SINGULAR SUPPORT OF COHERENT SHEAVES

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The course will describe my joint work with D. Arinkin.

Agenda:

1) Introduction
2) Definition of singular support
3) Functoriality
4) Microlocalization

Abstract:

Let Y be an algebraic variety (or, rather, a derived scheme). In recent years much attention has been devoted to the study of the *singularity category* of Y; the latter is by definition the triangulated category quotient of $\mathrm{Coh}(Y)$, the category of coherent complexes on Y, by $\mathrm{Perf}(Y)$, the category of perfect complexes.

Assume now that Y is *quasi-smooth*, which is another word for (derived) locally complete intersection. In this case, to an object $\mathcal{F} \in \mathrm{Coh}(Y)$ one can associate a geometric invariant, denoted $\mathrm{sing.\,supp.}(\mathcal{F})$, and called the *singular support* of $\mathcal{F}$. This invariant measures how far $\mathcal{F}$ is from belonging to $\mathrm{Perf}(Y)$. The singular support of $\mathcal{F}$ is a closed conical subvariety inside a certain algebraic variety $\mathrm{Sing}(Y)$, canonically attached to Y, which measures how much Y itself is far from being smooth. Thus, each closed conical subvariety $\mathcal{N} \subset \mathrm{Sing}(Y)$ gives rise to a category $\mathrm{Coh}_{\mathcal{N}}(Y)$, which is "sandwiched" between $\mathrm{Perf}(Y)$ and $\mathrm{Coh}(Y)$.

It turns out that categories of the form $\mathrm{Coh}_{\mathcal{N}}(Y)$ are of central importance to the Geometric Langlands program. Namely, when Y is taken to be the stack of local systems on a given algebraic curve, an appropriately defined stack $\mathcal{N}$ will capture the Arthur parameters.