Villa Olmo
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BINARY SYSTEMS AS TEST-BED OF GRAVITY THEORIES

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IHES

LECTURE No 4

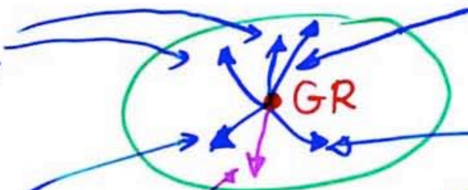
THEORY-SPACE APPROACH TO TESTING RELATIVISTIC GRAVITY WITH BINARY PULSAR DATA

ALTERNATIVE THEORIES OF GRAVITY

- IF YOU LOOK UP WILL'S BOOK YOU FIND A LONG LIST OF ALTERNATIVE THEORIES
- SOME OF THESE HAVE PLAYED A USEFUL RÔLE IN FINDING POSSIBLE PHENOMENOLOGICAL DIRECTIONS AWAY FROM EINSTEIN'S THY: 'POST-EINSTEIN PARAMS'

EDDINGTON'S $\overline{\gamma}, \overline{\beta}$ IN MOST THIES

WHITEHEAD'S ξ



NORDTVEDT'S 'η PARAMETER': VIOLATION OF SEP IN TENSOR-SCALAR THIES

$\alpha_1\, \alpha_2\, \alpha_3\ \zeta_1 \ldots \zeta_4$ PARAMETERS IN THIES ADMITTING PREFERRED FRAMES OR VIOLATING MOMENTUM CONSERVATION

ROSEN'S BIMETRIC THY: HAS $\overline{\gamma} = 0 = \overline{\beta}$ BUT DEVIATES IN STRONG-FIELD (Will, Eardley '77)

- HOWEVER, MOST SUCH THEORIES ARE:
  - ILL-MOTIVATED FROM FUNDAMENTAL PHYSICS POINT OF VIEW
  - 'SICK' WRT BASIC TENETS OF FIELD THEORY ('GHOSTS', CAUSALITY PROBLEMS, ...)

- HERE, WE SHALL FOCUS INSTEAD ON A GENERAL CLASS OF THEORIES WHICH ARE:
  - MOTIVATED BY FUNDAMENTAL PHYSICS (KALUZA-KLEIN, STRING THEORY): $D > 4$; DILATON, MODULI: PARTNERS OF $g_{\mu\nu}$
  - OFTEN DEALT WITH IN COSMOLOGY (INFLATION, QUINTESSENCE, ...)
  - CONSISTENT WITH BASIC TENETS OF FIELD THEORY
  - SIMPLE ENOUGH TO STUDY IN GREAT DETAIL, ESPECIALLY FOR STRONG-FIELD EFFECTS
  - RICH ENOUGH TO EXHIBIT INTERESTING DEVIATIONS FROM G.R.

TENSOR-SCALAR THEORIES

$\tilde{R} \equiv R(\tilde{g}_{\mu\nu})$

ACTION

KINETIC TERM OF Φ — POTENTIAL — COUPLING TO SCALAR CURVATURE

$$\boxed{S = \frac{c^4}{16\pi G_*}\int \frac{d^4x}{c}\sqrt{\tilde{g}}\left[-Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - U(\Phi) + F(\Phi)\tilde{R}\right] + S_{MATTER}[\psi_M ; \tilde{g}_{\mu\nu}]}$$

'PHYSICAL METRIC' COUPLED TO ORDINARY MATTER

FOR INSTANCE:

- JORDAN('49) – FIERZ('56) – BRANS-DICKE('61)

$$Z(\Phi) = \frac{\omega}{\Phi}, \quad U(\Phi) = 0, \quad F(\Phi) = \Phi$$

ω ← SINGLE DIMENSIONLESS PARAMETER

- NON-MINIMALLY COUPLED MASSIVE SCALAR

$$Z(\Phi) = 1, \quad U(\Phi) = m_\Phi^2 \Phi^2, \quad \overbrace{F(\Phi) = 1 + \xi\Phi^2}^{\underbrace{\tilde{R} + \xi\Phi^2\tilde{R}}_{\nearrow}}$$

HERE WE ASSUME

- $\tilde{g}_{\mu\nu}$ COUPLES UNIVERSALLY TO ORDINARY MATTER ('WEAK EQUIVALENCE PRINCIPLE')

  HOWEVER, ONE GENERALLY EXPECTS VIOLATIONS OF WEP (Damour, Polyakov '94) and THE COUPLING TO DARK MATTER COULD BE QUITE ≠ (Damour, Gibbons, Gundlach '90; Damour, Piazza, Veneziano '02)

- THE POTENTIAL $U(\Phi)$ IS USED ONLY TO DETERMINE THE PRESENT COSMOLOGICAL VEV Φ_0 OF Φ, BUT CORRESPONDS TO $\approx m_\Phi \sim H_0$, TOTALLY NEGLIGIBLE IN BINARY PSRS (see, however, Khoury, Veltman '03)

- ONLY ONE SCALAR FIELD Φ. THE MULTI-SCALAR CASE CAN BE DEALT WITH RATHER EASILY (Damour, Esposito-Farèse '92)

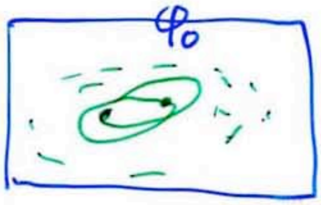
CANONICAL FORM OF TENSOR-SCALAR THEORIES

- CAN USE BOTH $\Phi \rightarrow \varphi = f(\Phi)$
- AND CONFORMAL TRANSFORMATIONS: $\tilde{g}_{\mu\nu} \rightarrow g^*_{\mu\nu} = g(\Phi)\tilde{g}_{\mu\nu}$

TO PUT ACTION IN CONVENIENT FORM:

$$\boxed{\begin{aligned} S = \frac{c^4}{16\pi G_*}\int \frac{d^4x}{c}\sqrt{g_*}\left[R_* - 2g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right] \\ + S_{MATTER}\left[\psi_m ; \tilde{g}_{\mu\nu} = e^{2a(\varphi)}g^*_{\mu\nu}\right] \end{aligned}}$$

- HERE, $g^*_{\mu\nu} \equiv$ 'EINSTEIN CONFORMAL FRAME' $\quad R_* \equiv R(g_*)$
- KINETIC TERM OF φ FIXED TO $-2(\nabla\varphi)^2$
- $V(\varphi)$ ASSUMED NEGLIGIBLE APART FROM FIXING φ_0



- $R(g_*)$ = SPIN-2 EXCITATIONS + $-2(\nabla\varphi)^2$ = SPIN-0 EXCITATIONS
- DEFINITION OF TENSOR-SCALAR THEORY FULLY CONTAINED IN

$$\boxed{\begin{gathered} \text{COUPLING FUNCTION} \longrightarrow a(\varphi) \\ \tilde{g}_{\mu\nu} = e^{2a(\varphi)}\, g^*_{\mu\nu} \end{gathered}}$$

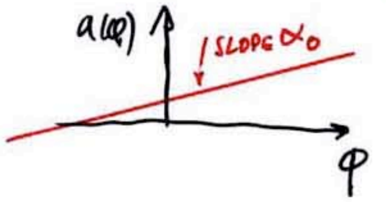
PHYSICAL METRIC: MEASURED BY ATOMIC CLOCKS, RODS, TEST PARTICLES

EINSTEIN METRIC: PURE TT GRAVIT. WAVES (spin 2)

COUPLING FUNCTION AND COUPLING PARAMETERS

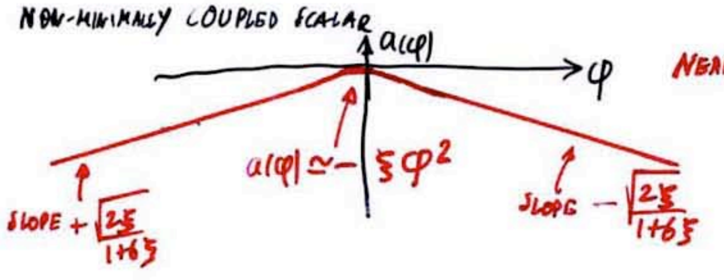
EXAMPLE:

JORDAN-FIERZ-BRANS-DICKE: $a^{JFBD}(\varphi) = \underset{\text{CST}}{a_0} + \underset{\text{SLOPE = COUPLING}}{\alpha_0}\, \varphi$



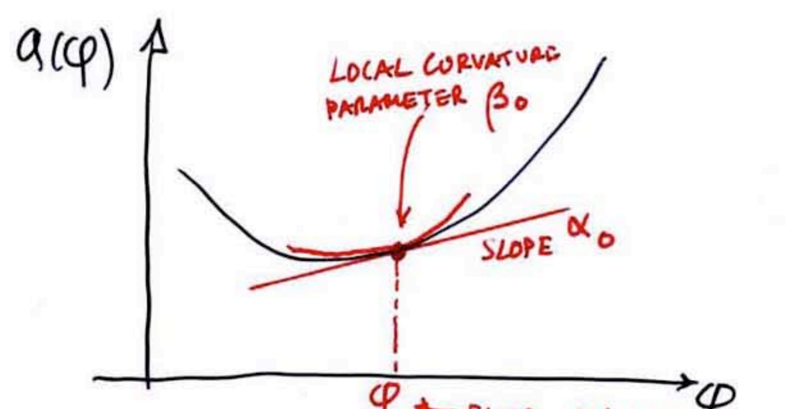
$$\alpha_0^2 = \frac{1}{2\omega+3}$$

NON-MINIMALLY COUPLED SCALAR



NEAR $\varphi = 0$: $a(\varphi) \simeq \underbrace{-\xi\varphi^2}_{\text{PARABOLIC SHAPE}}$

GENERAL COUPLING FUNCTION



LOCALLY

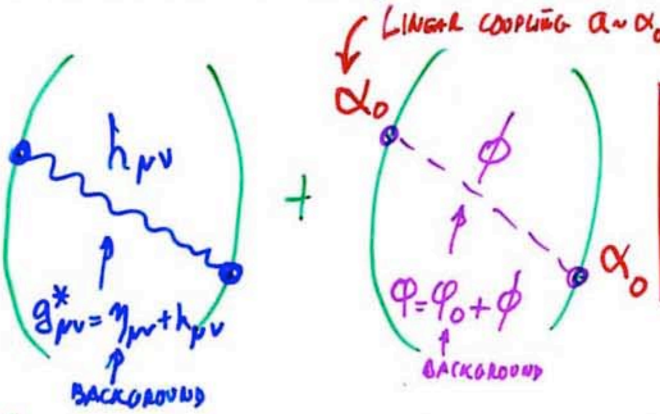
$$\boxed{a(\varphi) \simeq a(\varphi_0) + \alpha_0(\varphi-\varphi_0) + \frac{1}{2}\beta_0(\varphi-\varphi_0)^2}$$

TWO BASIC COUPLING PARAMETERS

- SUFFICE TO DESCRIBE FIRST POST-NEWTONIAN EFFECTS
- ALLOW TO PARAMETRIZE INTERESTING STRONG-FIELD EFFECTS, TESTABLE BY BINARY PULSARS

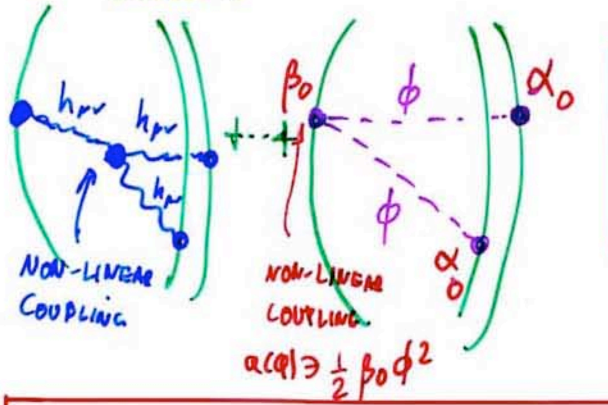
PPN FORMALISM AND TENSOR-SCALAR

$$\bar{\gamma} \equiv \gamma^{PPN} - 1$$



$$\boxed{\bar{\gamma} = -\frac{2\alpha_0^2}{1+\alpha_0^2}}$$

$$\bar{\beta} \equiv \beta^{PPN} - 1$$



$$\boxed{\bar{\beta} = +\frac{1}{2}\frac{\alpha_0 \beta_0 \alpha_0}{(1+\alpha_0^2)^2}}$$

IN ADDITION: $$\boxed{G^{OBS}_{NEWTON} = G_* \, e^{2a(\varphi_0)}(1+\alpha_0^2)}$$

$\Rightarrow$ SUGGESTS TO CONSIDER 'MINIMAL TWO-PARAMETER CLASS OF TENSOR-SCALAR THEORIES' $T(\alpha_0, \beta_0)$ DEFINED BY

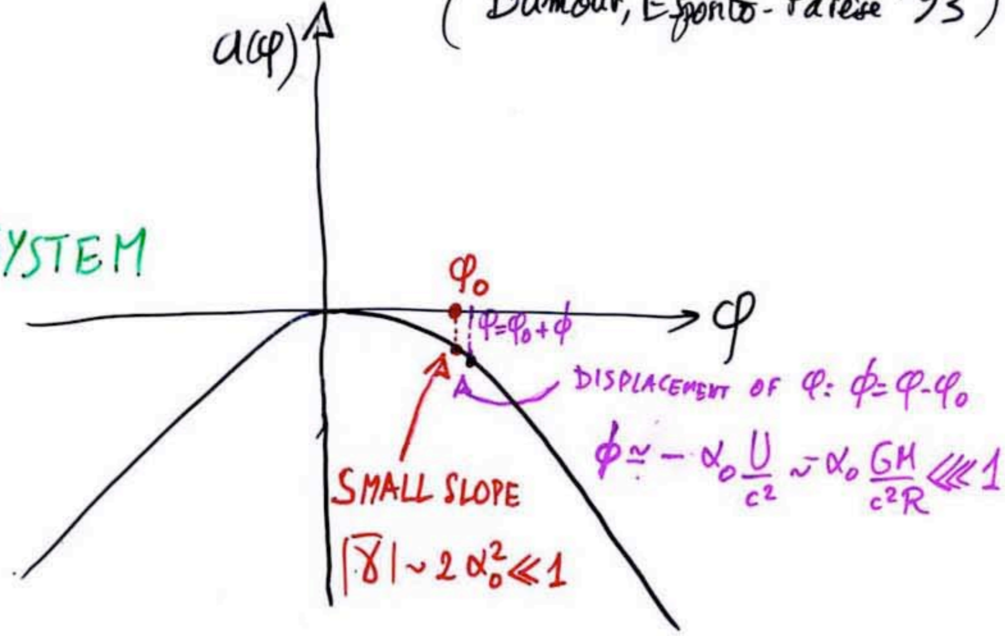
$$\overset{T(\alpha_0 \beta_0)}{a(\varphi)} = \alpha_0(\varphi - \varphi_0) + \frac{1}{2}\beta_0(\varphi - \varphi_0)^2$$

- SUFFICIENT TO PARAMETRIZE $\bar{\gamma}, \bar{\beta} \leftrightarrow \alpha_0, \beta_0$
- CONTAINS NON-PERTURBATIVE STRONG-FIELD EFFECTS WHEN $\beta_0 \lesssim -4$

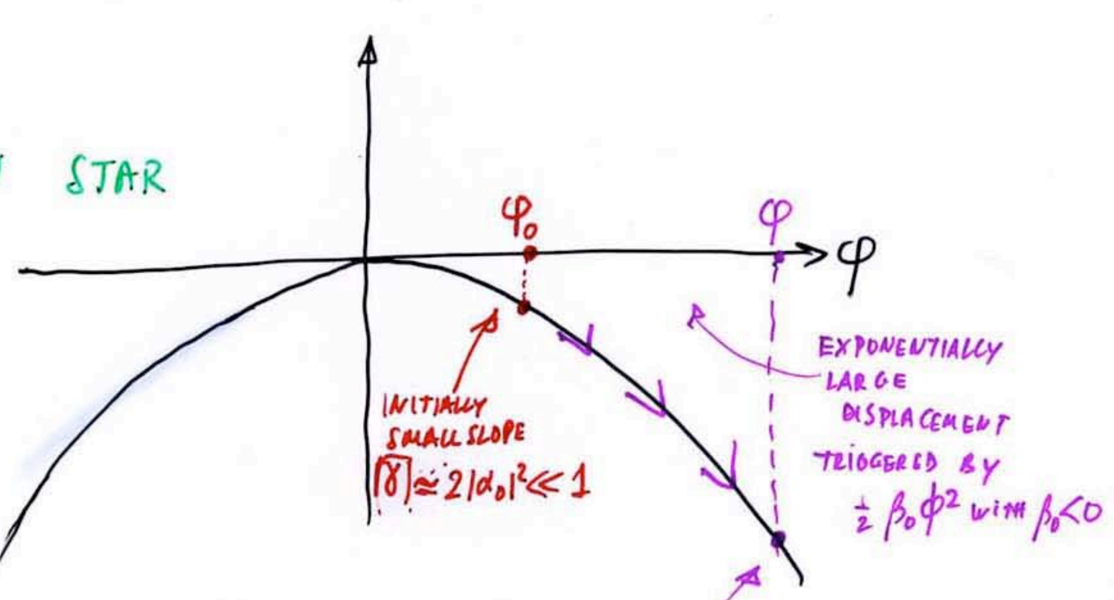
WEAK-FIELD (SOLAR SYSTEM) VERSUS STRONG-FIELD (NEUTRON STARS)

(Damour, Esposito-Farèse '93)

SOLAR SYSTEM



NEUTRON STAR



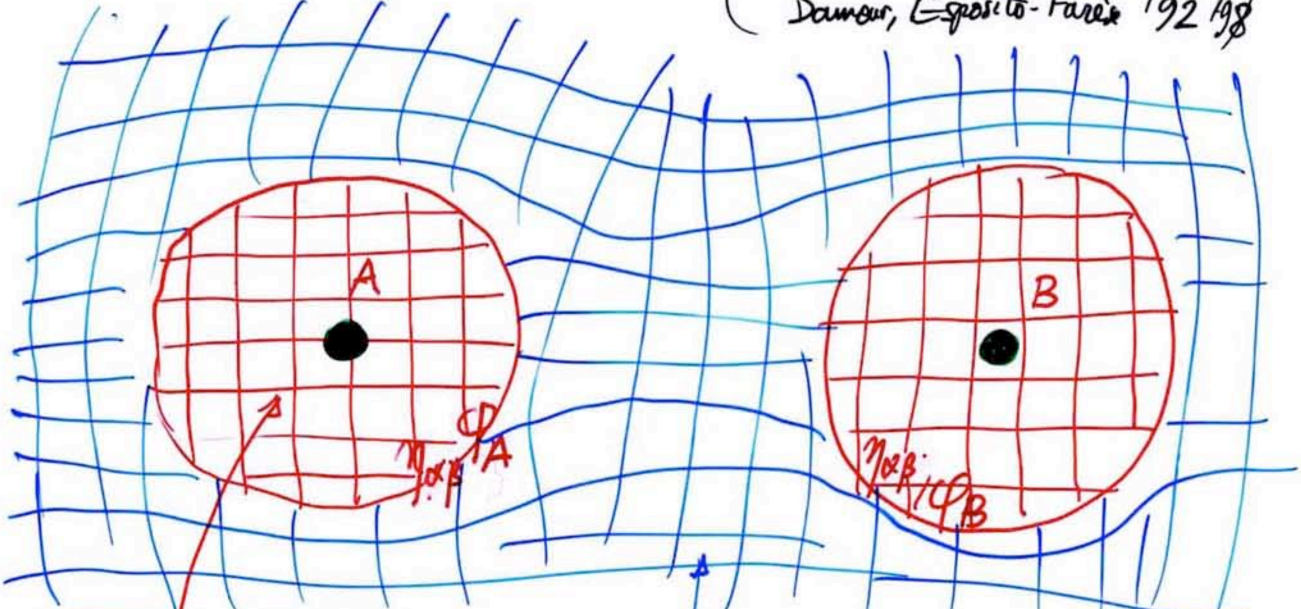
CAN REACH LARGE SLOPES $\left|\frac{da(\varphi)}{d\varphi}\right| \sim 1$

I.E. LARGE DEVIATIONS FROM GR IN NEUTRON STARS

NON-PERTURBATIVE STRONG-FIELD EFFECTS

MOTION AND RADIATION OF COMPACT BODIES IN TENSOR-SCALAR THEORIES

GENERALIZE THE SIMPLIFIED 'POINT-PARTICLE' APPROACH, MODIFIED BY MAIN EXPECTED EFFECT OF MATCHING (Eardley '75, Will Eardley '77, Damour, Esposito-Farèse '92 '98)



LOCAL CHART X^α_A

$$G^*_{\alpha\beta}(X^\gamma_A) = G^{(0)\,\text{ISOLATED}}_{\alpha\beta}(X^\gamma_A) + \dots \longrightarrow \lim_{|X|\to\infty} G^{(0)}_{\alpha\beta} = \eta_{\alpha\beta}$$

$$\varphi(X) = \varphi^{(0)\,\text{ISOLATED}}(X) + \dots$$

WITH $\lim_{|X|\to\infty} \varphi^{(\text{ISOL})}(X) = \varphi_A \simeq \varphi_0 +$ EFFECT OF COMPANION

GLOBAL COORDINATES x^μ

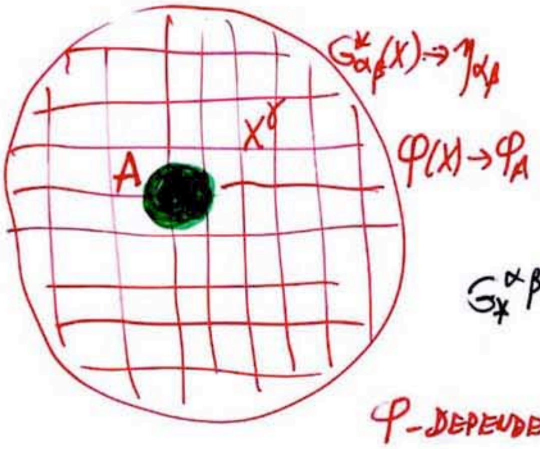
$$g^*_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

$$\varphi(x) = \varphi_0 + \phi(x)$$

$$\boxed{S = -\sum_a \int c\, m_a\left(\varphi_a = \tilde{\varphi}(z_a)\right)\sqrt{-\tilde{g}^*_{\mu\nu}(z^\lambda_a)\, dz^\mu_a dz^\nu_a} + \frac{c^4}{16\pi G_*}\int \frac{d^4x}{c}\sqrt{g_*}\left[R_* - 2 g_*^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi\right]}$$

ONE MUST DETERMINE THE EFFECTIVE STRONG-SELF-GRAVITY φ-DEPENDENT 'MASS' $m_a(\varphi_a)$ BY SOLVING THE STRUCTURE OF ISOLATED NEUTRON STARS IN TENSOR-SCALAR GRAVITY

NEUTRON STARS IN TENSOR-SCALAR GRAVITY



NEED TO SOLVE THE EXACT, FULLY NON-LINEAR FIELD EQUATIONS

$$R_{\alpha\beta}(G^*) = 2\partial_\alpha\varphi\partial_\beta\varphi + \frac{8\pi G_*}{c^4}\left(T^*_{\alpha\beta} - \frac{1}{2}T^* G^*_{\alpha\beta}\right)$$

$$G_*^{\alpha\beta}\nabla_\alpha\nabla_\beta\varphi = -\frac{4\pi G_*}{c^4}\alpha(\varphi)\,T^*$$

φ-DEPENDENT COUPLING STRENGTH

$$\boxed{\alpha(\varphi) \equiv \frac{\partial a(\varphi)}{\partial\varphi}}$$

NUMERICALLY DETERMINE

TOTAL MASS-ENERGY (IN EINSTEIN UNITS) OF NS SPACE-TIME (INCLUDING ALL STRONG-SELF-GRAVITY EFFECTS, AND ENERGY IN $(\nabla\varphi)^2$...)

$$\boxed{m_A = m_A(\varphi_A)}$$

EFFECTIVE COUPLING STRENGTH OF φ TO NS

$$\alpha_A \equiv \frac{\partial \ln m_A(\varphi_A)}{\partial\varphi_A}$$

$$\beta_A \equiv \frac{\partial\alpha_A}{\partial\varphi_A}$$

INERTIA MOMENT (IN EINSTEIN UNITS) (SLOW ROTATION)

$$I_A \equiv J_A/\Omega = I_A(\varphi_A)$$

$$k_A \equiv -\frac{\partial \ln I_A}{\partial\varphi_A}$$

THEORETICAL PREDICTIONS FOR MOTION AND RADIATION OF COMPACT BINARIES IN TENSOR-SCALAR GRAVITY

(Damour Esposito-Farèse '92 ...)

SOLVE

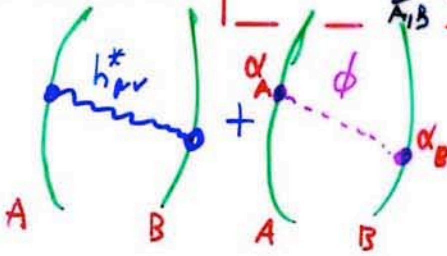
$$S = -\sum_a c \int m_a\left(\varphi_a = \tilde{\varphi}(z_a)\right)\sqrt{-g^*_{\mu\nu}(z_a)\,dz_a^\mu dz_a^\nu} + \frac{c^4}{16\pi G_*}\int \frac{d^4x}{c}\sqrt{g_*}\left[R_* - 2g_*^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi\right]$$

CONTAINS STRONG SELF-GRAVITY

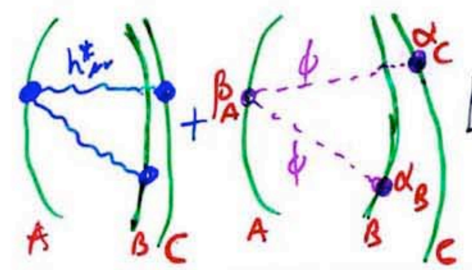
TOTAL MASS-ENERGY

MOTION

$$\boxed{L = -\sum_{A,B} m_A(\varphi_0)\,c^2\sqrt{1-\frac{v_A^2}{c^2}} + L^{2\text{-BODY}} + L^{3\text{-BODY}}}$$



$$L^{2\text{-BODY}}_{AB} = \frac{G_{AB}\, m_A^{(0)} m_B^{(0)}}{r_{AB}}\left[1 + \frac{3}{2c^2}(\vec{v}_A^2 + v_B^2) - \frac{7}{2c^2}(\vec{v}_A\cdot\vec{v}_B) - \frac{1}{2c^2}(n_{AB}v_A)(n_{AB}v_B) + \bar{\gamma}_{AB}\frac{(\vec{v}_A-\vec{v}_B)^2}{c^2}\right]$$



$$L^{3\text{-BODY}} = -\frac{1}{2}\sum_{B\neq A\neq C}\frac{G_{AB}G_{AC}\, m_A^{(0)} m_B^{(0)} m_C^{(0)}}{r_{AB}\, r_{AC}\, c^2}\left[1 + 2\bar{\beta}^A_{BC}\right]$$

EFFECTIVE NEWTON'S CONSTANT →

EFFECTIVE $\bar{\gamma} \equiv \gamma^{PPN} - 1$ →

EFFECTIVE $\bar{\beta} \equiv \beta^{PPN} - 1$ →

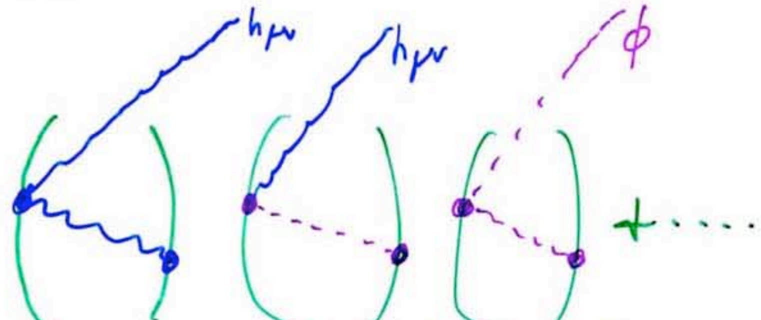
$$\boxed{\begin{aligned} G_{AB} &= G_*\left[1 + \alpha_A\alpha_B\right] \\ \bar{\gamma}_{AB} &= -2\,\frac{\alpha_A\alpha_B}{1+\alpha_A\alpha_B} \\ \bar{\beta}^A_{BC} &= +\frac{1}{2}\,\frac{\alpha_B\,\beta_A\,\alpha_C}{[1+\alpha_A\alpha_B][1+\alpha_A\alpha_C]} \end{aligned}}$$

ASSUMING FOR SIMPLICITY $a(\varphi_0) = 0$, IF NOT NEED TO RESCALE UNITS BY $e^{a(\varphi_0)}$

STRONG-FIELD EFFECTS CONTAINED IN $G_{AB} \neq G_*$, $\bar{\gamma}_{AB} \neq \bar{\gamma}$, $\bar{\beta}^A_{BC} \neq \bar{\beta}$

THEORETICAL PREDICTIONS FOR RADIATION AND TIMING OF COMPACT BINARIES IN TENSOR-SCALAR GRAVITY

RADIATION



CANNOT DO EASILY THE FULL $v^2/c^2 + v^4/c^4 + v^5/c^5$ EOM $\Rightarrow$

MORE HEURISTICALLY

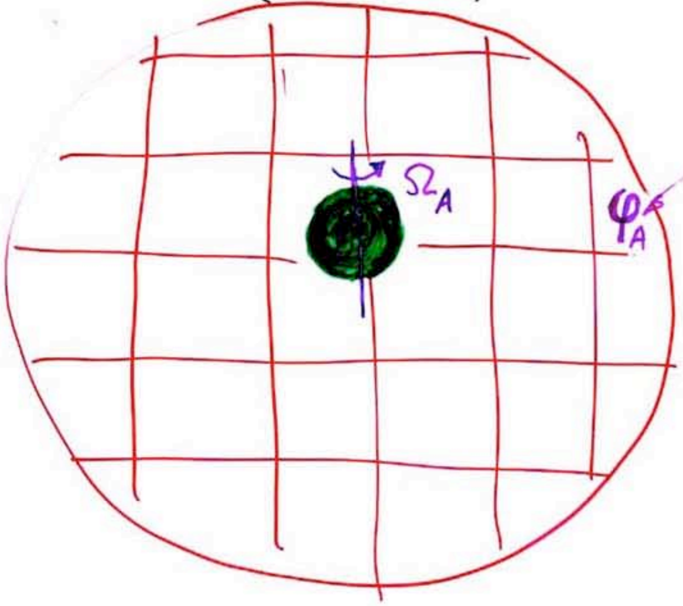
ENERGY FLUX AT ∞:
$$F = F_\varphi^{\text{MONOPOLE}} + F_\varphi^{\text{DIPOLE}} + F_\varphi^{\text{QUADRUPOLE}} + F_h^{\text{QUADRUPOLE}}$$

$\sim \frac{v^5}{c^5} f(\alpha_A \alpha_B \beta_A \beta_0)$ $\quad \sim \frac{v^3}{c^3}(\alpha_A - \alpha_B)^2$ $\quad \sim \frac{v^5}{c^5} f(\alpha_A \alpha_B \beta_A \ldots)$ $\quad$ GR RESULT BUT $G_N \to G_{AB}$

$$\Rightarrow \boxed{\begin{aligned}\dot{P}_b = &- \frac{2\pi\nu}{1+\alpha_A\alpha_B}\,\frac{G_{AB}Mm}{c^3}\,\frac{1+e^2/2}{(1-e^2)^{5/2}}\,(\alpha_A-\alpha_B)^2 \\ &- \frac{192\pi\nu}{5\,[1+\alpha_A\alpha_B]}\left(\frac{G_{AB}Mm}{c^3}\right)^{5/3}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}} \\ &- \text{SEVERAL OTHER TERMS} \propto \left(\frac{G_{AB}Mm}{c^3}\right)^{5/3} \ldots\end{aligned}}$$

THEORETICAL PREDICTION FOR HOW THE PULSAR CLOCK TICKS IN TENSOR-SCALAR GRAVITY

(Eardley '75, Will's book, Damour Esposito Farèse '96)



$\varphi_A(t)$ VARIES BECAUSE THE DISTANCE TO COMPANION CHANGES

ADIABATICALLY INVARIANT PULSAR ANGULAR MOMENTUM $\rightarrow$ $J_A = I_A \Omega_A = I_A \dfrac{d\phi}{d\tau^*_A}$ ROTATIONAL PHASE OF PULSAR

VARIES BECAUSE $I_A = I_A(\varphi_A(t))$

$\Rightarrow$ MODIFIES THE GR EXPRESSION FOR γ_{TIMING}

$$\boxed{\Delta_E = \gamma \sin u}$$

eccentric anomaly

$X_B \equiv \dfrac{m_B}{M} = 1 - X_A$

$$\boxed{\gamma = \frac{e}{n}\frac{X_B}{1+\alpha_A\alpha_B}\left(\frac{G_{AB}(m_A+m_B)n}{c^3}\right)^{2/3}\left[X_B(1+\alpha_A\alpha_B)+1+k_A\alpha_B\right]}$$

$k_A \equiv -\dfrac{\partial \ln I_A}{\partial \varphi_A}$

TIMING FORMULA IN TENSOR-SCALAR GRAVITY

- THE UNIVERSAL DD TIMING FORMULA HOLDS UNCHANGED

$$T_{ARRIVAL} = T + \Delta_R(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T)$$

$$= F\left[T\,;\, P_b, T_0, e_0, \omega_0, x_0\,;\, k, \gamma, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\right]$$

$\Rightarrow$ • PHENOMENOLOGICAL ANALYSIS (PPK FORMALISM) AND PHENOM. TESTS CAN BE DONE INDEPENDENTLY OF THY

- BUT THE THEORETICAL PREDICTIONS

$$\boxed{P_{PK}^{OBS} = f^{THEORY}(pk\,;\, m_A, m_B)}$$

GET MODIFIED, NOTABLY BY SELF-GRAVITY EFFECTS

$$k^{TH} \equiv \frac{\langle\dot{\omega}\rangle}{n} = \frac{3}{1-e^2}\left(\frac{G_{AB}Mn}{c^3}\right)^{2/3}\left[\frac{1-\frac{1}{3}\alpha_A\alpha_B}{1+\alpha_A\alpha_B} - \frac{X_A\beta_B\alpha_A^2 + X_B\beta_A\alpha_B^2}{6(1+\alpha_A\alpha_B)^2}\right]$$

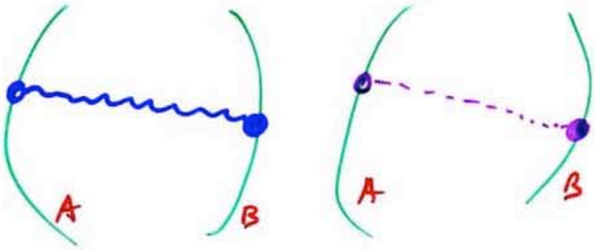
$$\gamma^{TH} = \frac{e}{n}\,\frac{X_B}{1+\alpha_A\alpha_B}\left(\frac{G_{AB}Mn}{c^3}\right)^{2/3}\left[X_B(1+\alpha_A\alpha_B) + 1 + k_A\alpha_B\right]$$

$$\dot{P}_b^{TH} = -\frac{2\pi X_A X_B}{1+\alpha_A\alpha_B}\left(\frac{G_{AB}Mn}{c^3}\right)\frac{1+e^2/2}{(1-e^2)^{5/2}}(\alpha_A-\alpha_B)^2 - X_A X_B\left(\frac{G_{AB}Mn}{c^3}\right)^{5/3}[\phi\ldots]$$

$$\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet$$

PHENOMENOLOGICAL TESTS OF THE STRONG EQUIVALENCE PRINCIPLE

WE HAVE SEEN THAT



→ NEWTON'S FORCE : $G_{AB} \dfrac{m_A m_B}{r_{AB}^2}$

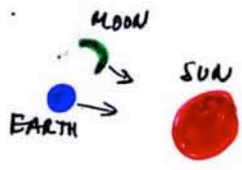
EFFECTIVE NEWTON'S CONSTANT, MODIFIED BY SELF-GRAVITY EFFECTS

- WAS EMPHASIZED BY NORDTVEDT (1968) WITHIN PPN FORMALISM

1PN LEVEL

$$G_{AB} = G\left[1 + \eta\left(\frac{E_A^{GRAV}}{m_A c^2} + \frac{E_B^{GRAV}}{m_B c^2}\right)\right]$$

$$\eta = 4\bar{\beta} - \bar{\gamma}$$



AND NORDTVEDT POINTED OUT OBSERVABLE EFFECTS IN 3-BODY SYSTEMS

- DAMOUR AND SCHÄFER (1991) POINTED OUT THAT THERE COULD BE FURTHER SELF-GRAVITY VIOLATIONS OF SEP OF THE TYPE

$$\boxed{\frac{m_A^{GRAV}}{m_A^{INERTIAL}} = 1 + \eta \frac{E_A^{GRAV}}{m_A c^2} + \eta' \left(\frac{E_A^{GRAV}}{m_A c^2}\right)^2 + \cdots}$$

SIZABLE ONLY IN NEUTRON STARS

AND THAT THEY WOULD HAVE SPECIFIC OBSERVABLE EFFECTS IN CERTAIN BINARY SYSTEMS

GRAVITATIONAL 'STARK EFFECT'

(Damour, Schäfer '91)

RELATIVE MOTION IN NEWTONIAN APPROXIMATION $\quad \vec{r} = \vec{z}_A - \vec{z}_B$

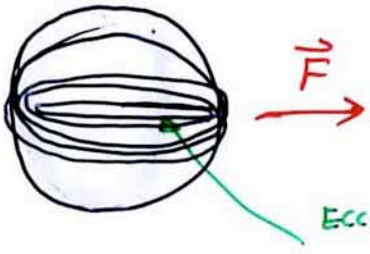
IF ~~SEP~~: $\dfrac{m_A^{GRAV}}{m_A^{IN}} = 1 + \Delta_A \qquad \dfrac{m_B^{GRAV}}{m_B^{IN}} = 1 + \Delta_B$

$$\boxed{\Delta \equiv \Delta_A - \Delta_B}$$

$$\boxed{\frac{d^2\vec{r}}{dt^2} + G_{AB} M \frac{\vec{r}}{r^3} = \vec{F} \equiv \Delta\, \vec{g}}$$

SIMILAR TO A UNIFORM, CONSTANT ELECTRIC FIELD

SECULAR
$\Rightarrow$ INSTABILITY
OF RELATIVE ORBIT



ECCENTRICITY ↗ UP TO 1!
COLLISION!

- MUST TAKE INTO ACCOUNT RELATIVISTIC EFFECTS $\mathcal{O}(v^2/c^2)$

NOTABLY: RELATIVISTIC ADVANCE OF PERIASTRON: $\omega_R = nk = n \dfrac{3 G_{AB} M}{c^2 a (1-e^2)}$ ~~$\frac{7}{4}$~~ 1 INGR

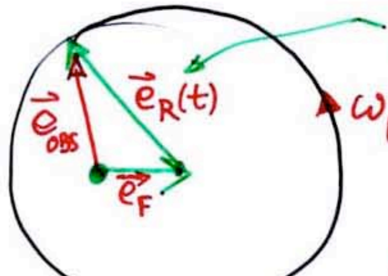
TOTAL ECCENTRICITY VECTOR

RELATIVISTICALLY PRECESSING ECCENTRICITY

$$\boxed{\vec{e}(t) = \vec{e}_F + \vec{e}_R(t)}$$

$$\boxed{\vec{e}_F = \frac{3}{2} \frac{\Delta g_\perp}{\omega_R n a}}$$

"FORCED" ECCENTRICITY VECTOR



UNKNOWN MAGNITUDE OF $|e_R|$
+
UNKNOWN ANGLE BETWEEN
$\vec{e}_F$, $\vec{e}_R$

PHENOMENOLOGICAL LIMITS ON $\Delta \equiv \left(\frac{m_g}{m_i}\right)_A - \left(\frac{m_g}{m_i}\right)_B$

USING. LONG-PERIOD, QUASI-CIRCULAR ($e \ll 1$)
NEUTRON STAR – WHITE DWARF ($\Delta_B \simeq 0 \Rightarrow \Delta \simeq \Delta_A$) SYSTEMS

Damour, Schäfer '91	$\lvert\Delta\rvert < 1.1 \times 10^{-2}$	90% CL
Arzoumanian '95	$\lvert\Delta\rvert < 1.4 \times 10^{-3}$	90% CL
Wex '97	$\lvert\Delta\rvert < 4 \times 10^{-3}$	90% CL
Wex '00	$\lvert\Delta\rvert < 9 \times 10^{-3}$	95% CL
Lorimer, Freire '04	$\lvert\Delta\rvert < 9 \times 10^{-4}$	90% CL
Stairs et al. '05	$\lvert\Delta\rvert < 5.3 \times 10^{-3}$	95% CL
	5.1×10^{-3} [WITH BAYESIAN ANALYSIS]	90% CL

SIMILAR PHENOMENOLOGICAL LIMITS CAN BE SET ON OTHER, CONCEIVABLE, SELF-GRAVITY-MODIFIED 'POST-EINSTEIN' PARAMETERS APPEARING IN BINARY PSRS.
E.G. $\hat{\alpha}_3$ ($\sim$ PPN α_3) ASSOCIATED TO VIOLATION OF MOMENTUM CONSERVATION

Will's book	$\lvert\hat{\alpha}_3\rvert < 2 \times 10^{-10}$	
Bell, Damour '96	$\lvert\hat{\alpha}_3\rvert < 2.2 \times 10^{-20}$	90% CL
Wex '00	$\lvert\hat{\alpha}_3\rvert < 1.5 \times 10^{-19}$	95% CL
Stairs et al '05	$\lvert\hat{\alpha}_3\rvert < 4.0 \times 10^{-20}$	95% CL

FOR MORE ON SUCH PHENOMENOLOG. LIMITS SEE. I. STAIRS'S, LIVING REVIEW NOTABLY ON $\dot{G}$

TWO WAYS OF PARAMETRIZING THE THEORY-SPACE

① GENERALIZE THE PPN FORMALISM TO HIGHER-ORDER TERMS IN $\frac{GM}{c^2 r}$

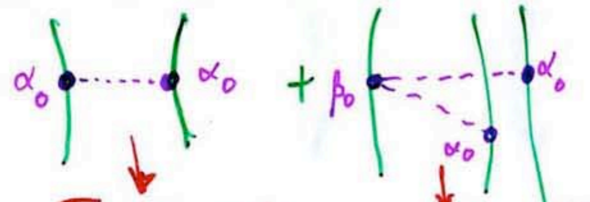
② CONSIDER SOME SPECIFIC, PARAMETER-DEPENDENT CLASSES OF TENSOR-SCALAR GRAVITY

GENERALIZED PPN FORMALISM 4.17

TENSOR-SCALAR GRAVITY : $a(\varphi) = \alpha_0 \phi + \frac{1}{2}\beta_0 \phi^2 + \dots$ (Damour, Esposito-Farèse '96)

MOST GENERAL COUPLING FUNCTION

AT FIRST PN APPROXIMATION



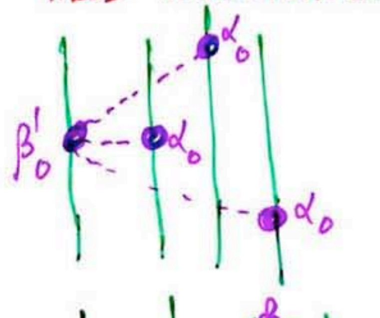
$\bar{\gamma} \equiv \gamma^{PPN} - 1 \propto \alpha_0^2$ $\qquad$ $\bar{\beta} \equiv \beta^{PPN} - 1 \propto \alpha_0 \beta_0 \alpha_0$

$$\bar{\gamma} = -2 \frac{\alpha_0^2}{1+\alpha_0^2}$$

$$\bar{\beta} = +\frac{1}{2} \frac{\alpha_0 \beta_0 \alpha_0}{(1+\alpha_0^2)^2}$$

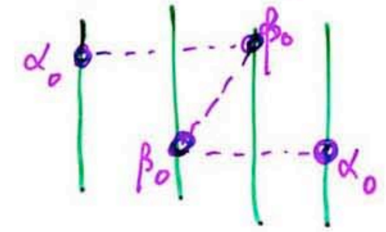
AT SECOND POST-NEWTONIAN APPROXIMATION

NEED HIGHER-ORDER EXPANSION: $a(\varphi) = \alpha_0 \phi + \frac{1}{2}\beta_0 \phi^2 + \frac{1}{6}\beta_0' \phi^3 + \dots$



$\rightarrow$

$$\boxed{\epsilon \equiv \frac{\beta_0' \alpha_0^3}{(1+\alpha_0^2)^3}}$$



$\rightarrow$

$$\boxed{\zeta \equiv \frac{\alpha_0 \beta_0 \beta_0 \alpha_0}{(1+\alpha_0^2)^3}}$$

- ONLY TWO 2PN PARAMETERS ϵ, ζ
- UNMEASURABLE IN SOLAR SYSTEM, EVEN WITH μarcsec LIGHT DEFLECTION BY SUN
- PARAMETRIZE THE (NEXT-TO) LEADING SELF-GRAVITY EFFECTS IN MOTION AND RADIATION OF BINARY PULSARS

USING PULSARS TO CONSTRAIN THE 2PN PARAMETERS ϵ, ζ

ONE PROVES (Damour Esposito-Farèse '92, '96)

$$\frac{G_{AB}}{G} = 1 + (4\bar{\beta} - \bar{\gamma})\left(\frac{E_A^g}{m_A c^2} + \frac{E_B^g}{m_B c^2}\right) + 4\zeta \frac{E_A^g}{m_A c^2}\frac{E_B^g}{m_B c^2} + \left(\frac{\epsilon}{2} + \zeta\right)\frac{\langle U^2\rangle_A + \langle U^2\rangle_B}{c^4} + \dots$$

$$\Rightarrow \boxed{\Delta_A \simeq \frac{1}{2}(4\bar{\beta} - \bar{\gamma})\, c_A + \left(\frac{\epsilon}{2} + \zeta\right) 1.03\, c_A^2}$$

COMPACTNESS $c_A \equiv -2 \frac{\partial \ln \tilde{m}_A}{\partial \ln G} \simeq 0.21 \frac{m_A}{M_\odot}$ EOS-DEPENDENT

FOR PULSARS ($m_A \simeq 1.4 M_\odot$) ONE EXPECTS $c_A \simeq 0.3$

SOLAR-SYSTEM (LUNAR LASER RANGING) EXPERIMENTS

$$\Rightarrow \quad 4\bar{\beta} - \bar{\gamma} = (4.4 \pm 4.5) \times 10^{-4}$$

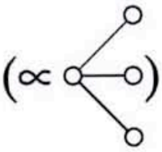
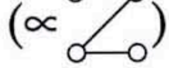
Williams, Turyshev, Boggs '04

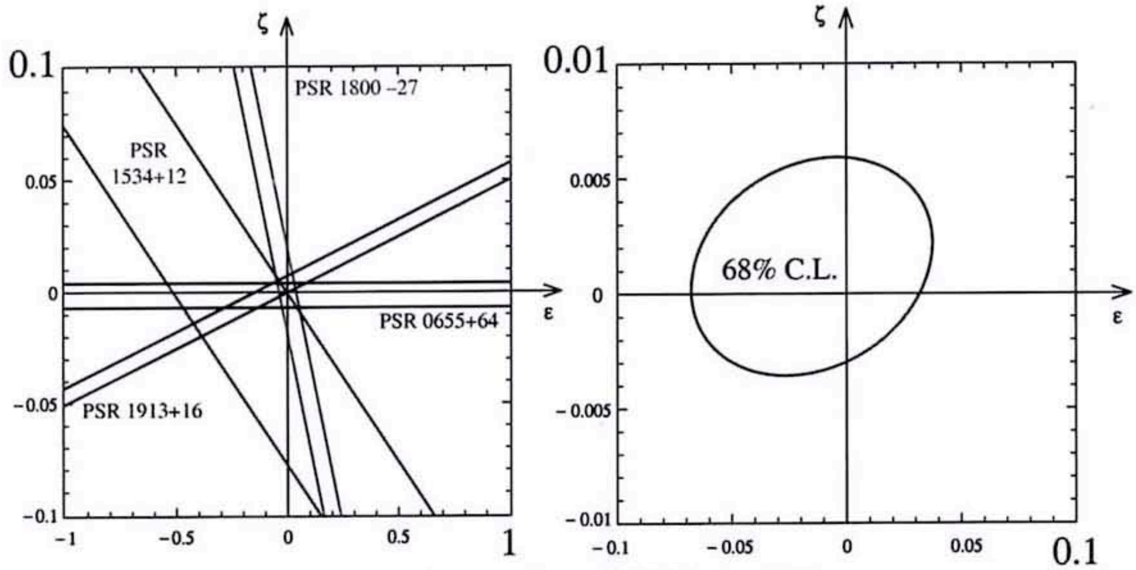
MOREOVER ϵ, ζ PARAMETRIZE ALSO

$$\bar{\gamma}_{AB} \sim \bar{\gamma} + \eta \cdot (c_A + c_B) + \epsilon\, \mathcal{O}(c^2) + \zeta\, \mathcal{O}(c^2)$$

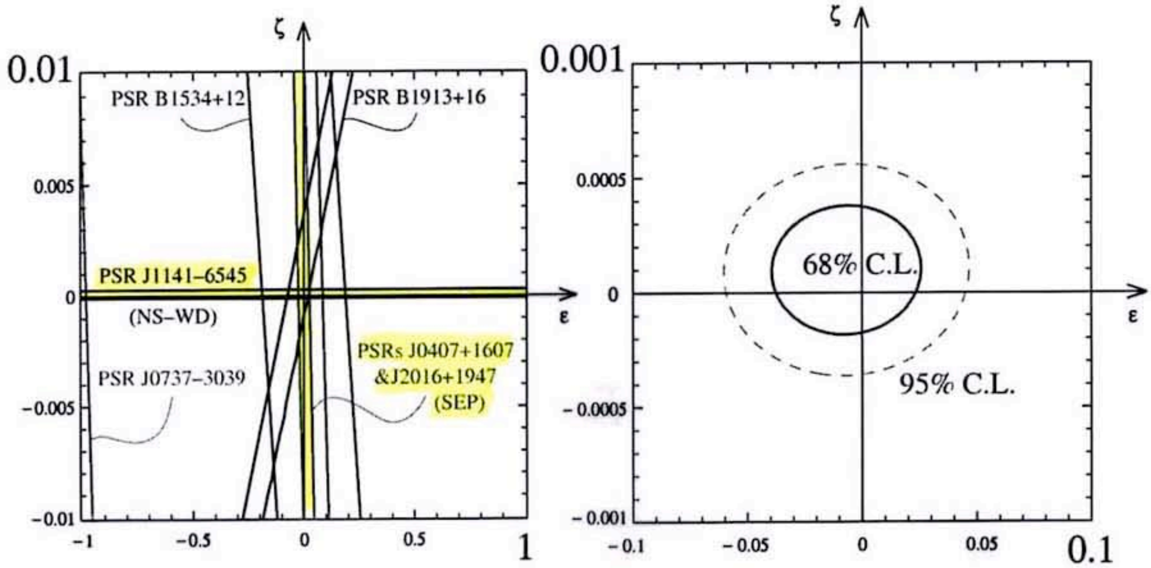
$$\bar{\beta}^A_{BC} \sim \bar{\beta} + \epsilon\, \mathcal{O}(c) + \zeta\, \mathcal{O}(c)$$

Binary pulsar constraints on the 2PN parameters

ε (∝ ) and ζ (∝ )



[Damour & Esposito-Farèse, PRD **53** (1996) 5541]



situation in 2004 [T.D. & G.E-F, in preparation]

⇒ 2× tighter constraints on ε ; 15× tighter constraints on ζ

$-4\times10^{-2} < \varepsilon < 3\times10^{-2}$ $\quad -2\times10^{-4} < \zeta < +4\times10^{-4}$

SECOND APPROACH TO THEORY-DEPENDENT ANALYSIS

CHOOSE A SPECIFIC, PARAMETRIZED CLASS OF TENSOR-SCALAR THEORIES OF GRAVITY

SIMPLE CHOICE $T(\alpha_0, \beta_0)$ DEFINED BY

COUPLING FUNCTION (Damour Esposito-Farèse '96)

$$\boxed{a(\varphi) = \alpha_0 (\varphi - \varphi_0) + \frac{1}{2} \beta_0 (\varphi - \varphi_0)^2}$$

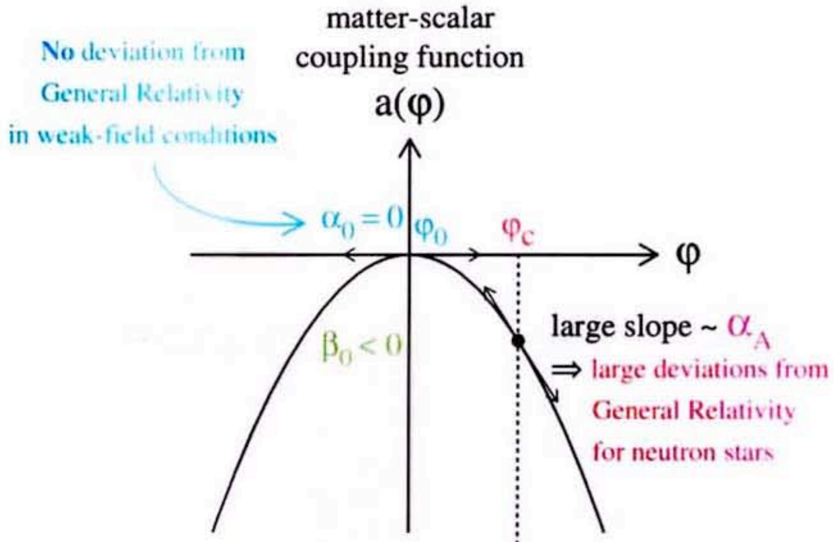
- SIMPLE GENERALIZATION OF JORDAN-FIERZ-BRANS-DICKE

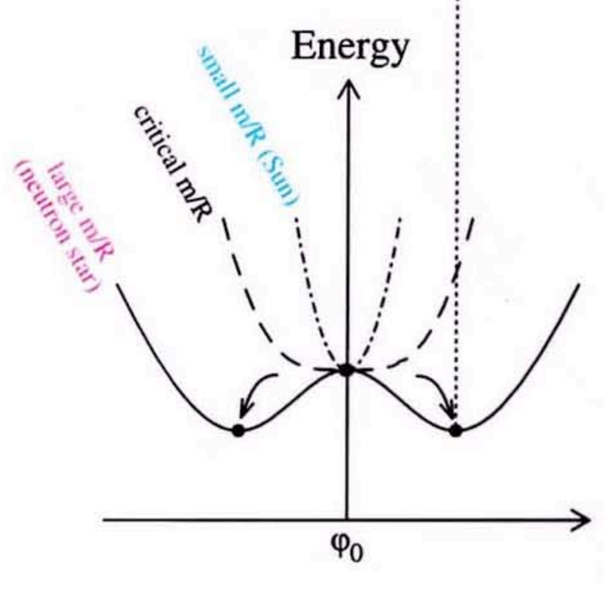
$$a^{JFBD}(\varphi) = \alpha_0 (\varphi - \varphi_0) \; ; \quad \alpha_0^2 = \frac{1}{2\omega + 3}$$

- MINIMAL THEORY LEADING TO PPN PARAMETERS:

$$\bar{\gamma} = -\frac{2\alpha_0^2}{1+\alpha_0^2} \; ; \; \bar{\beta} = \frac{1}{2} \frac{\beta_0 \alpha_0^2}{(1+\alpha_0^2)^2} \; ; \left[\epsilon = 0 \; ; \; \zeta \sim -8 \bar{\beta}^2/\bar{\gamma} \cdots\right]$$

- FEATURES INTERESTING NON-PERTURBATIVE STRONG-FIELD EFFECTS





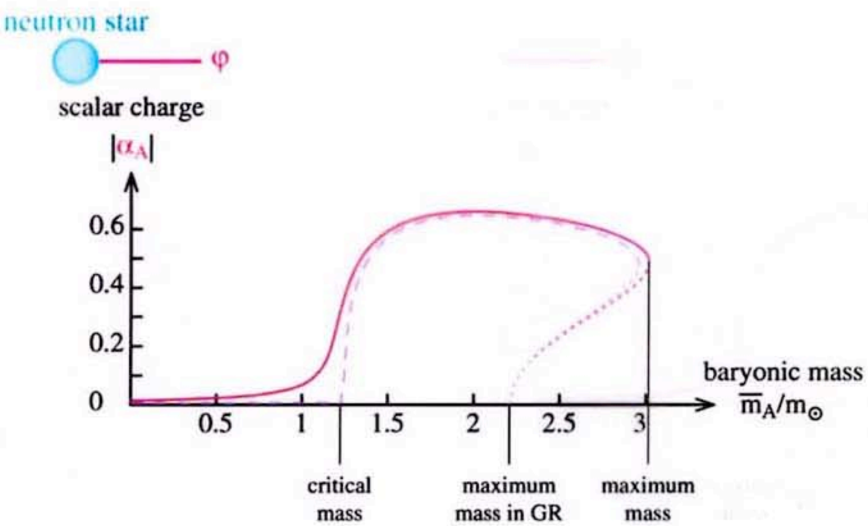
$$E \approx \int \left[\tfrac{1}{2} (\vec{\nabla}\varphi)^2 + \rho \, e^{\beta_0 \varphi^2/2} \right]$$

$$\downarrow \qquad\qquad \downarrow$$

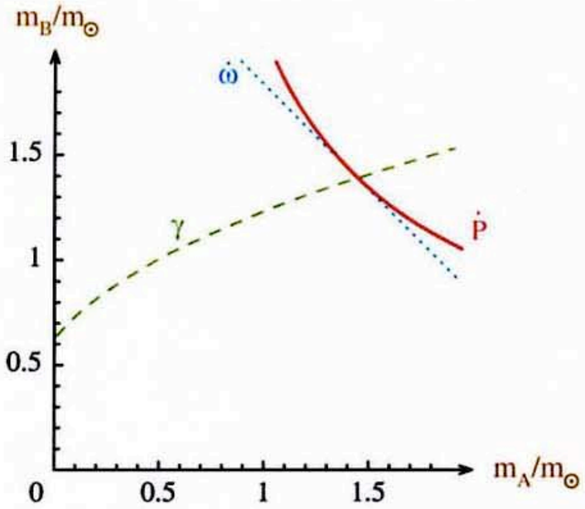
$$\tfrac{1}{2} R \, \varphi_c^{\,2} \; + \; m \, e^{\beta_0 \varphi_c^{\,2}/2}$$

parabola   Gaussian
if $\beta_0 < 0$

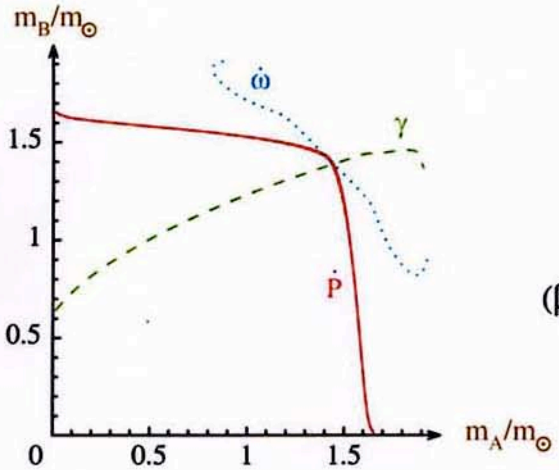
"spontaneous scalarization" [T. Damour & G.E-F 1993]



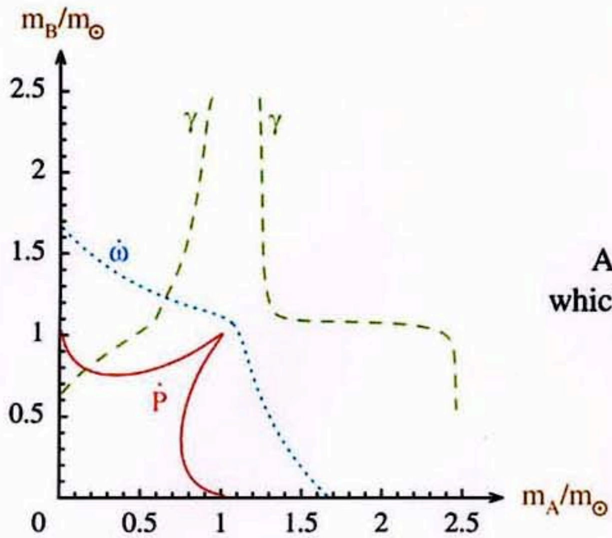
PSR B1913+16 in scalar-tensor theories



General relativity
passes the test

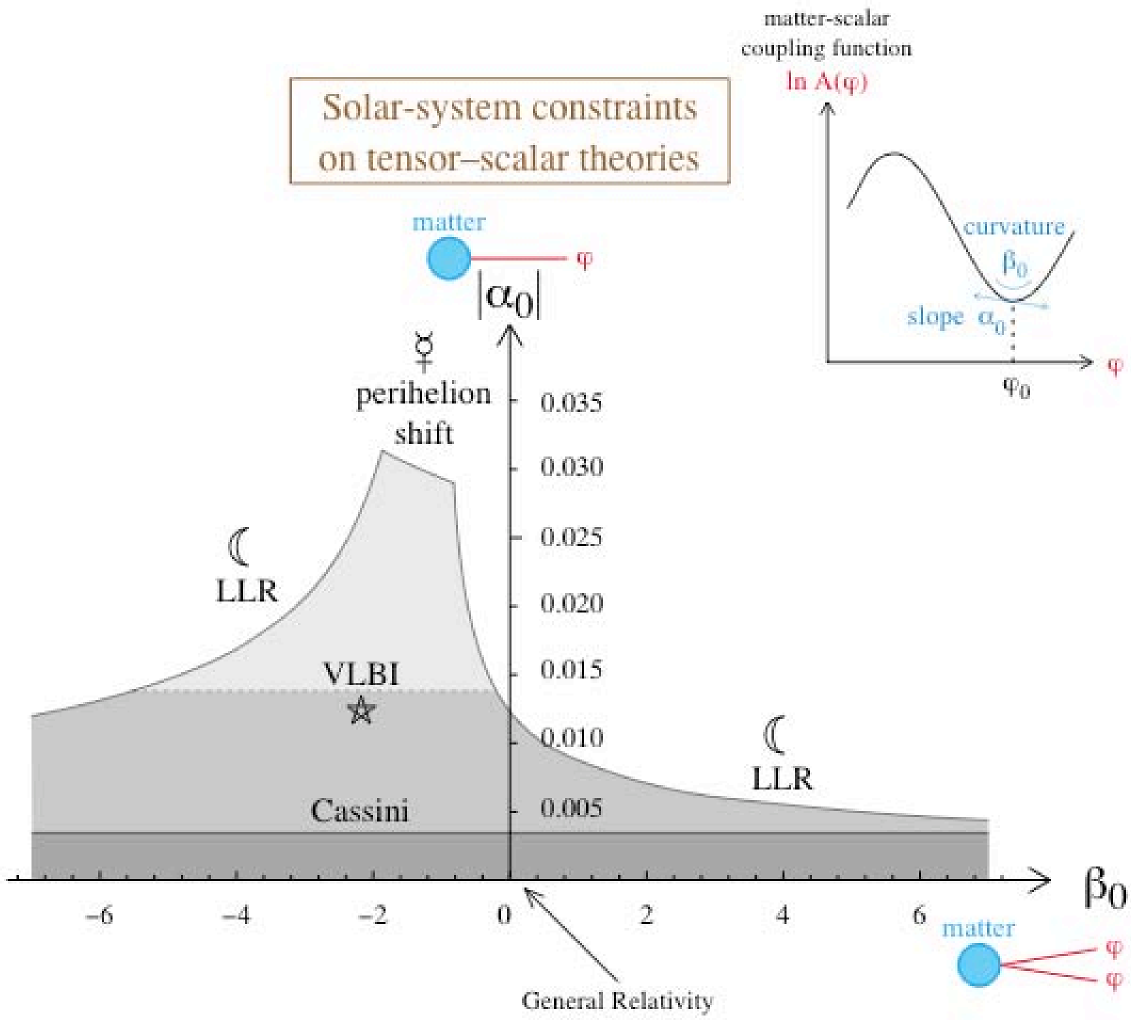


A tensor–scalar theory
which **passes the test**
($\beta_0 = -4.5$, α_0 small enough)



A tensor–scalar theory
which **does not pass the test**
($\beta_0 = -6$, any α_0)

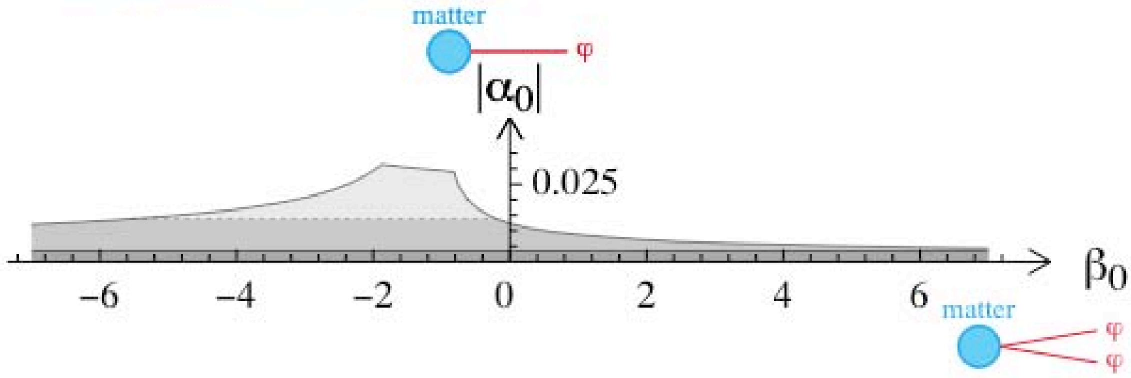
Solar-system constraints on tensor–scalar theories

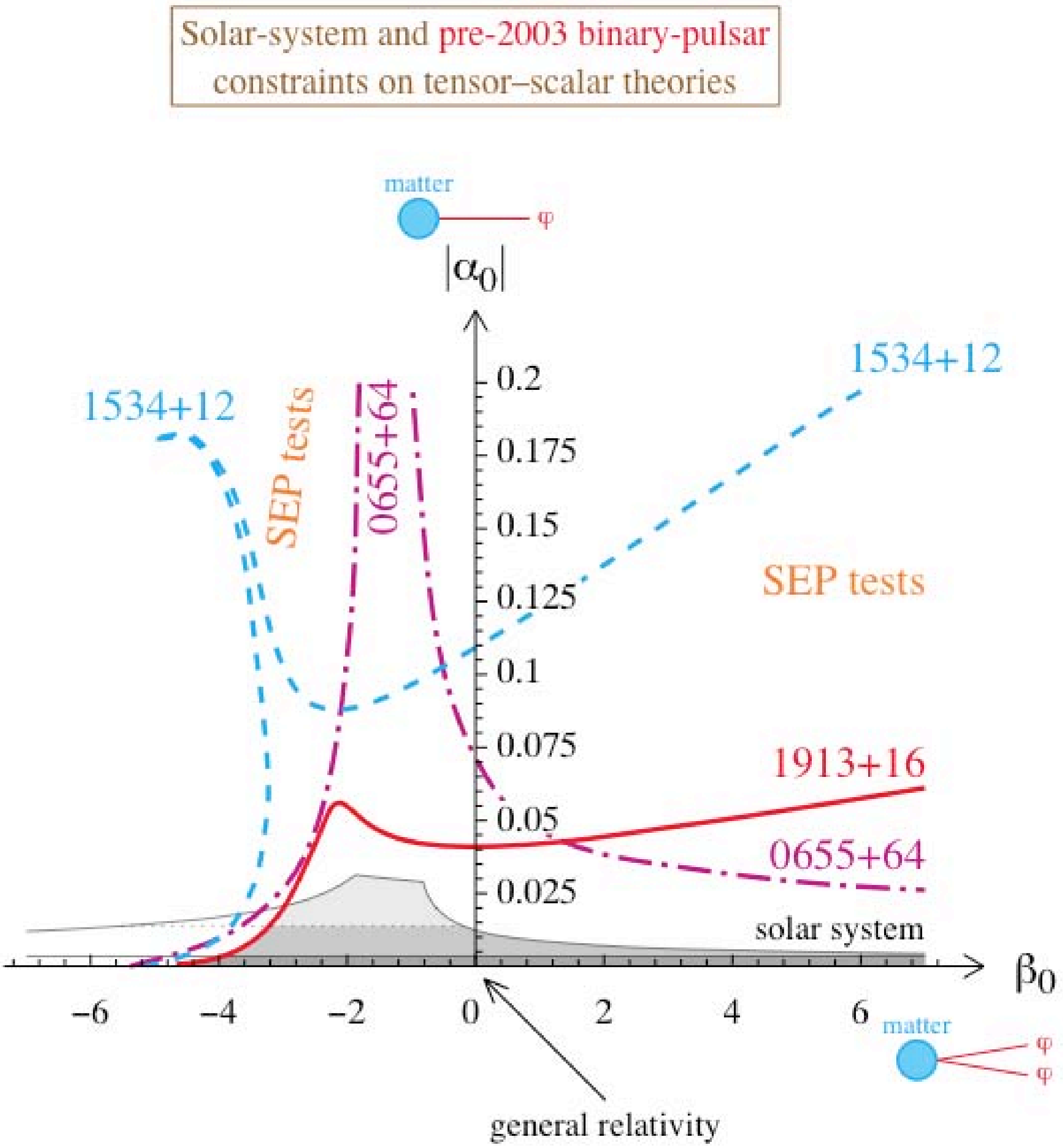


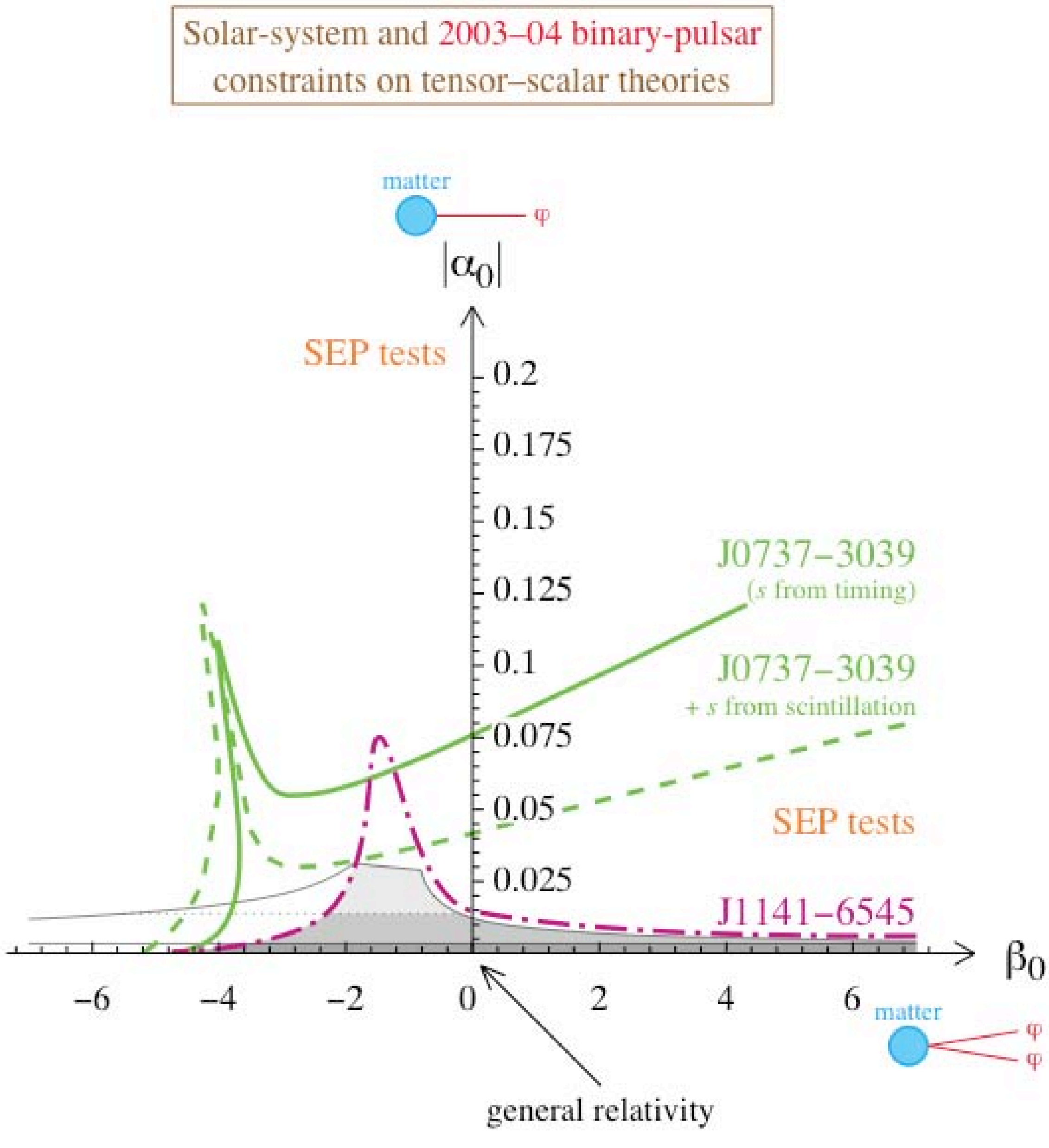
Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \frac{1}{2\,\omega_{BD} + 3}$

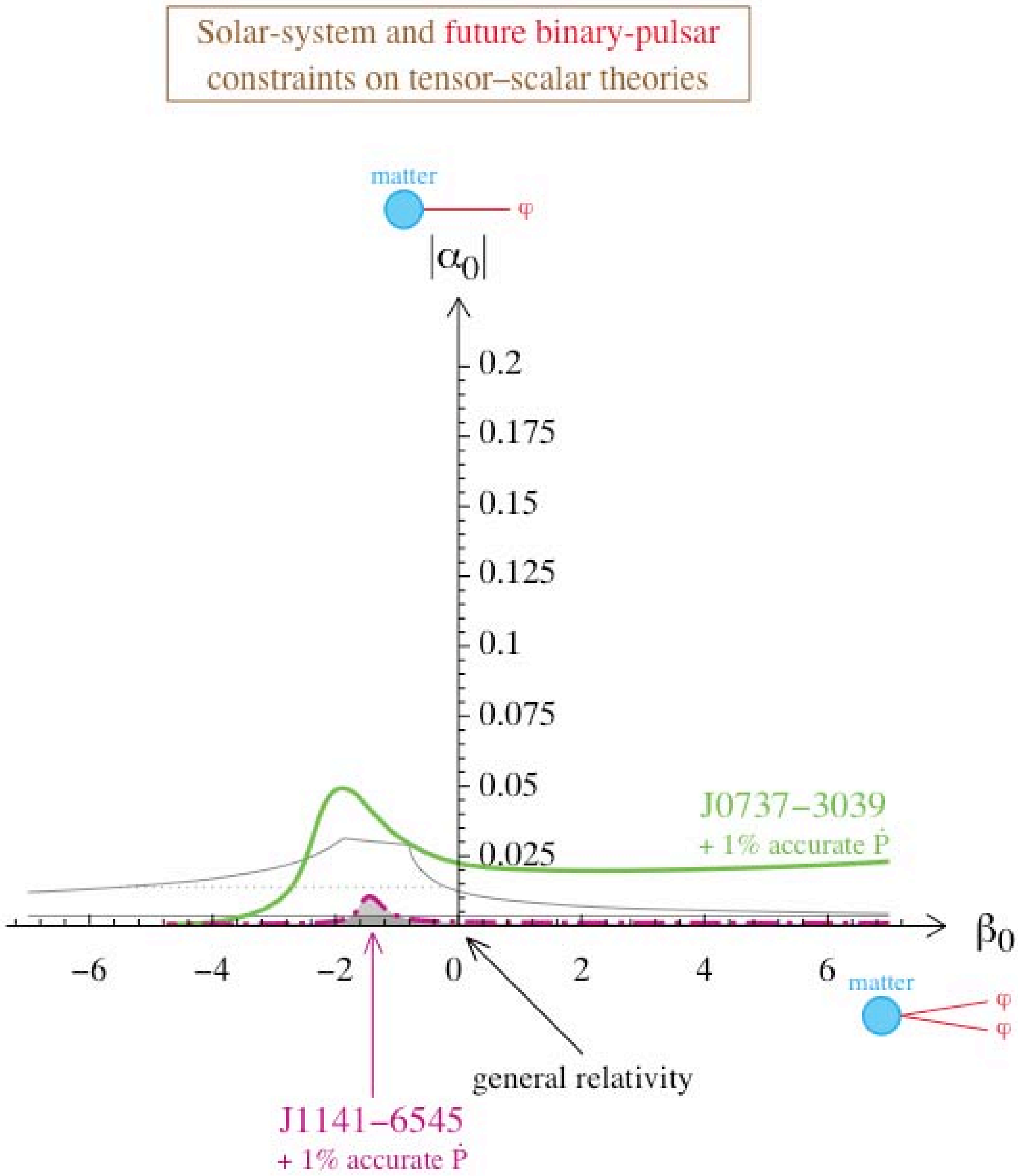
Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.

N.B.: Scale used in next plots

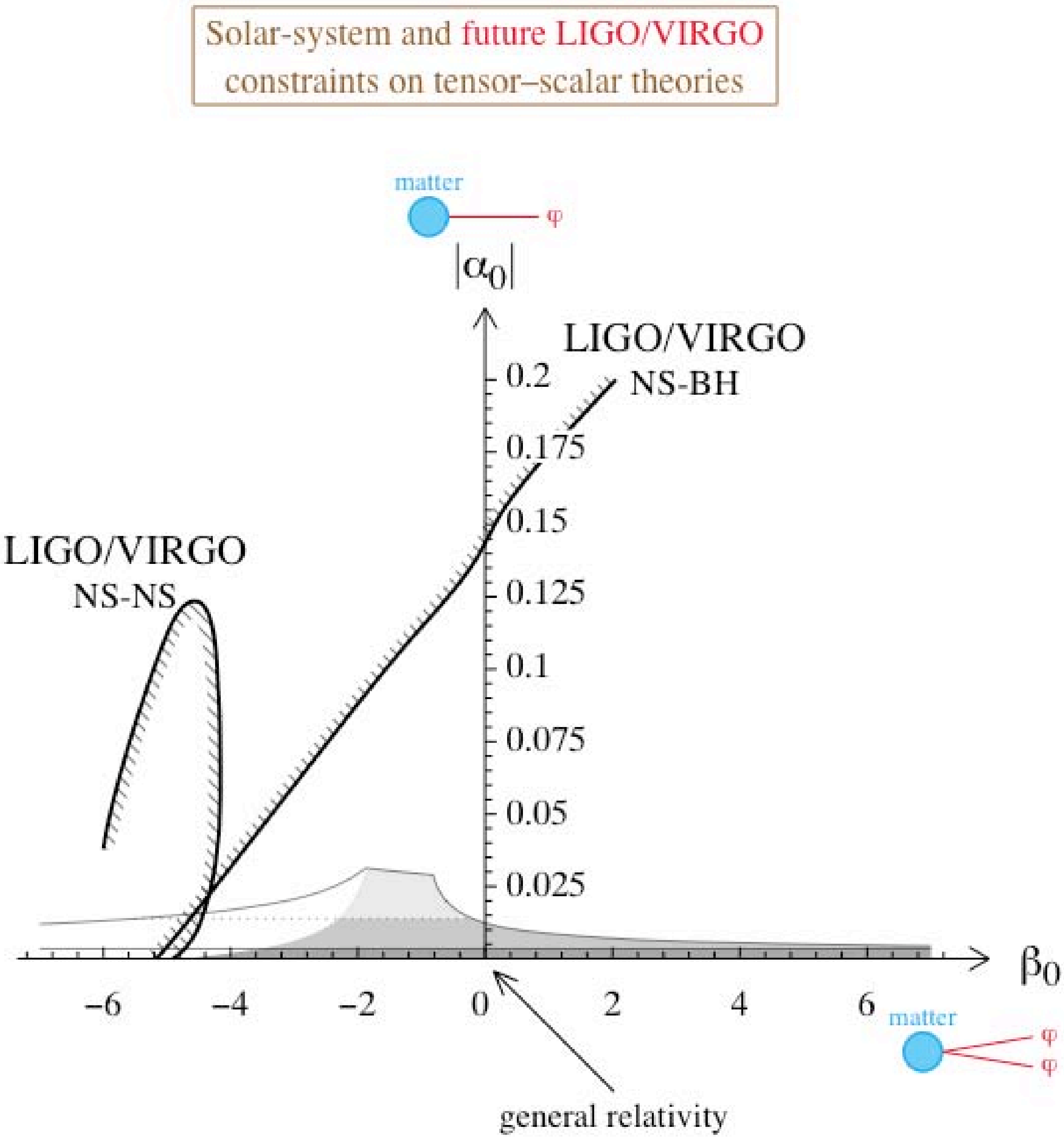



Solar-system and pre-2003 binary-pulsar
constraints on tensor–scalar theories
matter
φ
|α0|
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
1534+12
1534+12
SEP tests
0655+64
SEP tests
1913+16
0655+64
solar system
β0
−6
−4
−2
0
2
4
6
matter
φ
φ
general relativity


Solar-system and 2003–04 binary-pulsar constraints on tensor–scalar theories
matter
φ
$|\alpha_0|$
SEP tests
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
J0737−3039
(s from timing)
J0737−3039
+ s from scintillation
SEP tests
J1141−6545
β_0
−6
−4
−2
0
2
4
6
matter
φ
φ
general relativity



Solar-system and future binary-pulsar constraints on tensor–scalar theories
matter
φ
|α0|
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
J0737−3039
+ 1% accurate Ṗ
β0
−6
−4
−2
0
2
4
6
matter
φ
φ
general relativity
J1141−6545
+ 1% accurate Ṗ


Solar-system and future LIGO/VIRGO
constraints on tensor–scalar theories
matter
φ
$|\alpha_0|$
LIGO/VIRGO
NS-BH
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
LIGO/VIRGO
NS-NS
β_0
−6
−4
−2
0
2
4
6
matter
φ
φ
general relativity

SUMMARIZING

- CONTRASTING GR WITH ALTERNATIVE THEORIES IS A WAY TO EXPLORE WHICH STRUCTURES OF GR ARE TESTED BY SOME OBSERVATIONS

- THE CLASS OF TENSOR-SCALAR THEORIES OF GRAVITY IS
  - MOTIVATED BY FUNDAMENTAL PHYSICS (AND COSMOLOGY)
  - CONSISTENT WITH BASIC REQUIREMENTS (NO GHOSTS)
  - SIMPLE ENOUGH TO BE WORKED OUT IN DETAIL, EVEN FOR STRONG-FIELD EFFECTS
  - RICH ENOUGH TO EXHIBIT INTERESTING DEVIATIONS FROM GR, NOTABLY IN STRONG-FIELD REGIME

- THE DD (PPK) TIMING FORMULA PHENOMENOLOGICALLY DESCRIBES THE TIMING OF BINARY PULSARS IN THE MOST GENERAL TENSOR SCALAR THY

- THE RELATIONS $P_{PK}^{OBS} = f(p_K; m_A, m_B)$ ARE GENERICALLY MODIFIED BY SELF-GRAVITY EFFECTS (EXCEPT $R = x_B/x_A$)

- SUCH THEORY-DEPENDENT ANALYSES PROVE THAT THE PROBING POWER OF PSR TESTS IS QUALITATIVELY DIFFERENT FROM SOLAR-SYSTEM ONES:
  - ϵ, ζ ANALYSES: 2PN LEVEL UNREACHABLE IN SOLAR-SYSTEM
  - $T(\alpha_0, \beta_0)$ ANALYSES: $\beta_0 \lesssim -4.5$ EXCLUDED EVEN IF $\alpha_0 \lll 1$!

- PRESENT THEORY-DEPENDENT ANALYSES MAY HAVE EXHAUSTED THEIR POSSIBILITIES BUT THE VALUE OF THEORY-INDEPENDENT (PPK) TESTS REMAINS VERY HIG: ANY NEW TEST IS POTENTIALLY LETHAL TO GR (AND POSSIBLY TO TENSOR-SCALAR GRAVITY!)

- ALL THESE SUCCESSFUL RADIATIVE + STRONG-FIELD TESTS GIVE A VERY HIGH CONFIDENCE IN USING GR IN RELATIV. ASTROPHYS. (GW, COSMO, ...)

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1998 Phys Rev D 58, 042001

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G. Esposito-Farèse, gr-qc/0402007

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For GR Tests with 1141-6545 see Bailes et al. astro-ph/0307468

For recent results on 1913+16 see Weisberg, Taylor astro-ph/0211217

" " " 1534+12 see Stairs et al. astro-ph/0208357

For the "gravitational Stark effect" and limits on Δ see

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N. Wex 1996 [gr-qc/9511017] and 2000 [gr-qc/0002032]

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