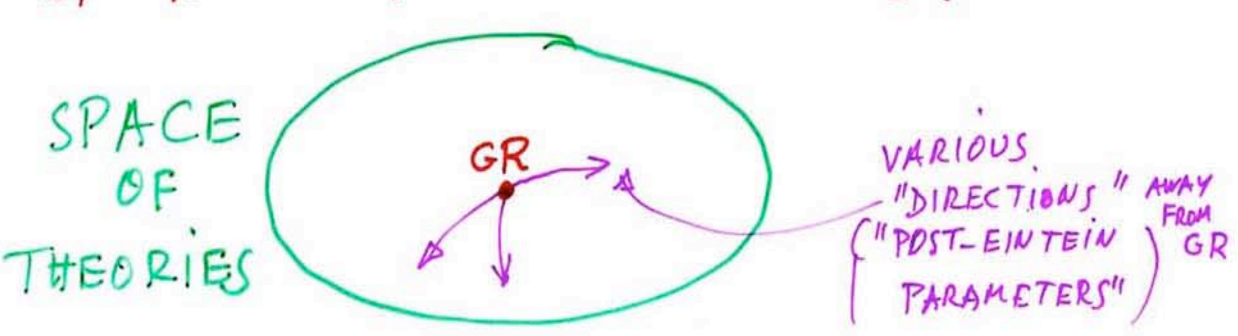
ADVANTAGES OF PHENOMENOLOGICAL APPROACH

- CAN CONFIRM OR INVALIDATE A THEORY WITHOUT MAKING ASSUMPTIONS ABOUT OTHER THEORIES
- GR HAS NO PARAMETER ⇒ <u>ANY TEST IS POTENTIALLY LETHAL</u>

DISADVANTAGES

- DOES NOT TELL US WHICH PART OF THE THEORY IS CONFIRMED [RECALL THAT SRONG SELF-GRAVITY EFFECTS ARE EFFACED IN GR ⇒ GR-BASED VIEW CAN MISS SOME POSSIBILITIES]
- IN CASE OF FAILURE, DOES NOT TELL US WHICH PART SHOULD BE MODIFIED

⇒ USEFUL TO COMPLEMENT THE PHENOMENOLOGICAL APPROACH BY A THEORY-DEPENDENT ANALYSIS:

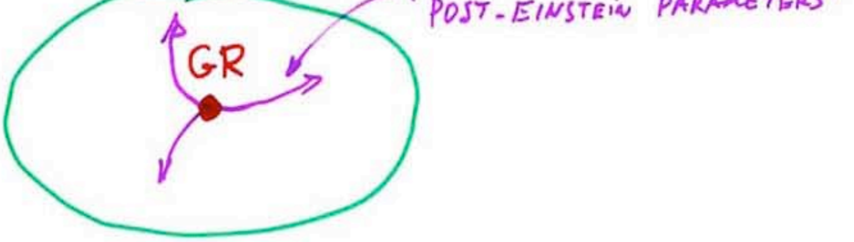


3.18

THEORY-SPACE APPROACHES

'CONTRAST' GR WITH ~~GR~~

? HOW TO DEFINE A USEFUL 'SPACE OF THEORIES'



- WEAK-FIELD GRAVITY: PARAMETRIZED POST-NEWTONIAN FORMALISM
  Eddington'24, Schiff'60, Baierlein'67, Nordtvedt'68, Will'71
- CONTRAST GR TO SOME SPECIFIC THEORIES OF GRAVITY IN STRONG-FIELD REGIME
  ONE-PARAMETER JORDAN-FIERZ-BRANS-DICKE Eardley'75 'ROSEN'S THY Will, Eardley'77
- CONTRAST RADIATIVE STRUCTURE OF GR TO MANY THEORIES OF GRAVITY
  GENERICALLY EXPECT ∃ DIPOLE RADIATION $\mathcal{O}\left(\frac{v^3}{c^3}\right) \gg$ QUADRUPOLE $\mathcal{O}\left(\frac{v^5}{c^5}\right)$
  Will'77
- CONTRAST $G=$ cst IN GR TO POSSIBLE $\dot{G} \neq 0$ (Damour Gibbons Taylor'88 1913+16)
- CONTRAST STRONG EQUIVALENCE PRINCIPLE IN GR TO $m_g^{PSR}/m_i^{PS} = 1+\Delta \neq 1$ (Damour Schäfer '91)
- STUDY VERY GENERAL CLASSES OF TENSOR-SCALAR THEORIES

→ DEFINE NEW DIRECTIONS IN TH.SPACE ⊥ TO PPN ONES (Damour, Esposito-Farèse '92)

→ GENERALIZE PPN FORMALISM TO SECOND POST-NEWTONIAN LEVEL $T(\beta', \beta'')$ Damour Esposito-Farèse '96

→ ∃ NON-PERTURBATIVE STRONG-FIELD EFFECTS IN TENSOR-SCALAR THEORIES Damour, Esposito-Farèse '93

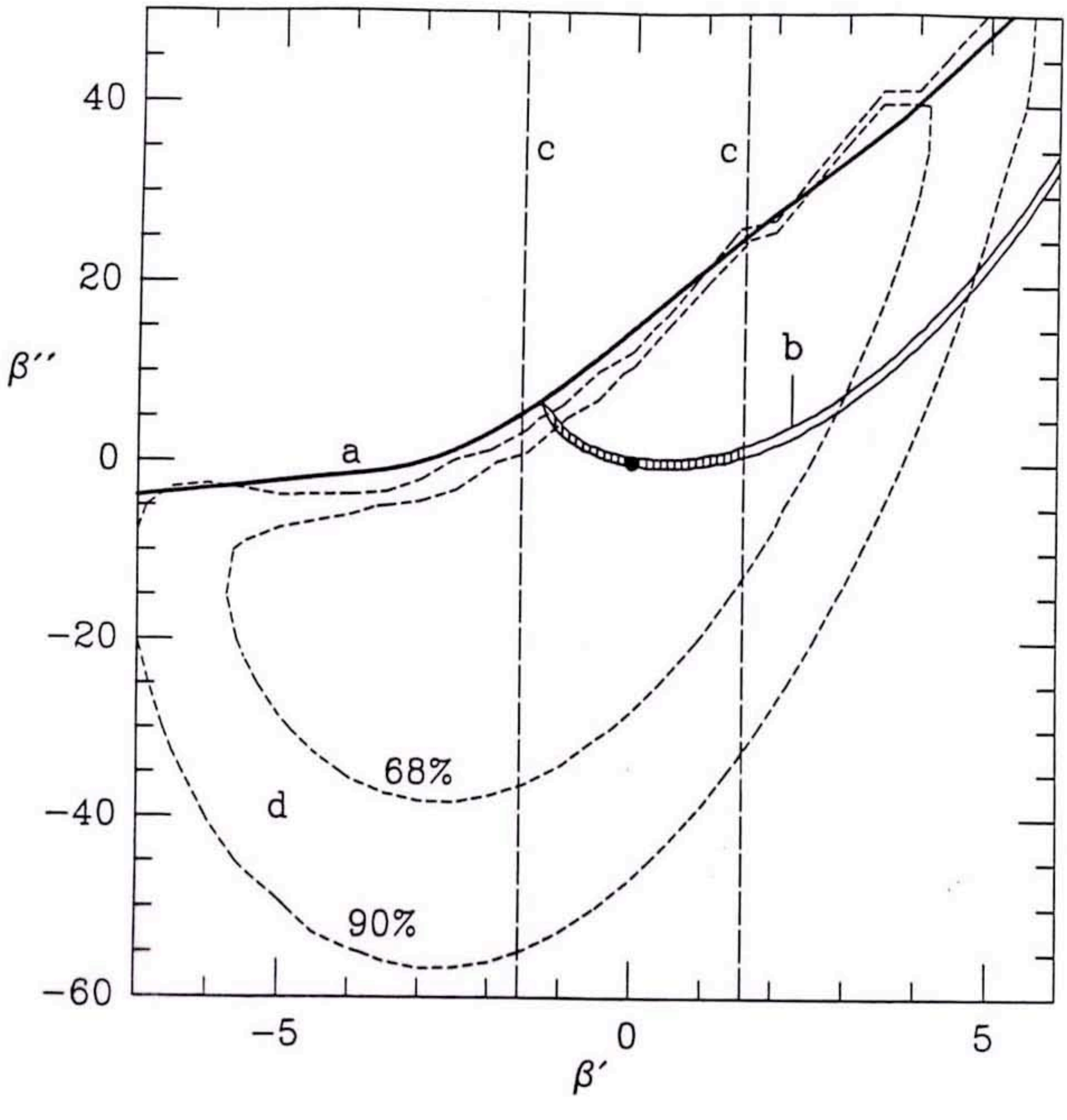


FIG. 3. Regions of the parametrized theory-space (β', β'') lying above curve (a) are incompatible with the observed values of $\dot{\omega}$, γ, and the Keplerian parameters of the PSR 1913+16 system. Contour (b) encloses the region allowed by our theory-dependent analysis of data for this pulsar (see text). Vertical lines (c) delimit the range of β' values consistent with the orbital parameters of PSR 1855+09, based on an upper limit for its violation of the strong equivalence principle [7]. Contours (d) outline the region consistent with timing observations of PSR 1534+12. All contours have been computed for 90% confidence levels, except that an additional 68% contour is included for PSR 1534+12 to help illustrate how the diagram may evolve when additional data are available. The intersection of regions allowed by all four tests is cross-hatched for emphasis, and the black dot at the origin corresponds to general relativity.

(Taylor, Wolszczan, Damour, Weisberg '92)

PPN FORMALISM

$\gamma, \beta, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4$ (Will, Nordtvedt '72)

$$\begin{cases} g_{00} = -1 + 2\frac{U}{c^2} - 2\beta\frac{U^2}{c^4} - 2\xi\frac{\Phi_W}{c^4} + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\frac{\Phi_1}{c^4} \\ \qquad + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\frac{\Phi_2}{c^4} + 2(1+\zeta_3)\Phi_3 + \cdots \\ g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi) V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi) W_i \\ g_{ij} = \left(1 + 2\gamma\frac{U}{c^2}\right)\delta_{ij} \end{cases}$$

$$\begin{cases} U = \int \frac{\rho' d^3x'}{|\vec{x}-\vec{x}'|} \; ; \quad \Phi_1 = \int \frac{\rho' \vec{v}'^2}{|\vec{x}-\vec{x}'|} \; , \quad \Phi_2 = \int \frac{\rho' U' d^3x'}{|\vec{x}-\vec{x}'|} , \ldots \\ V_i = \int \frac{\rho' v'_i}{|\vec{x}-\vec{x}'|} \end{cases}$$

LAGRANGIAN FOR INTERACTING POINT MASSES

KEEPING ONLY γ, β

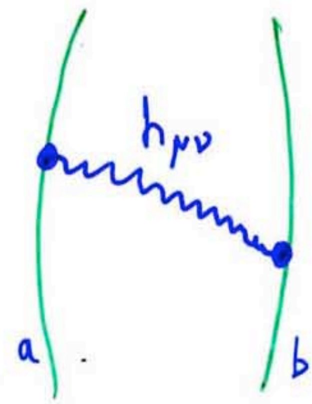
$$\boxed{\begin{aligned} L &= -\sum m_a c^2 \sqrt{1 - \vec{v}_a^2/c^2} + L^{2\text{-BODY}}(\gamma) + L^{3\text{-BODY}}(\beta) \\ L^{2\text{-BODY}} &= L^{2\text{-BODY}}_{GR} + \frac{\bar{\gamma}}{2}\sum_{a \neq b} \frac{G m_a m_b}{r_{ab}} \frac{(\vec{v}_a - \vec{v}_b)^2}{c^2} \\ L^{3\text{-BODY}} &= (1 + 2\bar{\beta}) L^{3\text{-BODY}}_{GR} \end{aligned}}$$

$$\boxed{\begin{aligned} \bar{\gamma} &\equiv \gamma - 1 \\ \bar{\beta} &\equiv \beta - 1 \end{aligned}}$$

('POST-EINSTEIN') PARAMETERS ($= 0$ IN GR)

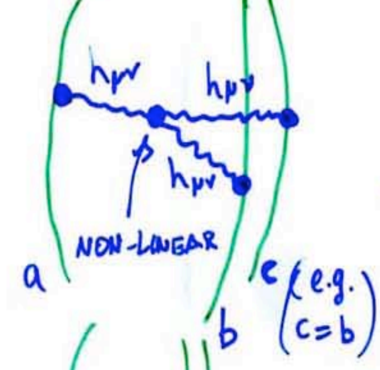
IN DIAGRAMMATIC REPRESENTATION

GR

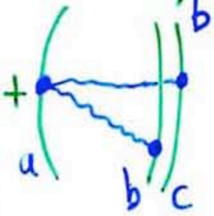


$$L^{2\text{-BODY}}_{GR} = \frac{G m_a m_b}{r_{ab}}\left[1 + \mathcal{O}\left(\frac{v^2}{c^2}\right)\right]$$

SOME DEFINITE TERMS LINKED TO $h_{\mu\nu}$ EXCHANGE



$$L^{3\text{-BODY}}_{GR} = -\frac{1}{2}\sum_{b\neq a\neq c}\frac{G^2 m_a m_b m_c}{r_{ab}\, r_{ac}\, c^2}$$



PPN

$$L^{2\text{-BODY}}_{PPN} = L^{2\text{-BODY}}_{GR} + \bar{\gamma}\,\frac{G m_a m_b}{r_{ab}}\,\frac{(\vec{v}_a - \vec{v}_b)^2}{c^2}$$

$$L^{3\text{-BODY}}_{PPN} = \left(1 + 2\bar{\beta}\right) L^{3\text{-BODY}}_{GR}$$

SIMPLEST FIELD-THEORY INTERPRETATION OF PPN PARAMETERS $\bar{\gamma}, \bar{\beta}$

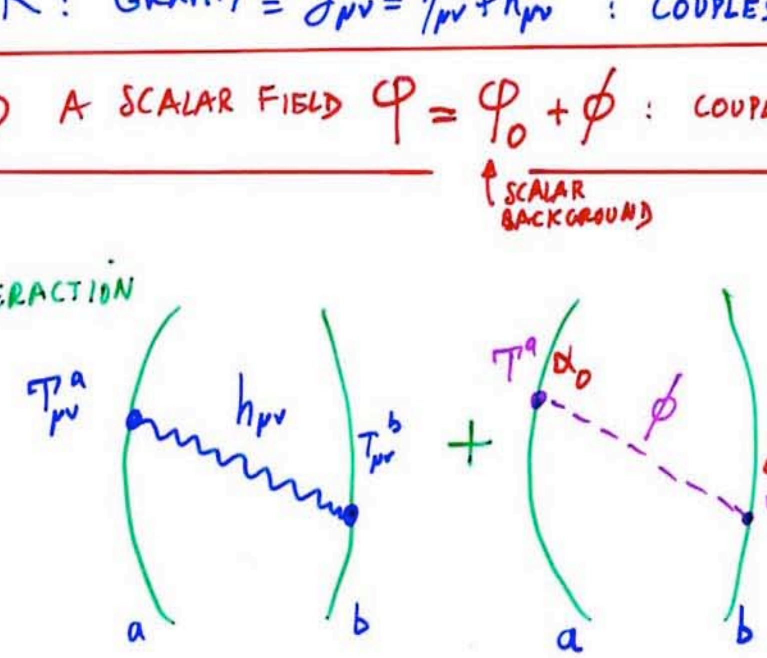
$$\bar{\gamma} \equiv \gamma^{PPN} - 1 \qquad \bar{\beta} \equiv \beta^{PPN} - 1$$

(Damour, Esposito-Farèse '92 ...)

GR: GRAVITY $\equiv g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$: COUPLES: $h_{\mu\nu} T^{\mu\nu}$ (BACKGROUND: $\eta_{\mu\nu}$)

$$\boxed{\text{ADD A SCALAR FIELD } \varphi = \underbrace{\varphi_0}_{\text{SCALAR BACKGROUND}} + \phi \text{ : COUPLES: } \underbrace{\alpha_0}_{\text{LINEAR COUPLING STRENGTH WRT } h_{\mu\nu} \text{ GRAVITY}} \phi\, T + \frac{1}{2} \underbrace{\beta_0}_{\text{NON-LINEAR COUPLING}} \phi^2 T}$$

INTERACTION



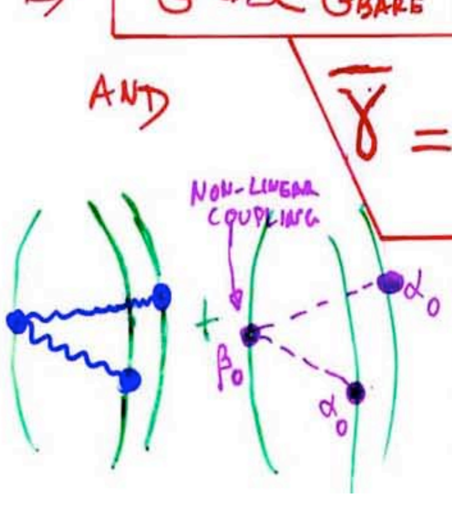
GREEN FUNCTION $\square^{-1}$: $\square G = -4\pi\delta$

$$\Rightarrow L^{2\text{-BODY}} = G_{BARE} \iint ds_a\, ds_b\, m_a m_b \left[u_a^\mu u_a^\nu (2\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma})\, G\, u_b^\rho u_b^\sigma + \alpha_0\, G\, \alpha_0 \right]$$

$$\Rightarrow \boxed{G^{OBS} \propto G_{BARE}\,(1 + \alpha_0^2)}$$

AND

$$\boxed{\bar{\gamma} = -2\,\frac{\alpha_0^2}{1+\alpha_0^2}}$$



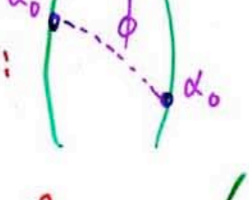
$$\boxed{\bar{\beta} = \frac{1}{2}\,\frac{\alpha_0 \beta_0 \alpha_0}{(1+\alpha_0^2)^2}}$$

SUMMARIZING

- ∃ TWO BASIC APPROACHES FOR TESTING GRAVITY WITH PULSARS:
  - (PHENOMENOLOGICAL) : 'COMPARE OBS TO GR'
  - (THEORY-SPACE) : 'CONTRAST GR TO ALTERNATIVES'

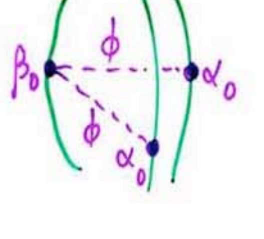
  COMPLEMENTARY
- HISTORY SHOWS THEY ARE BOTH USEFUL ($\dot{\omega}_{MERCURY}$)
- PHENOMENOLOGICAL : 'PARAMETRIZED POST KEPLERIAN'
  - TIMING SINGLE-LINE BP
  - PULSE-STRUCTURE DATA
  - TIMING DOUBLE-LINE BP
- RICHNESS OF SPIN-ORBIT EFFECTS
  - DIRECT ON PULSE-STRUCT. PRECESSION
  - DIRECT ORBITAL EFFECTS
  - INDIRECT EFFECTS ON SOME PPK
- REINTERPRETATION OF BASIC PPN PARAMETERS γ^{PPN} AND β^{PPN}:

$\bar{\gamma} \equiv \gamma^{PPN} - 1 \propto \alpha_0^2$:



$\alpha_0 \phi T$

$\bar{\beta} \equiv \beta^{PPN} - 1 \propto \beta_0 \alpha_0^2$:



$\frac{1}{2} \beta_0 \phi^2 T$

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