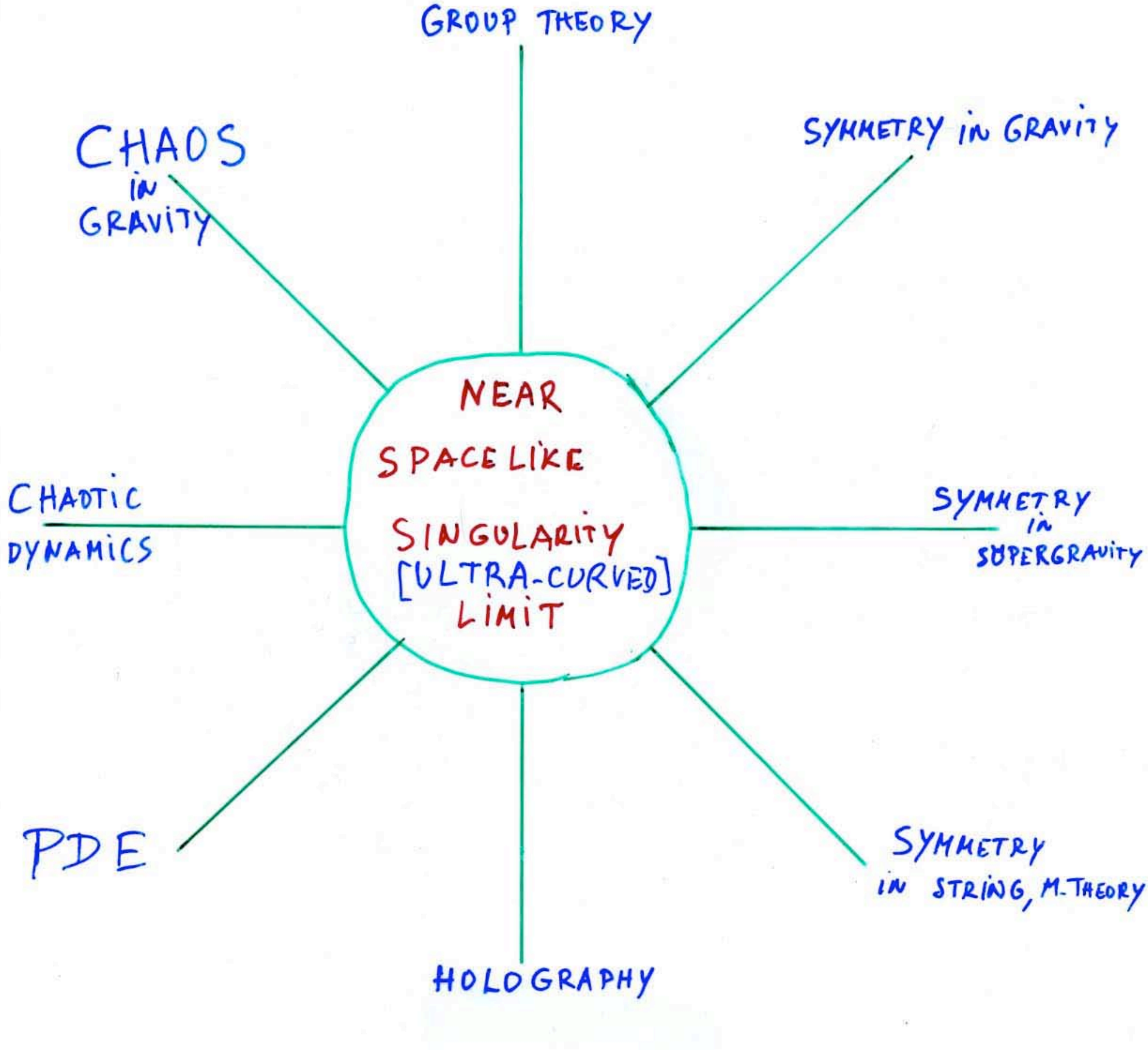
MICROSYMPOSIUM ON CYCLIC AND BOUNCING UNIVERSES
PRINCETON CENTER FOR THEORETICAL SCIENCE,
PRINCETON, NOVEMBER 7, 2008

DE-EMERGENCE OF SPACE AT A BIG CRUNCH

Thibault Damour
IHES

WORK WITH: HENNEAUX, NICOLAI, KLEINSCHMIDT, ...

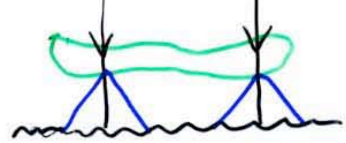

GROUP THEORY
SYMMETRY in GRAVITY
CHAOS in GRAVITY
NEAR SPACE LIKE SINGULARITY [ULTRA-CURVED] LIMIT
CHAOTIC DYNAMICS
SYMMETRY in SUPERGRAVITY
PDE
SYMMETRY in STRING, M-THEORY
HOLOGRAPHY

COSMOLOGICAL SINGULARITIES

- NOT AS A MODEL OF THE EARLY UNIVERSE

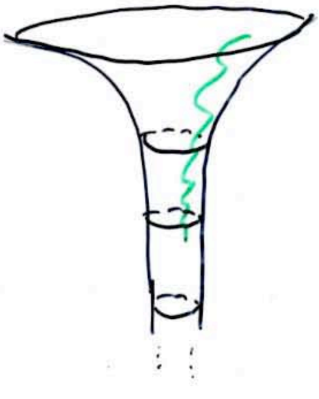
- BUT AS A TOOL FOR PROBING THE STRUCTURE OF M-THY, AND, IN PARTICULAR, FOR SEARCHING FOR HIDDEN SYMMETRIES

NEAR SPACELIKE SINGULARITY LIMIT



? $\updownarrow$

NEAR HORIZON LIMIT

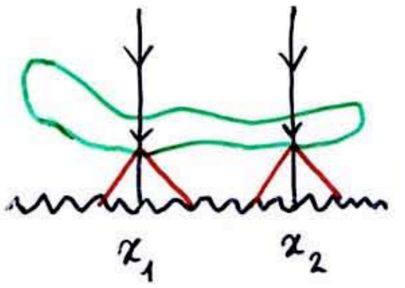


$\downarrow$

HOLOGRAPHIC CORRESPONDENCE

STRINGS ON $AdS_5 \times S_5 \longleftrightarrow CFT_4$

NEAR SPACELIKE SINGULARITY LIMIT AND A $SUGRA_{11} / [E_{10}/K(E_{10})]_1$ CORRESPONDENCE



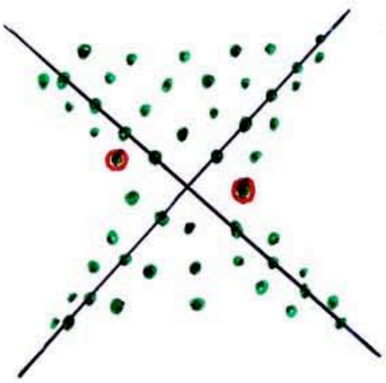
$T=0$

GRADIENT EXPANSION (BKL)

($\sim$ SMALL TENSION EXPANSION : $\alpha' \to \infty$)

$$\partial_{x^1}^{k_1} \partial_{x^2}^{k_2} \cdots \partial_{x^{10}}^{k_{10}} \ll \partial_T^{k_1+k_2+\cdots+k_{10}}$$

$$\updownarrow$$



HEIGHT EXPANSION IN KAC-MOODY ALGEBRA

ROOT : $\alpha = n_0 \alpha_0 + n_1 \alpha_1 + \cdots + n_9 \alpha_9$

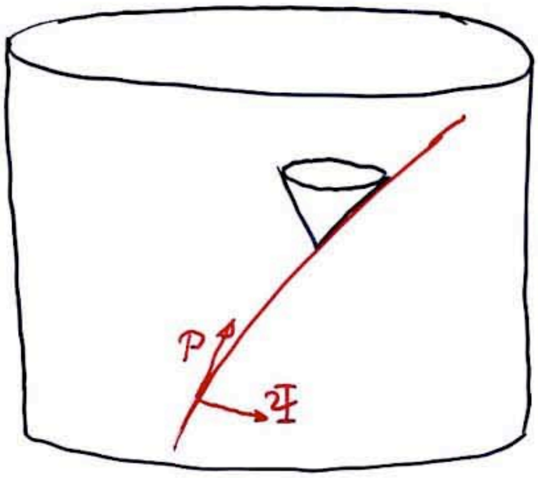
CONJECTURED CORRESPONDENCE

SUGRA$_{11}$ (OR M-THEORY)

MASSLESS SPINNING PARTICLE ON COSET $E_{10}/K(E_{10})$

$$G_{\mu\nu}(t,\vec{x})$$
$$A_{\mu\nu\lambda}(t,\vec{x})$$
$$\psi_\mu(t,\vec{x})$$

$\longleftrightarrow$



$$S_{11} = \int d^{11}x \Big\{ \frac{E}{4} R(G) - \frac{E}{48}(dA_3)^2 + \frac{2}{(12)^4} F_4 \wedge F_4 \wedge A_3 - \frac{i}{2}\bar{\psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_\rho - \frac{i}{96}\left(\bar{\psi}_\mu \Gamma^{\mu\alpha\beta\gamma\delta\nu}\psi_\nu + 12\,\bar{\psi}^\alpha \Gamma^{\beta\gamma}\psi^\delta\right) F_{\alpha\beta\gamma\delta} + \cdots \Big\}$$

+ LOOP CORRECTIONS

$\longleftrightarrow$

$$S_1^{\text{COSET}} = \int dt \Big\{ \frac{1}{4n}\langle P(t) \,|\, P(t)\rangle - \frac{i}{2}\left(\Psi(t) \,|\, \overset{vs}{D}\, \Psi(t)\right)_{vs} + \left(\chi(t) \,|\, P(t) \odot \Psi(t)\right)_s \Big\}$$

T.D., Henneaux, Nicolai '02 ; T.D., Kleinschmidt, Nicolai '06 ; de Buyl, Henneaux, Paulot '06

BASIC IDEA

TWO 'DUAL' OR 'COMPLEMENTARY' DESCRIPTIONS

SUPERGRAVITY DESCRIPTION	COSET DESCRIPTION
$G_{\mu\nu}(t,\vec{x}),\ A_{\mu\nu\lambda}(t,\vec{x}),\ \psi_\mu(t,\vec{x})$	$g(t) \in E_{10}/K(E_{10})$
$\mathcal{R} \ll \ell_P^{-2}$	$\mathcal{R} \gg \ell_P^{-2}$

$$1 \qquad \ell_P^2 \mathcal{R}$$

THE 'SINGULARITY' IS 'RESOLVED' BY THE EFFECTIVE 'DISAPPEARANCE' OF SPACE, AND THE REPLACEMENT OF DYNAMICAL FIELDS, $g_{ij}(t,\vec{x}), A_{ijk}(t,\vec{x}), \dots$ BY A LIE-ALGEBRAIC VARIABLE $g(t) \in E_{10}/K_{10}$

HOLOGRAPHIC OR PHOTOGRAPHIC (POLYAKOV) CORRESPONDENCE

$$\left.\begin{array}{l} g_{ij}(t,\vec{x}) \\ A_{ijk}(t,\vec{x}) \\ \psi_i(t,\vec{x}) \end{array}\right\} \longleftrightarrow \left\{ g(t) = e^{h^a{}_b(t) K^b{}_a}\, e^{A_{abc}(t) E^{abc} + \partial_{[a} A_{bcd]} E^{\cdots} + \cdots} \right.$$

KAC-MOODY ALGEBRAS

SU(2) ≅ A_1 IS SL(2)

$$[\underset{\substack{\uparrow\\ \text{CARTAN GENERATOR (DIAGONAL)}}}{J_z}, \underset{\substack{\uparrow\\ \text{RAISING GENERATOR}}}{J_+}] = +J_+ \qquad [\underset{\substack{\uparrow\\ \text{CARTAN GENERATOR}}}{J_z}, \underset{\substack{\uparrow\\ \text{LOWERING GENERATOR}}}{J_-}] = -J_-$$

CARTAN SUBALGEBRA : LINEAR SPACE $\mathbb{R}^{r}$ (RANK r) $\mathfrak{h} = \{ \beta^a h_a \,;\, a = 1, \dots, r \}$

β^a: coordinates in Cartan space: $h = \sum_a \beta^a h_a$

h_a: r independent Cartan gators

$$[h', h''] = 0$$

<u>TRIANGULAR DECOMPOSITION:</u>

$$\mathfrak{g} = \underset{\substack{\uparrow\\ \text{LOWERING GATORS}\\ F_\alpha}}{\mathfrak{n}^-} \oplus \underset{\substack{\uparrow\\ \text{CARTAN}\\ h}}{\mathfrak{h}} \oplus \underset{\substack{\uparrow\\ \text{RAISING GATORS}\\ E_\alpha}}{\mathfrak{n}^+}$$

$$\boxed{[\underset{\substack{\uparrow\\ \text{Cartan}\\ h = \sum_a \beta^a h_a}}{h}, \underset{\substack{\uparrow\\ \text{RAISING GENERATOR(S)}}}{E_\alpha^{(s)}}] = \alpha(h)\, E_\alpha^{(s)}}$$

(s): DEGENERACY INDEX

$\alpha(h)$: ROOT ≡ EIGENVALUE OF ad_h AS A LINEAR FORM OF $h \in$ CSA

$$h = \beta^a h_a \to \alpha(h) = \alpha_a \beta^a \equiv \alpha(\beta)$$

$$\boxed{\begin{aligned} [E_\alpha^{(s)}, E_\beta^{(t)}] &= C_{\alpha\beta}^{(stu)} E_{\alpha+\beta}^{(u)} \\ [h, F_\alpha^{(s)}] &= -\alpha(h)\, F_\alpha^{(s)} \end{aligned}}$$

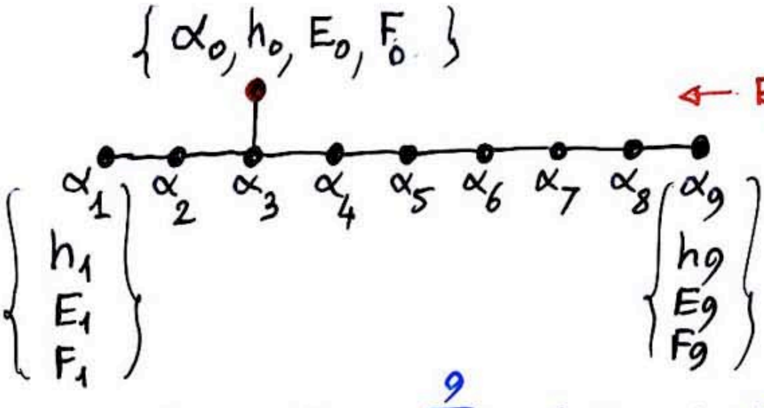
LOWERING GENERATORS : $F_\alpha^{(s)} \equiv E_{-\alpha}^{(s)}$

+ JACOBI + SERRE RELATIONS

E_{10}

rank 10 : $\dim \mathfrak{h} = 10$ AND $\exists$ 10 basic raising gators E_{α_i}

10 SIMPLE ROOTS



$\leftarrow E_0 = E^{123} \in E^{[abc]}$; $F_0 = F_{123} \in F_{[abc]}$

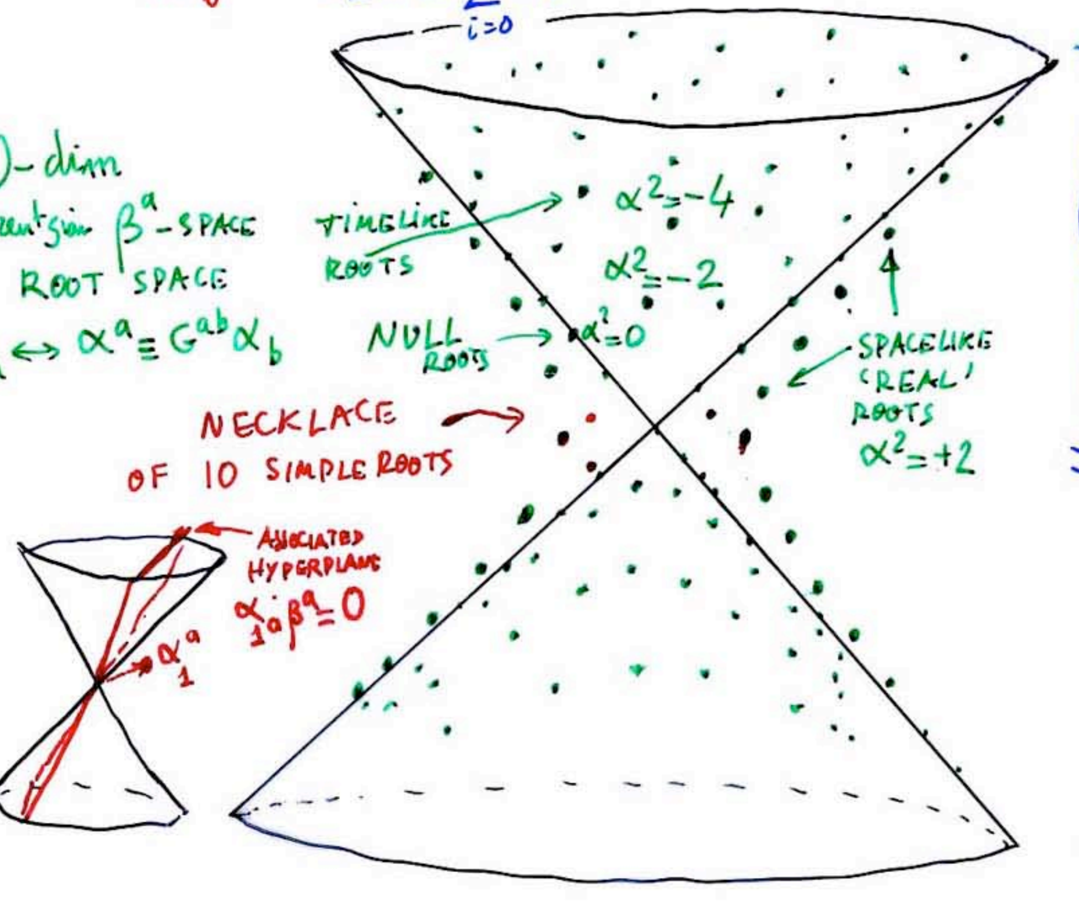
WITH h_0 DEFINES GL_{10} SUBALGEBRA

$$\{\text{ALL ROOTS}\} = \underbrace{\left\{\alpha = \sum_{i=0}^{9} n^i \alpha_i \,;\, n^i \in \mathbb{N}\right\}}_{\text{POSITIVE ROOTS}} \cup \underbrace{\left\{\alpha = -\sum_{i=0}^{9} n^i \alpha_i \,;\, n^i \in \mathbb{N}\right\}}_{\text{NEGATIVE ROOTS}}$$

height $ht[\alpha] = \sum_{i=0}^{9} n_i$

10-dim Lorentzian β^a-SPACE $\simeq$ ROOT SPACE

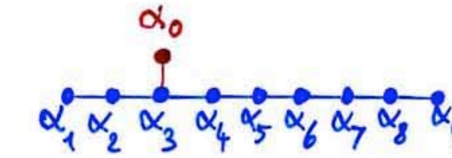
$\alpha_a \leftrightarrow \alpha^a \equiv G^{ab}\alpha_b$



POSITIVE ROOTS

NEGATIVE ROOTS

DECOMPOSING E_{10} W.R.T. $GL(10)$ SUBALGEBRA



← $SL(10) = A_9$ SUBALGEBRA

"LEVEL" ℓ : $\alpha = \ell\,\alpha_0 + \sum_{j=1}^{9} m^j \alpha_j$

$\ell = 0$ GL(10) GENERATORS $K^a{}_b$ $[K^a{}_b, K^c{}_d] = \delta^c_b K^a{}_d - \delta^a_d K^c{}_b$

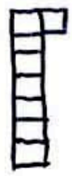
$\ell = \pm 1$ $E^{[a_1 a_2 a_3]}$, $F_{[a_1 a_2 a_3]}$

3 INDICES

$\ell = \pm 2$ $E^{[a_1 \cdots a_6]}$, $F_{[a_1 \cdots a_6]}$

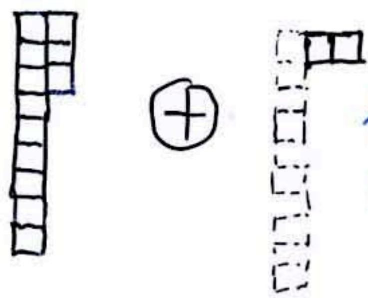
6 INDICES

$\ell = \pm 3$ $E^{[a_0 | a_1 \cdots a_8]}$, $F_{[a_0 | a_1 \cdots a_8]}$



9 INDICES

$\ell = \pm 4$

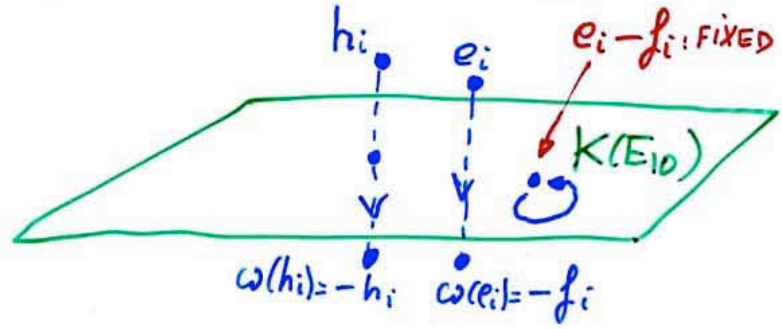


12 INDICES

$\vdots$

$K(E_{10})$: MAXIMAL COMPACT SUBGROUP OF THE CANONICAL REAL FORM OF E_{10}

FIXED SET OF
CHEVALLEY INVOLUTION ω



GL(10) DECOMPOSITION OF $K(E_{10})$

$$J^{ab} = K^a{}_b - K^b{}_a \quad : SO(10) \quad [J^{ab}, J^{cd}] = 4\,\delta^{[b}_{[c}\, J^{a]}_{d]}$$

$$J^{a_1a_2a_3} = E^{a_1a_2a_3} - F_{a_1a_2a_3} \qquad [J^{a_1a_2a_3}, J^{b_1b_2b_3}] = J^{a_1a_2a_3b_1b_2b_3} - 18\,\delta^{a_1b_1}\delta^{a_2b_2} J^{a_3b_3}$$

$$J^{a_1a_2\cdots a_6} = E^{a_1\cdots a_6} - F_{a_1\cdots a_6} \qquad [J^3, J^6] = J^9 + J^3$$

$$J^{a_0|a_1\cdots a_8} = E^{a_0|a_1\cdots a_8} - F_{a_0|a_1\cdots a_8} \qquad [J^3, J^9] = J^{12} + J^6$$

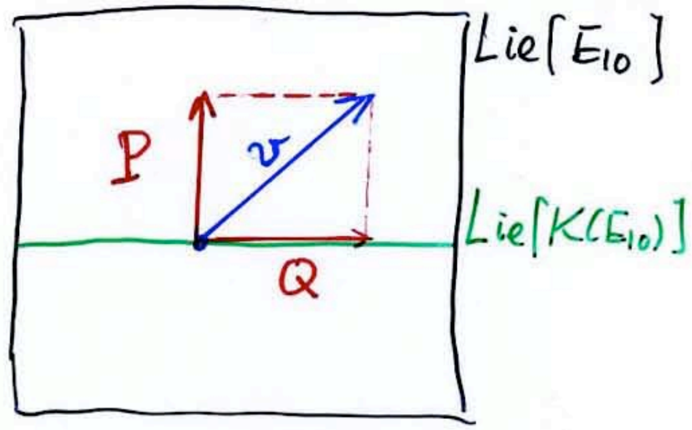
⋮

$D=1 \quad E_{10}/K(E_{10})$ COSET MODEL

GROUP ELEMENT $\quad g(t) \in E_{10}$

$Lie[E_{10}]$ 'VELOCITY': $\boxed{v \equiv \frac{dg}{dt} g^{-1}}$

DECOMPOSING v



$$\underset{Lie(E_{10})}{\overset{\cap}{v}} \equiv \underset{Lie(K(E_{10}))^{\perp}}{\overset{\cap}{P}} + \underset{Lie(K(E_{10}))}{\overset{\cap}{Q}}$$

'VERTICAL' 'HORIZONTAL'

COSET ACTION

BOSONIC PART

'VERTICAL' PART OF VELOCITY $v = \dot{g} g^{-1}$

$$\boxed{S_{1\,BOS}^{COSET} = \int dt \, \frac{1}{4n} \langle P(t) \mid P(t) \rangle}$$

UNIQUE INVARIANT QUADRATIC FORM OF $Lie(E_{10})$

SIGNATURE: $\underbrace{-+++++++++}_{CARTAN}$; $\underbrace{+++++\cdots\cdots}_{K(E_{10})^{\perp}}$; $\cancel{\underbrace{-----\cdots}_{K(E_{10})}}$

SYMMETRY: $\quad g(t) \longrightarrow \underset{\text{LOCAL } K(E_{10})}{k(t)} \, g(t) \, \underset{\text{GLOBAL } E_{10}}{g_0}$

$$P(t) \longrightarrow k(t) \, P(t) \, k(t)^{-1}$$

$$Q(t) \longrightarrow k(t) \, Q(t) \, k^{-1}(t) + \partial_t k \, k^{-1}$$

HORIZONTAL VELOCITY: $K(E_{10})$ CONNECTION

FERMIONIC PART

$$\boxed{S_{1\,FERM}^{COSET} = -\frac{i}{2} \int dt \left(\Psi(t) \mid \overset{VS}{D} \Psi(t) \right)_{VS} + \int dt \left(\chi(t) \mid P(t) \circ \Psi(t) \right)_{S}}$$

'VECTOR-SPINOR REPRES. OF $K(E_{10})$': $\quad \Psi = (\psi_a , \overset{?}{\cdots})$

EXPLICIT PARAMETRIZATION OF $E_{10}/K(E_{10})$

$$g(t) = e^{h^a{}_b(t) K^b{}_a} \; e^{\frac{1}{3!} A_{a_1a_2a_3}(t) E^{a_1a_2a_3} + \frac{1}{6!} A_{a_1\cdots a_6} E^{a_1\cdots a_6} + \frac{1}{9!} A_{a_0|a_1\cdots a_8} E^{a_0|a_1\cdots a_8} + \cdots}$$

$GL(10): K^a{}_b$ $\qquad$ $A_{a_1a_2a_3}$ $\qquad$ $A_{a_1\cdots a_6}$ $\qquad$ $A_{a_0|a_1\cdots a_8}$ $\qquad + \cdots$

h^a_b

$$g^{ab}(t) = (e^h)^a{}_c (e^h)^b{}_c$$

indices raised by g^{ab}

$$\boxed{S_1^{E_{10}/K(E_{10})} = \int \frac{dt}{n(t)} \Big[\frac{1}{4} \left(g^{ac} g^{bd} - g^{ab} g^{cd} \right) \dot{g}_{ab} \dot{g}_{cd} + \frac{1}{2}\frac{1}{3!} \dot{A}_{a_1a_2a_3} \dot{A}^{a_1a_2a_3} + \frac{1}{2}\frac{1}{6!} DA_{a_1\cdots a_6} DA^{a_1\cdots a_6} + \frac{1}{2}\frac{1}{9!} DA_{a_0|a_1\cdots a_8} DA^{a_0|a_1\cdots a_8} + \cdots}$$

$$DA_{a_1\cdots a_6} = \dot{A}_{a_1\cdots a_6} + 10\, A_{[a_1\cdots a_3} \dot{A}_{a_4\cdots a_6]}$$

$$DA_{a_0|a_1\cdots a_8} = \dot{A}_{a_0|a_1\cdots a_8} + 42\, A_{\langle a_1\cdots a_3} \dot{A}_{a_4\cdots a_9\rangle} - 42\, \dot{A}_{\langle a_1\cdots a_3} A_{a_4\cdots a_9\rangle} + 280\, A_{\langle a_1\cdots a_3} A_{a_4\cdots a_6} \dot{A}_{a_7\cdots a_9\rangle}$$

$\langle \cdots \rangle$ = projection on Young

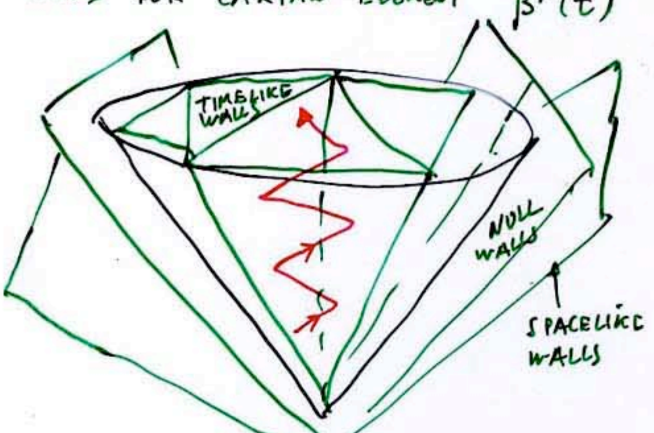
M24
~~ADI~~16

CORRESPONDENCE $E_{10}/K(E_{10})$ COSET $\Longleftrightarrow$ $SUGRA_{11}$

$$\mathcal{L}_1^{E_{10}} \sim (g^{-1}\dot g)^2 + (\dot A_3)^2 + (\dot A_6 + A_3 \dot A_3)^2 + (\dot A_9 + A_6 \dot A_3 + A_3 A_3 \dot A_3)^2 + \dots$$

$\Downarrow$

BILLIARD WITH INFINITE NUMBER OF EXPONENTIAL WALLS FOR CARTAN ELEMENT $\beta^{a}(t)$



$$\mathcal{H}_1 = \frac{1}{2} G^{\mu\nu}\pi_\mu\pi_\nu + \sum c_\alpha(q,p)\, e^{-2\alpha(\beta)}$$

$$\alpha(\beta) = \sum_i m^i \alpha_i(\beta)$$

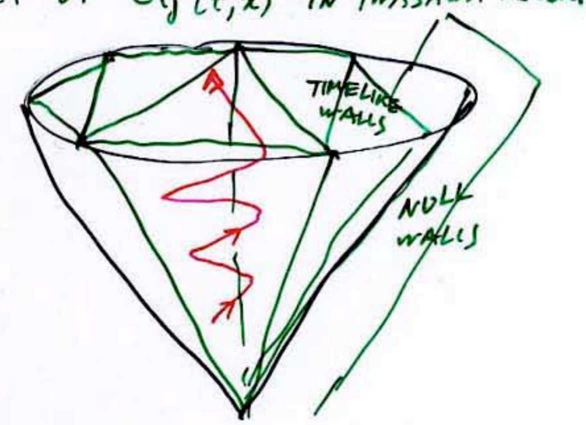
$m^i \in \mathbb{N}$ — SIMPLE ROOTS

$$\mathcal{F}_4 = d\mathcal{A}_3$$

$$\mathcal{L}_{11}^{SUGRA} = \int d^{11}x \left[\sqrt{-G}\left[R(G) - \frac{(d\mathcal{A}_3)^2}{48}\right] + \frac{1}{(12)^4}\mathcal{F}_4 \wedge \mathcal{F}_4 \wedge \mathcal{A}_3\right]$$

$\Downarrow$

BILLIARD WITH LARGE BUT FINITE # OF EXPONENTIAL WALLS FOR $\beta^a(t,x)$, DIAGONAL PART OF $G_{ij}(t,x)$ IN IWASAWA DECOMP.



$$\mathcal{H}_{11} = \frac{1}{2} G^{\mu\nu}\pi_\mu\pi_\nu + \sum_A c_A(Q,P,\partial_x\beta,\partial_x Q..) \times e^{-2 w_A(\beta)}$$

$$w_A(\beta) = \sum_i m^i w_i(\beta)$$

DOMINANT WALLS

$\boxed{\text{DICTIONARY}}$

$$g^{ab}(t) = (e^h)^a{}_c (e^h)^b{}_c = G^{ab}(t,\vec{x}_0)$$

WRT A SPECIAL FRAME

$$\dot A_{a_1 a_2 a_3}(t) = \mathcal{F}_{0 a_1 a_2 a_3}(t,\vec{x}_0)$$

$$\theta^a(x) = e^a_i(x)\, dx^i$$

$$DA^{a_1 \dots a_6}(t) = g^{a_1 a_1'} \cdots g^{a_6 a_6'}\left[\dot A_{a_1' \dots a_6'} + 10 A_{[3} \dot A_{3]}\right] = -\frac{1}{4!}\varepsilon^{a_1 \dots a_6 b_1 \dots b_4} \mathcal{F}_{b_1 \dots b_4}(t,\vec{x}_0)$$

$$DA^{b|a_1 \dots a_8}(t) = g^{\cdot\cdot}\left[\dot A_9 + 42 A_3 \dot A_6 + 280 A_3 A_3 \dot A_3\right] = +\frac{3}{2}\varepsilon^{a_1 \dots a_8 b_1 b_2} C^b{}_{b_1 b_2}(\vec{x}_0)$$

$$d\theta^a = \frac{1}{2} C^a{}_{bc}\, \theta^b \wedge \theta^c$$

THE CORRESPONDENCE WORKS FOR ALL TERMS OF HEIGHT $\leqslant 29$

$$\sum_i m^i \leq 29 \qquad \qquad \sum_i m^i \leqslant 29$$

HIGHER-ORDER M-THEORY CORRECTIONS AND E_{10}

(Damour, Nicolai, 2005)

$$S_M = \int \frac{d^{11}x \sqrt{-G}}{\ell_P^9} \Big[\underbrace{R - F^2 + AFF}_{\text{TWO DERIVATIVES } g\partial^2 g + \partial^2 A} \Big] + \text{HIGHER-ORDER CORRECTIONS}$$

FROM
- ONE STRING-LOOP CORRECTIONS TO $D=10$ IIA AMPLITUDES Green Schwarz '82, Sakai, Tanii '87
- DIVERGENCES IN $D=11$ SUGRA Deser, Seminara '99, '00
- M-THEORY LOOPS Green Vanhove '97, Green Gutperle Vanhove '97
- ANOMALY IN 5-BRANE Duff, Liu, Minasian '95

+ Tseytlin '00, Peeters, Vanhove, Westerberg '01

$\nu > 0$

$$S_1 = \frac{\nu}{\ell_P^3} \int d^{11}x \sqrt{-G} \Big[t_8 t_8 R^4 - \frac{1}{4} \varepsilon_8 \varepsilon_8 R^4 \quad \leftarrow \text{EXPECTED FROM ONE-LOOP (IIA) LIGHT-CONE 4-PT AMPLITUDES}$$

$$+ \frac{2}{4} \varepsilon_8 \varepsilon_8 R^4 \quad \leftarrow \text{CHANGES THE SIGN OF } \bar{E}_8$$

$$- 4 \varepsilon_{11} A_3 \left[\mathrm{tr} R^4 - \frac{1}{4} (\mathrm{tr} R^2)^2 \right] \quad \leftarrow \text{CHERN-SIMONS TYPE}$$

$$\underbrace{+ R^2 (DF)^2 + R (DF)^3 + (DF)^4}_{\text{NOT KNOWN IN NICE FORM}} + \underbrace{\cdots + F^8}_{\text{NOT KNOWN}} \Big]$$

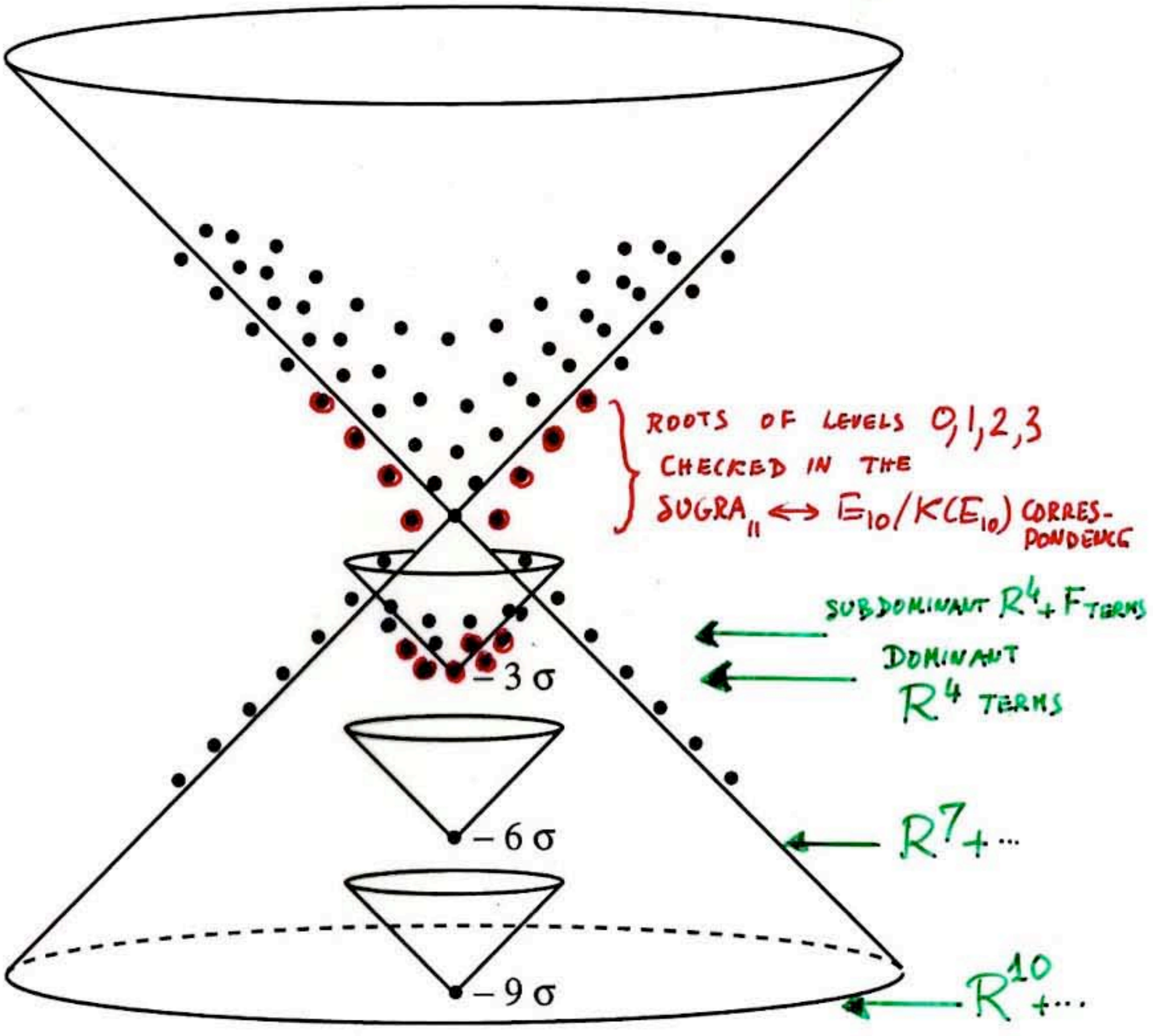
$$t_8 M^4 \equiv t_8^{\mu_1 \cdots \mu_8} M_{\mu_1\mu_2} M_{\mu_3\mu_4} M_{\mu_5\mu_6} M_{\mu_7\mu_8} \equiv 24\, \mathrm{tr} M^4 - 6 (\mathrm{tr} M^2)^2$$

ANTISYMMETRIC

$$\varepsilon_8 \varepsilon_8 R^4 \equiv \delta^{\nu_1\nu_2\nu_3\nu_4\nu_5\nu_6\nu_7\nu_8}_{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6\mu_7\mu_8} R^{\mu_1\mu_2}{}_{\nu_1\nu_2} R^{\mu_3\mu_4}{}_{\nu_3\nu_4} R^{\mu_5\mu_6}{}_{\nu_5\nu_6} R^{\mu_7\mu_8}{}_{\nu_7\nu_8}$$

$$\equiv \bar{E}_8 = \text{EULER-LOVELOCK DENSITY}$$

ROOTS OF E_{10}


ROOTS OF LEVELS 0,1,2,3
CHECKED IN THE
$SUGRA_{11} \leftrightarrow E_{10}/K(E_{10})$ CORRES-PONDENCE
SUBDOMINANT $R^4 + F$ TERMS
DOMINANT R^4 TERMS
-3σ
-6σ
-9σ
$R^7 + \cdots$
$R^{10} + \cdots$

SIGN OF $\delta_{R^4}\mathcal{H} = -C\, e^{-2 w_{R^4}(\beta)}$

$$C = \underbrace{C_1}_{t_8 t_8 R^4 - \frac{1}{4}\varepsilon_8\varepsilon_8 R^4} + \underbrace{C_2}_{+\frac{2}{4}\varepsilon_8\varepsilon_8 R^4}$$

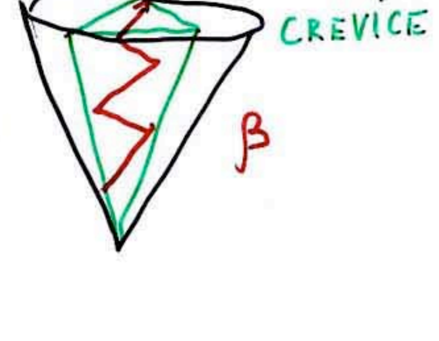
$$C_1 = \sum_{a,b}{}' v_a^2 v_b^2 \left(v_a \bar{v}_a + v_b \bar{v}_b - \bar{v}_a \bar{v}_b\right)^2 + \frac{1}{3}\sum_{a,b,c}{}'' v_a^2 v_b^2 v_c^2 \left(v_a + v_b + v_c\right)^2 \geq 0$$

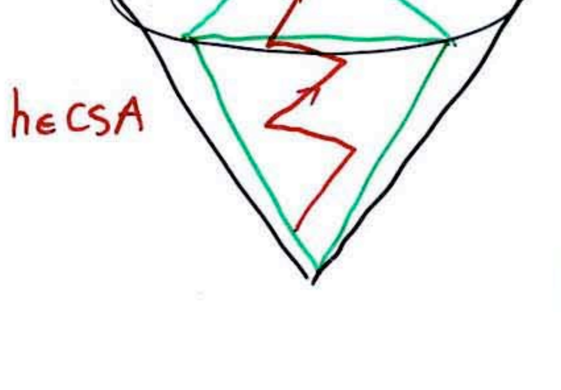
AND $\mathcal{O}(1) \times \left(\sum_a v_a\right)^8$

$$C_2 = -60 \sum_{a_1 < \cdots < a_8}{}' v_{a_1} v_{a_2} \cdots v_{a_8} : \; -0.0138 \leq \frac{C_2}{\left(\sum_a v_a\right)^8} \leq 0.0391$$

$$\boxed{\text{PROBABLY } \quad C = C_1 + C_2 \geq 0}$$

NO-BOUNCE IN LOOP-CORRECTED SUGRA $\xleftrightarrow{\text{OK}}$ NO-BOUNCE IN COSET MODEL





'E$_{10}$ PREDICTIONS' FOR HIGHER-LOOP CORRECTIONS IN M-THEORY

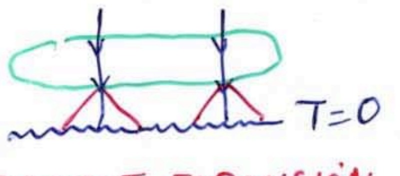
- ONLY $R^4, R^7, R^{10}, \dots, R^{3k+1}, \dots$ ARE COMPATIBLE WITH E_{10} AMONG R^N TERMS
  [OK WITH Russo, Tseytlin '97]
- SIMILARLY ONLY $R^{3k+1-n}(DF)^n$ IS OK WITH E_{10}

- EACH R^{3k+1} CORRECTION MUST BE ASSOCIATED WITH A $SL(10)$ SINGLET GENERATOR OF E_{10} AT LEVEL $\ell = -10k$

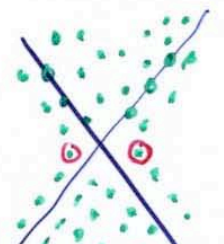
- E_{10} MIGHT ENCODE INFORMATION ABOUT THE ALGEBRAIC STRUCTURE OF LOOP CORRECTIONS

CONCLUSIONS

- THE 'NEAR COSMOLOGICAL SINGULARITY LIMIT' SUGGESTS $\exists$ HIDDEN $E_{10}(\mathbb{R})$ SYMMETRY OF $SUGRA_{11}$ (AND M-THEORY) [+ $AE_m(\mathbb{R})$ FOR GR_{m+1}?]



$\longleftrightarrow$



- BKL GRADIENT EXPANSION
$$\partial_{x^1}^{k_1}\partial_{x^2}^{k_2}\cdots\partial_{x^{10}}^{k_{10}} \ll \partial_T^{k_1+k_2+\cdots+k_{10}}$$

HEIGHT EXPANSION IN HYPERBOLIC KAC-MOODY ALGEBRA

ROOT: $\alpha = n_0\alpha_0 + n_1\alpha_1 + \cdots + n_9\alpha_9$

- SUGGESTS $\exists$ A 'HOLOGRAPHIC' (OR 'PHOTOGRAPHIC' (POLYAKOV)) CORRESPONDENCE

GRAVITY IN D=11 DYNAMICS $\longleftrightarrow$ D=1 PARTICLE DYNAMICS ON (∞-DIM) COSET SPACE

- OPTIMISTICALLY

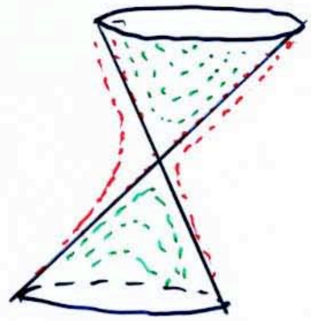
$\rightarrow$ BACKGROUND-INDEPENDENT FORMULATION OF (A SECTOR OF) M-THEORY

$\rightarrow$? NEW DESCRIPTION OF THE (QUANTUM) NATURE OF SPACE-(TIME) AT PLANCK SCALE VIA A 'DE-EMERGENCE' OF SPACE NEAR A SINGULARITY

'SPACE' I.E. $\begin{Bmatrix} G_{\mu\nu}(t,x) \\ A_{\mu\nu\lambda}(t,x) \\ \psi_\mu(t,x) \end{Bmatrix} \longrightarrow$ INFINITE TOWER OF LIE-ALGEBRAIC VARIABLES $\begin{Bmatrix} g(t) \in E_{10}/K(E_{10}) \\ \underline{\Psi}(t) \end{Bmatrix}$

OPEN ISSUES

- HOW TO GO BEYOND HEIGHT 29 IN TESTING THE CONJECTURE
- FERMIONS : NICELY COMPATIBLE WHEN $\partial_a \psi_i \to 0$; BEYOND?
  ? FAITHFUL REPS OF $K(E_{10})$?
- CONSTRAINTS: REMARKABLE SUGAWARA-LIKE STRUCTURE
  $\mathcal{L}^{(-m)} \sim J \otimes J$? VAST GENERALIZATION OF USUAL
  SUGAWARA $L_m \sim J_{AFFINE} \otimes J_{AFFINE}$ (Damour, Kleinschmidt, Nicolai '06)
  ? E_{10} COSETMODEL $\stackrel{?}{=}$ GAUGE-FIXED VERSION OF AN UNDERLYING GAUGE-INVARIANT ACTION
- ? INFINITE # OF CONSTRAINTS ?
  WELCOME FOR



- QUANTIZED COSET ACTION

$$\to \quad \Box \Psi(g) = 0$$

KLEIN-GORDON-(WHEELER-DEWITT)-TYPE EQ. ON $E_{10}(\mathbb{R})/K_{10}(\mathbb{R})$

- IF + TOROIDAL COMPACTIFICATION : $E_{10}(\mathbb{Z})$ (Hull, Townsend '95)

$\Rightarrow$ MODULAR FORM OVER $E_{10}(\mathbb{Z}) \backslash E_{10}(\mathbb{R}) / K_{10}(\mathbb{R})$

(Ganor '99, Brown, Ganor, Helfgott '04)

- RÔLE OF E_{11} ? (West, ...), OR OF OTHER HYPERBOLIC KM (Harvey, Moore, ... Verlinde, ...)