Homework 5

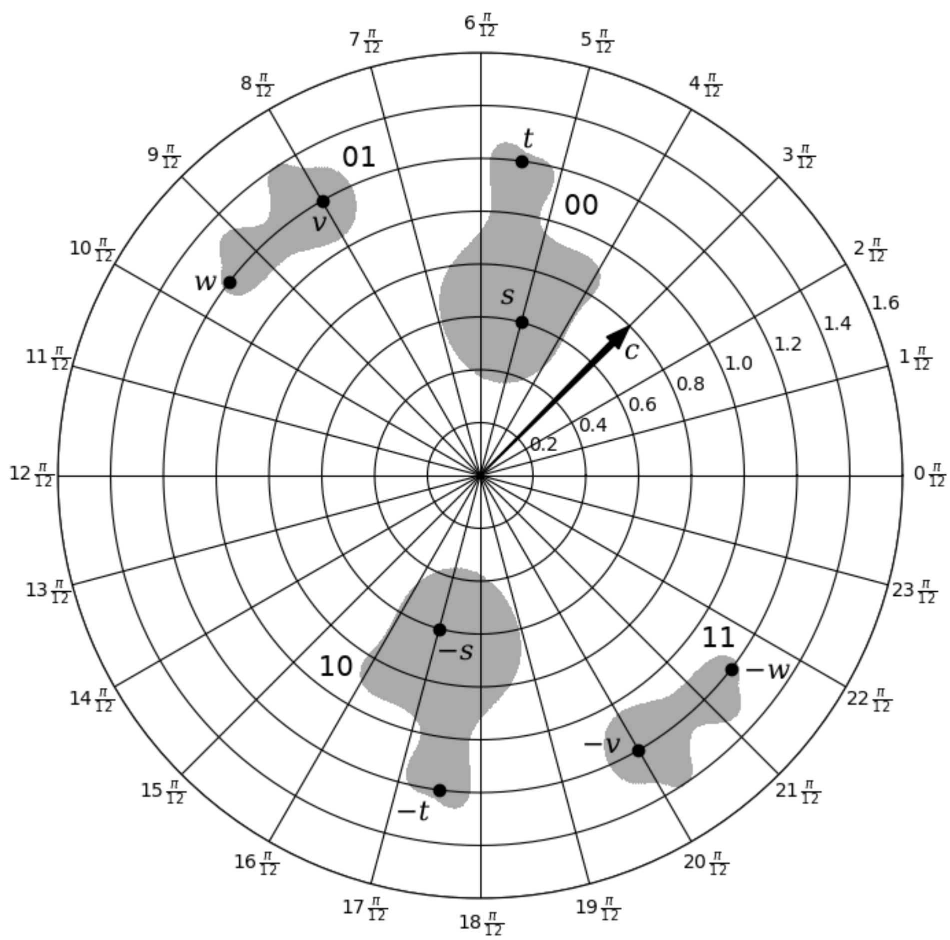
Due on Crowdmark
April 7, 11 p.m.

Chaos, fractals, and dynamics
MAT 335, Winter 2019

Show your calculations, and explain your reasoning. Your goal is for the graders to understand how you got your answers, and to be convinced that your reasoning makes sense.

1 Cantor dust

The dynamical map $Q_c\colon \mathbb{C} \to \mathbb{C}$ with $c = 0.8\,e^{i\,3\pi/12}$ has a filled Julia set of "Cantor dust" type. The plot below shows a "2nd-level" approximation of the filled Julia set.[1] Its pieces are labeled according to where they're sent by Q_c, as demonstrated for the V map in week 7 (slides 16–41) and for a complex quadratic map in week 10 (slides 121–139).



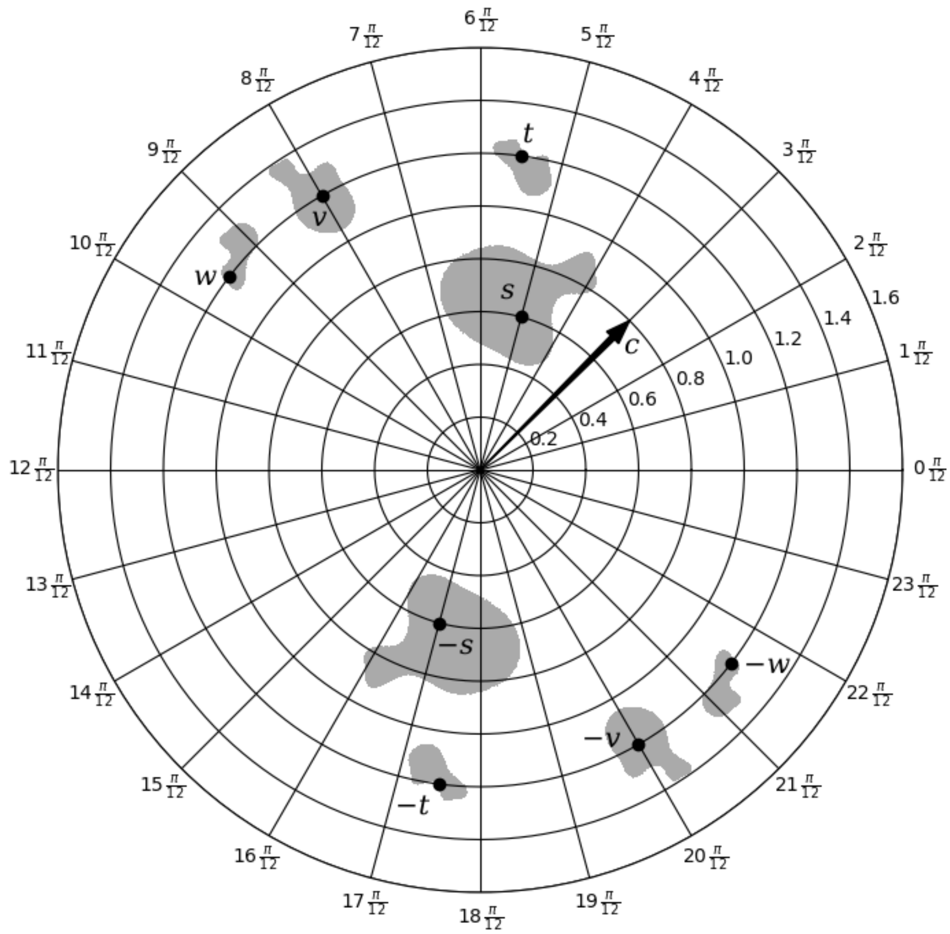
2nd-level approximation

[1] For anyone interestd in the details: I defined $L_0 \subset \mathbb{C}$ as the set of points whose distance from 0 is $\frac{1}{2} + \sqrt{\frac{1}{4} + 0.8}$ or more. The 2nd-level approximation is what remains after I cut out $L_0, \ldots, L_4$.

The plot below shows a 3rd-level approximation, with the points

$$s = 0.6\,e^{i\,5\pi/12} \qquad v = 1.2\,e^{i\,8\pi/12}$$
$$t = 1.2\,e^{i\,5.5\pi/12} \qquad w = 1.2\,e^{i\,9.5\pi/12}$$

marked. To help you with part c, I've also marked the negatives of these points.



3rd-level approximation

a. On the plot with the 2nd-level approximation, mark the points s^2, t^2, v^2, w^2. Label the points to show which is which.

HINT: You can do your calculations graphically, remembering that $(re^{i\theta})^2 = r^2 e^{i\,2\theta}$.

b. On the plot with the 2nd-level approximation, mark the points $Q_c(s)$, $Q_c(t)$, $Q_c(v)$, $Q_c(w)$. They don't have to be perfect—just be as accurate as you can. Draw arrows from s^2 to $Q_c(s)$, from t^2 to $Q_c(t)$, from v^2 to $Q_c(v)$ and from w^2 to $Q_c(w)$.

HINT: The arrow from 0 to c shows how adding c translates the complex plane.

c. Label the 3rd-level pieces according to where they're sent by Q_c, as demonstrated in the week 7 and week 10 slides.