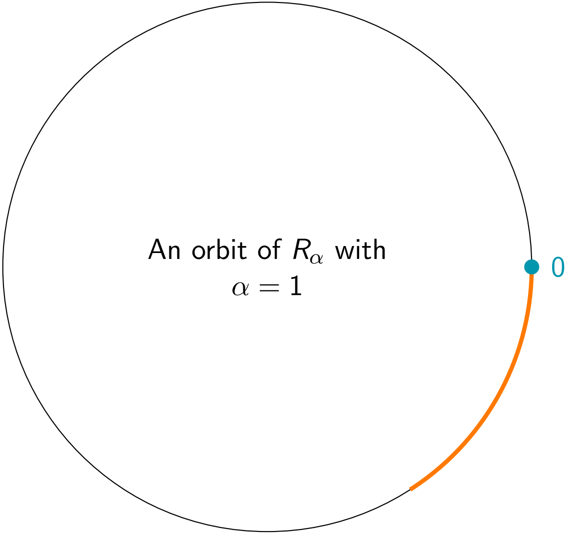
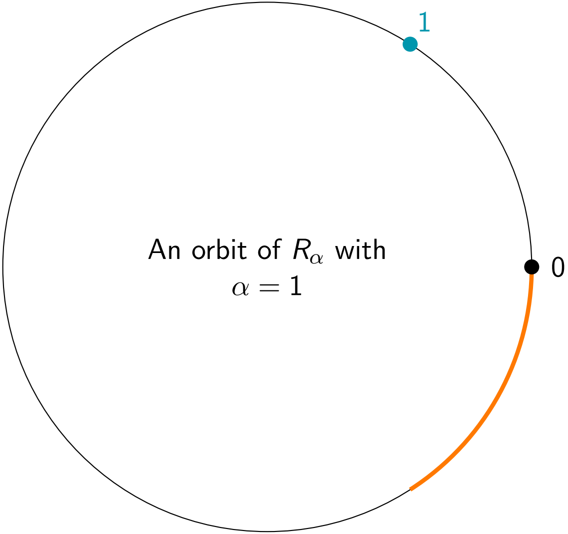
Visualizing rotation map orbits

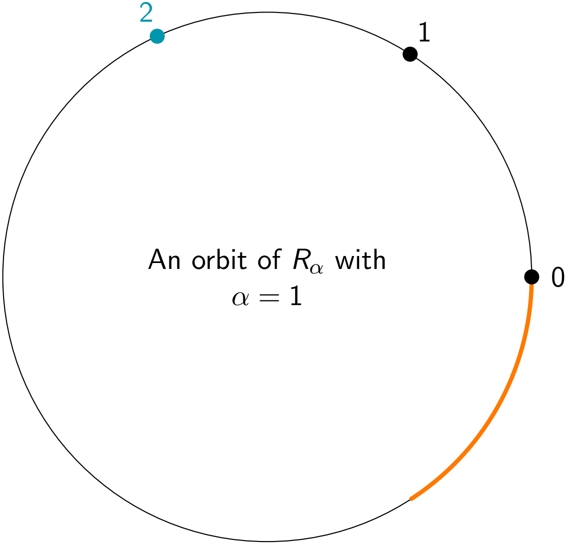


Visualizing rotation map orbits



An orbit of R_α with
$\alpha = 1$

Visualizing rotation map orbits



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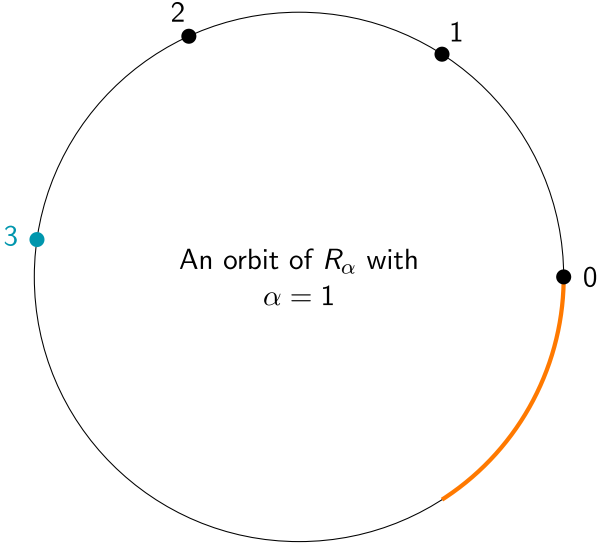

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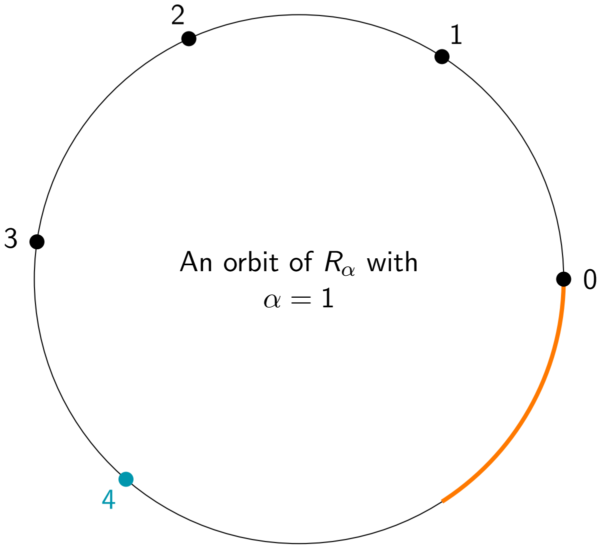

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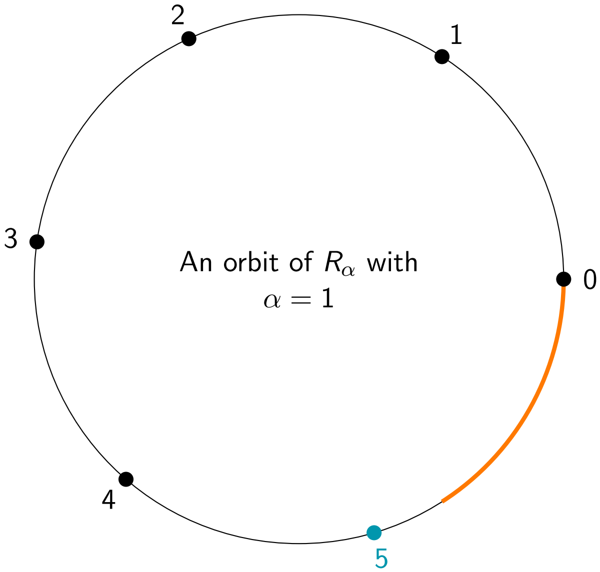

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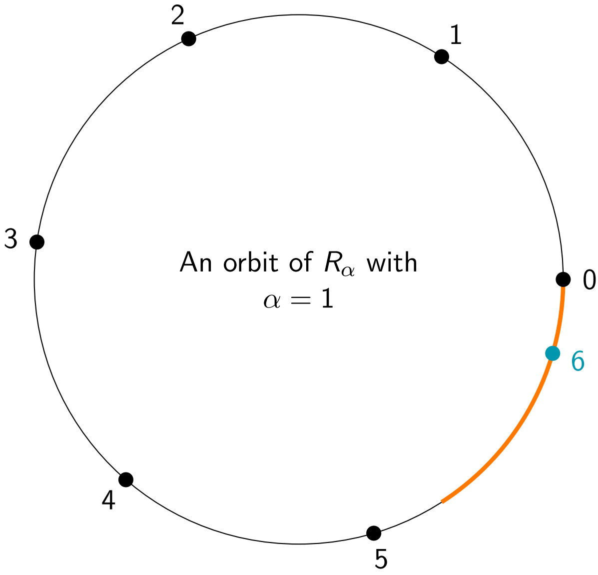

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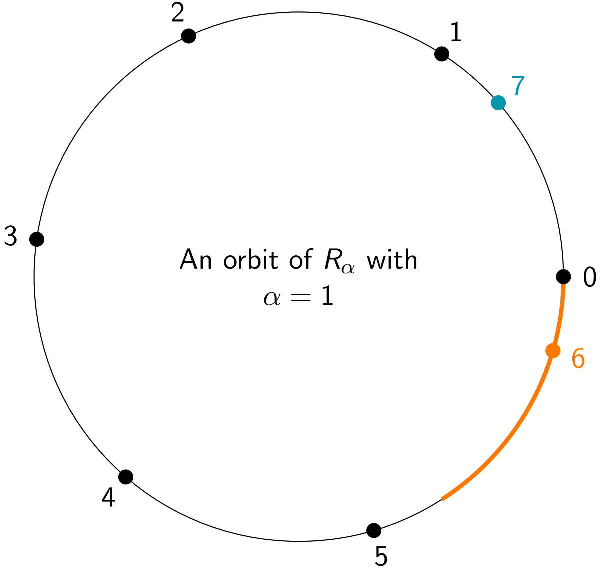

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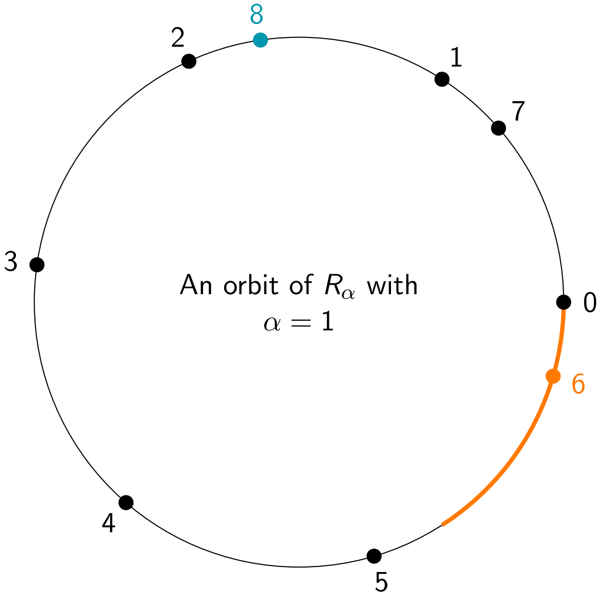

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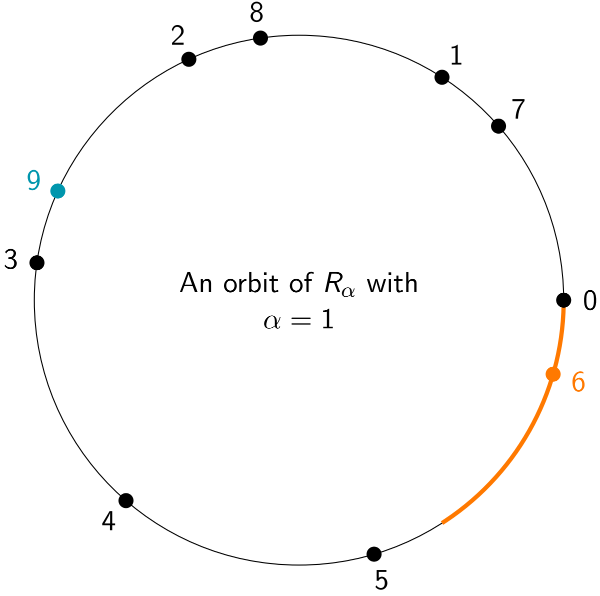

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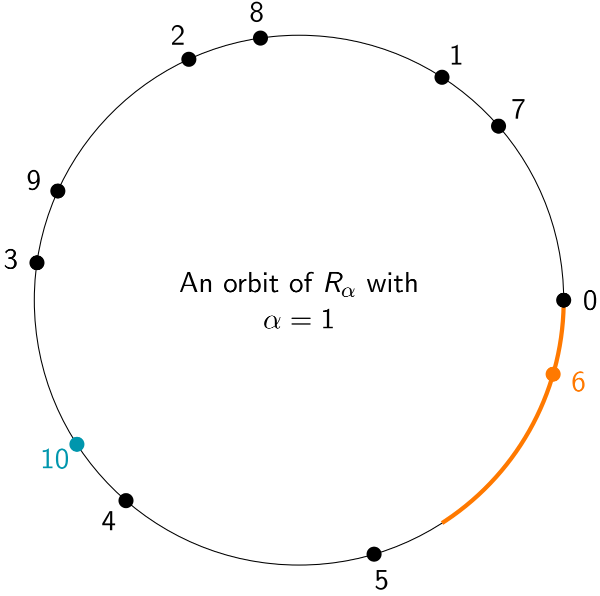

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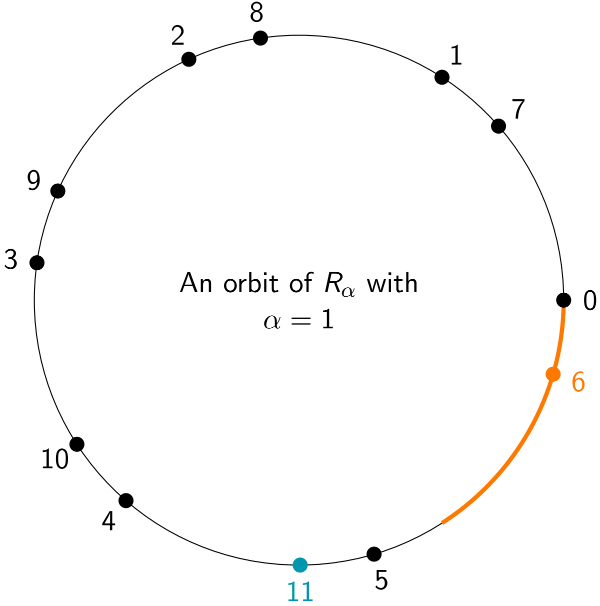

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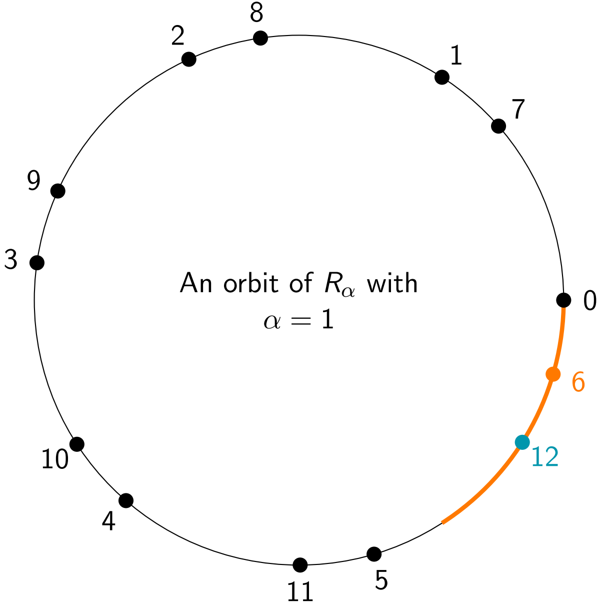

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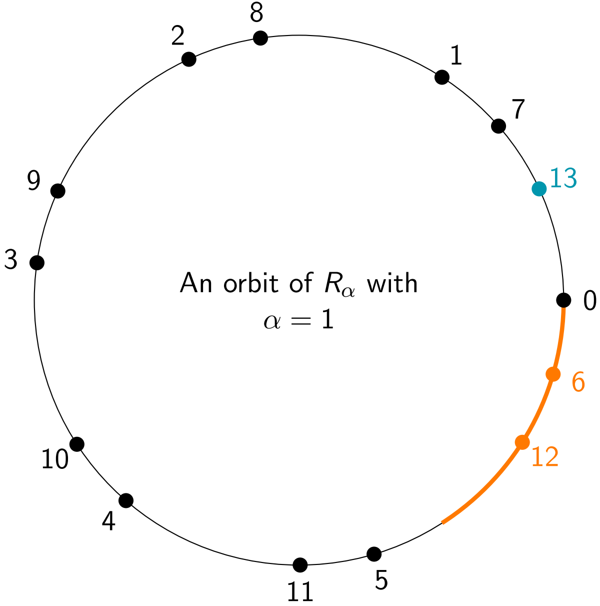

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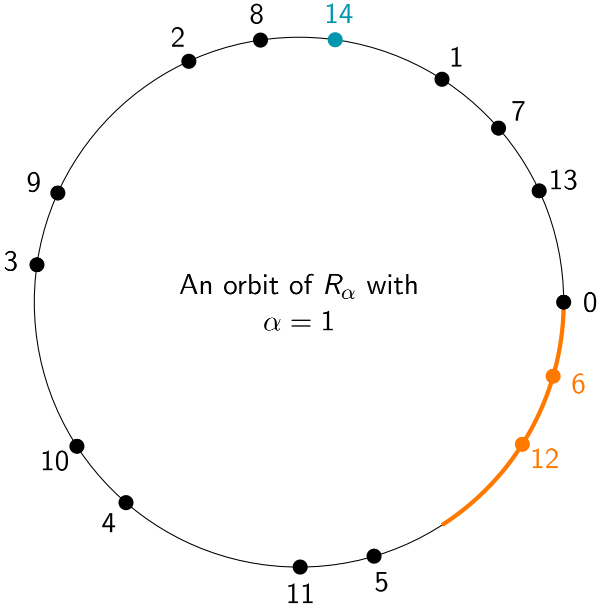

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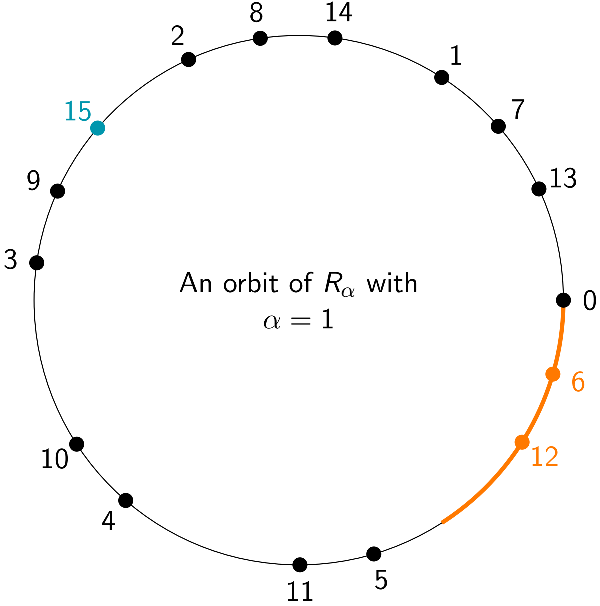

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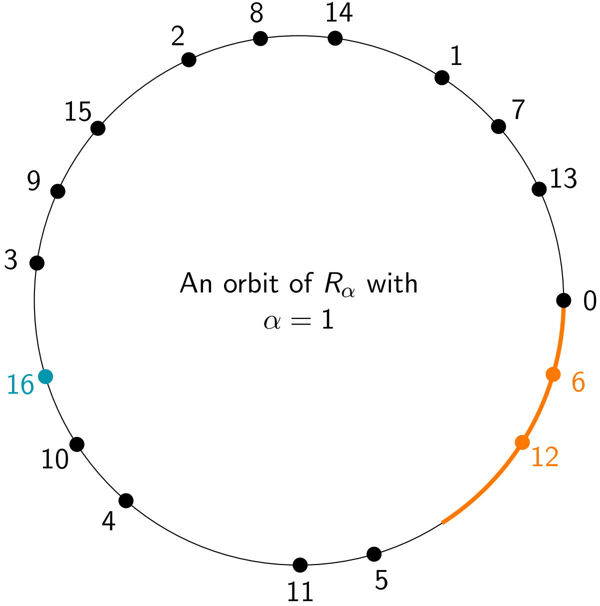

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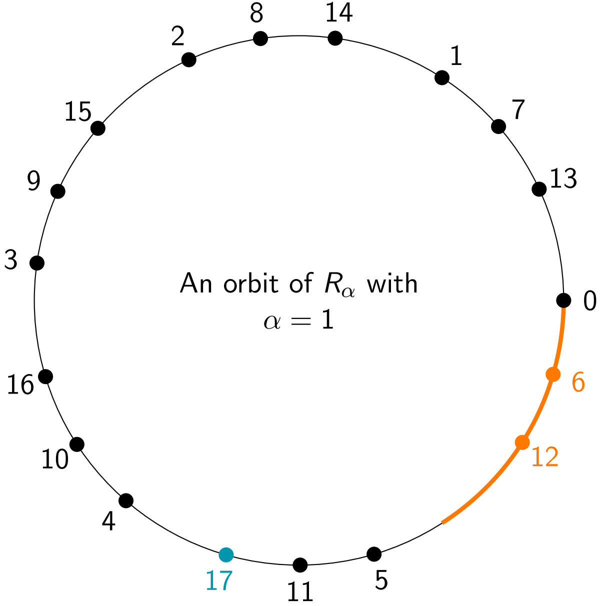

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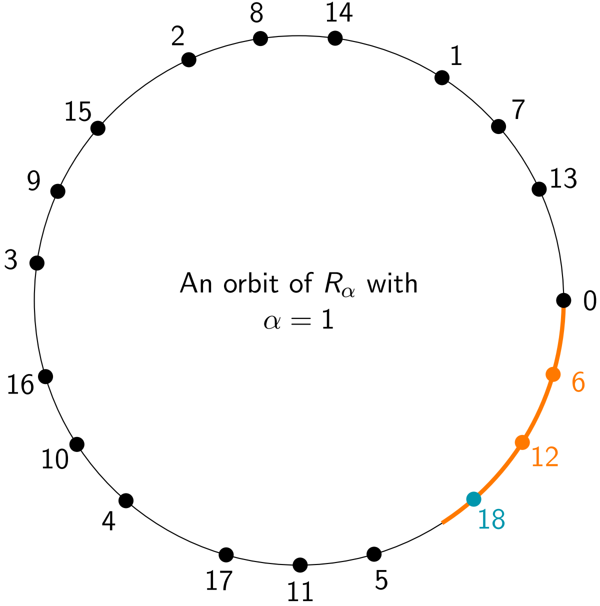

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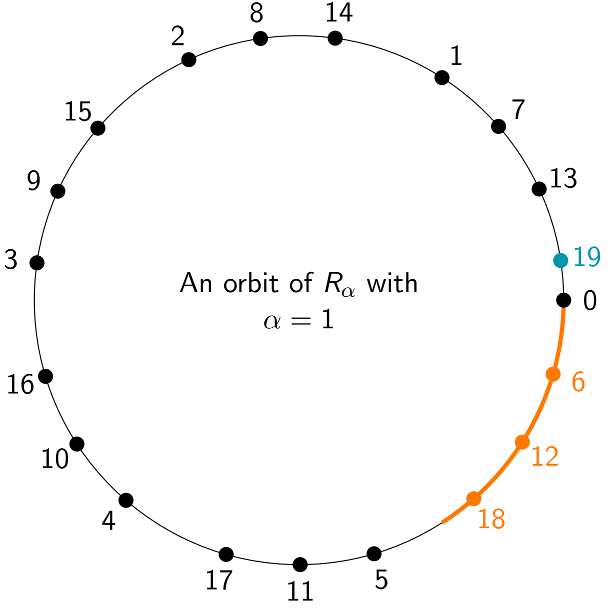

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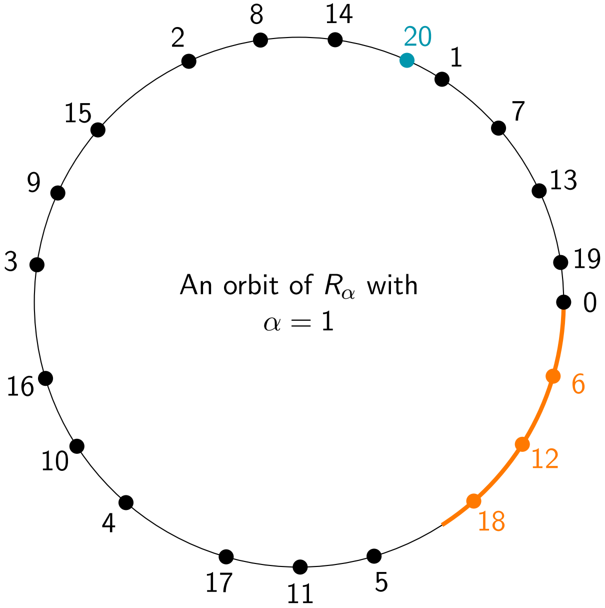

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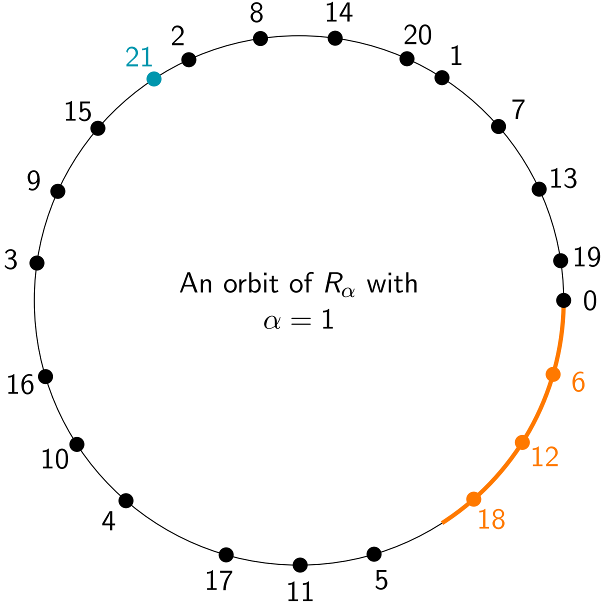

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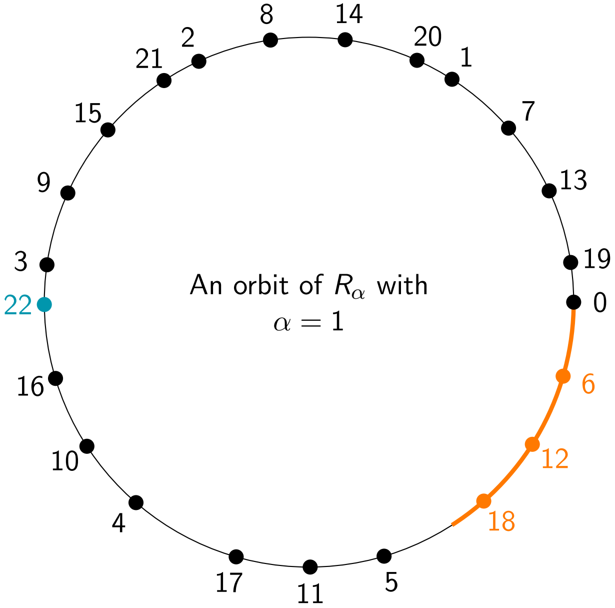

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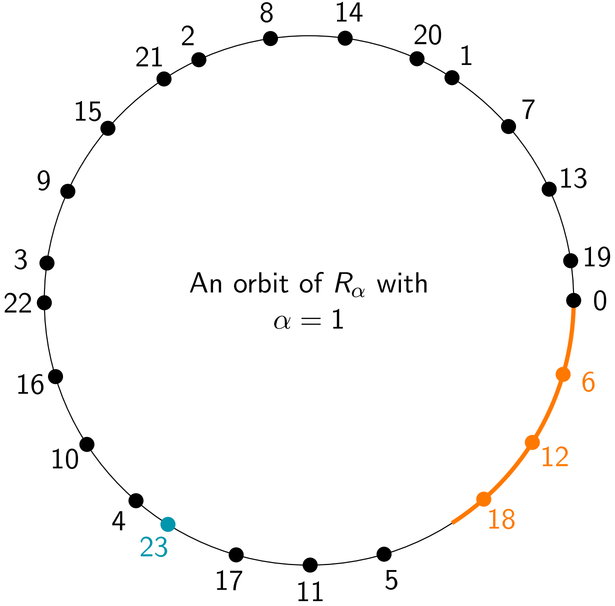

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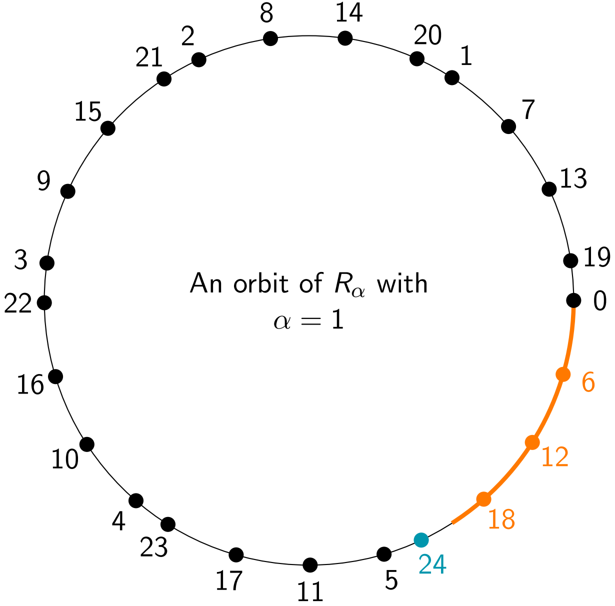

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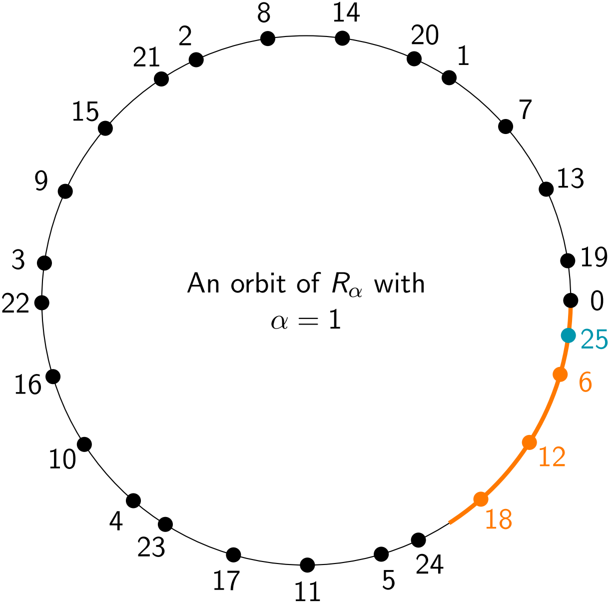

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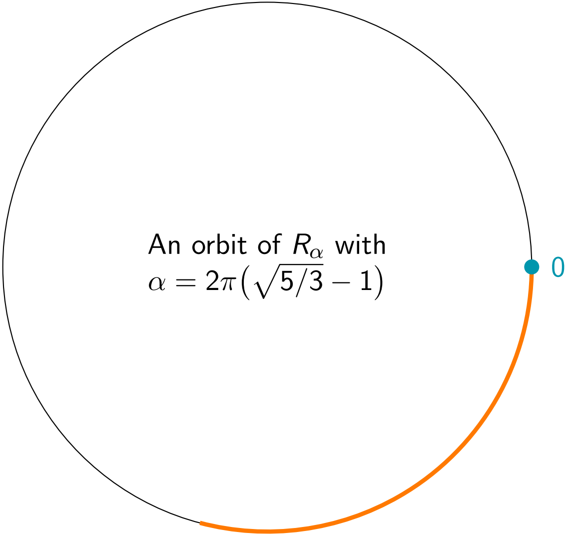

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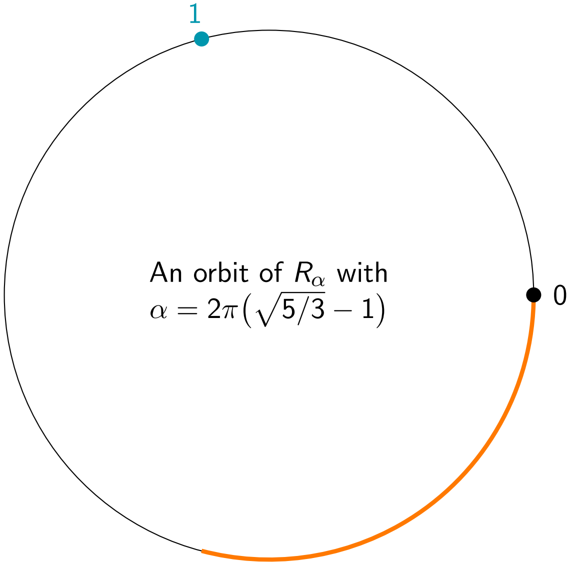

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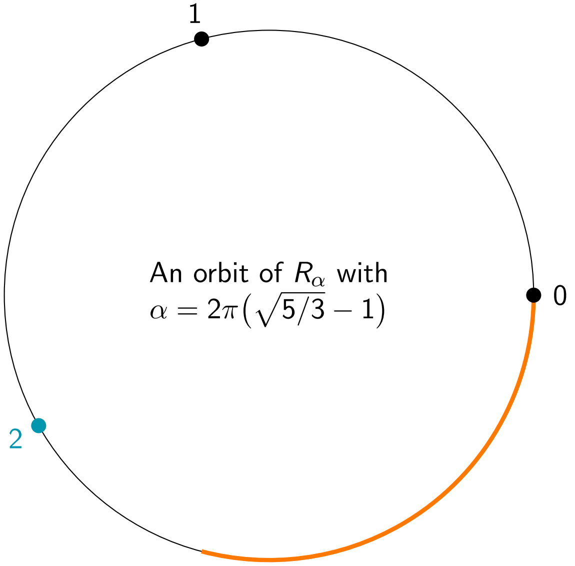

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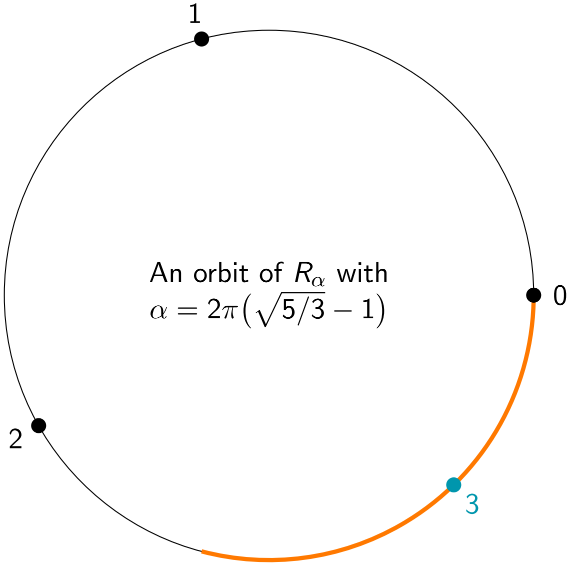
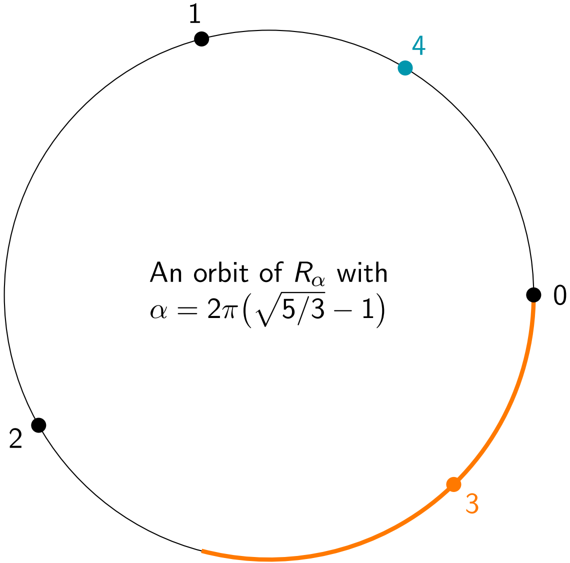


An orbit of R_α with
$\alpha = 2\pi\left(\sqrt{5/3} - 1\right)$

Visualizing rotation map orbits



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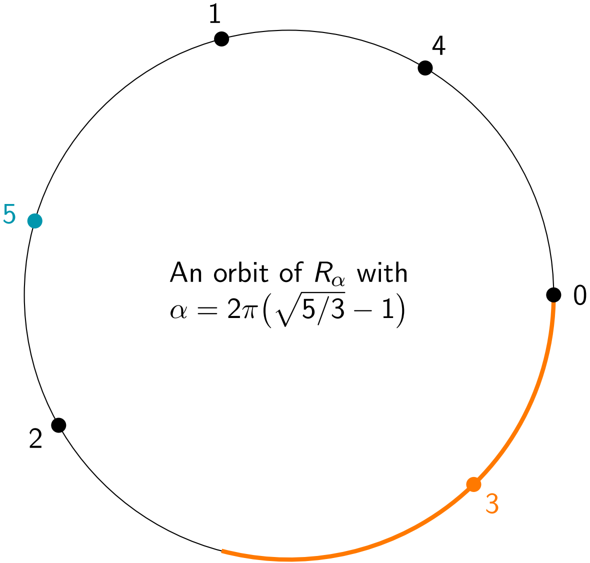

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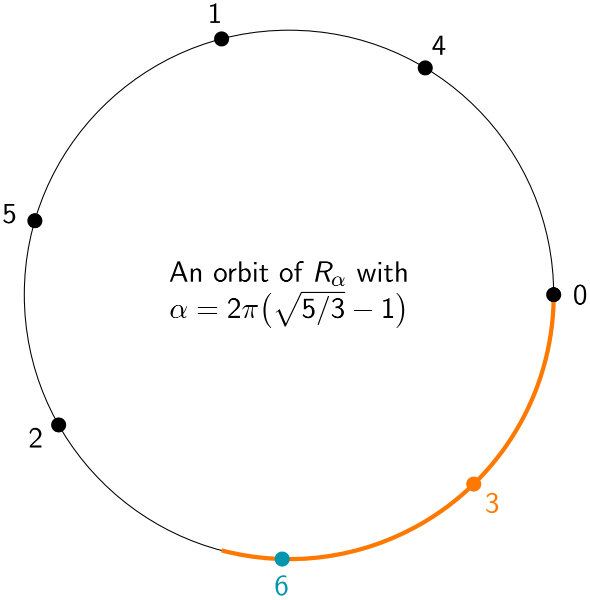

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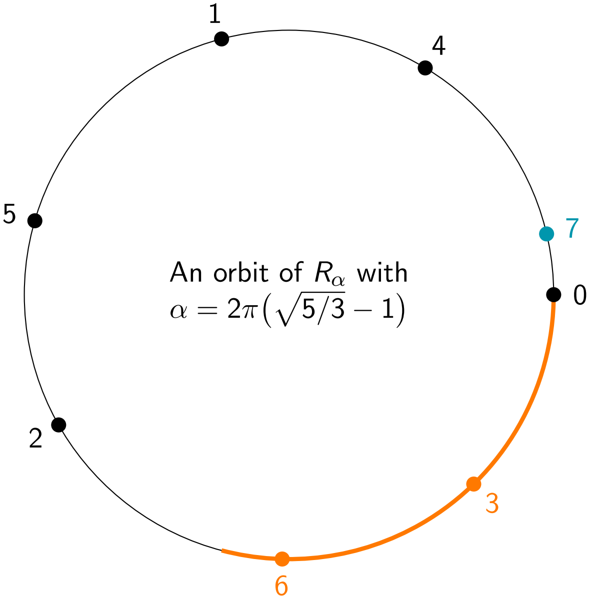

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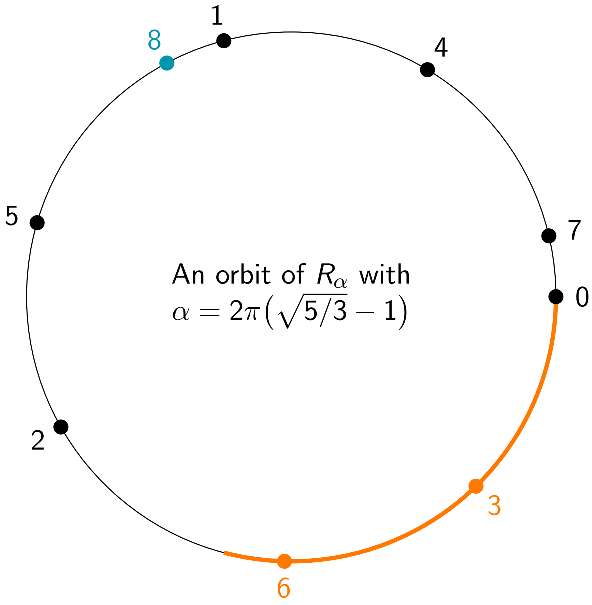

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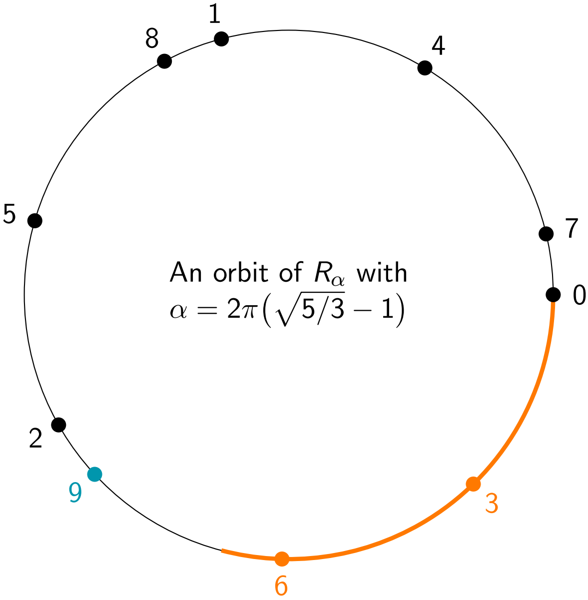

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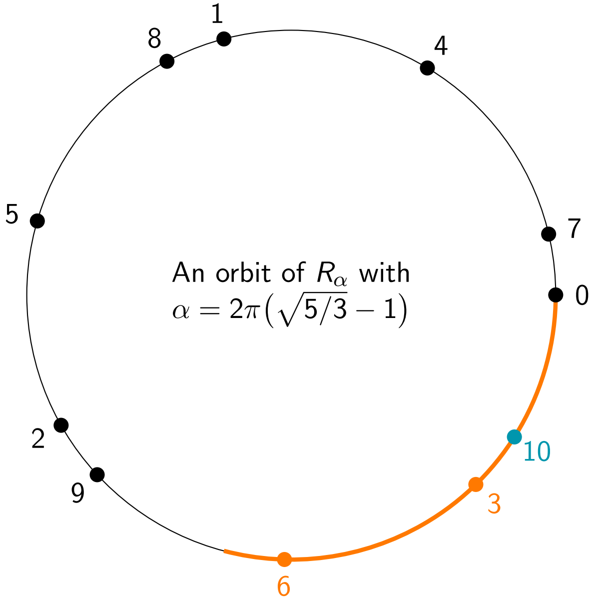

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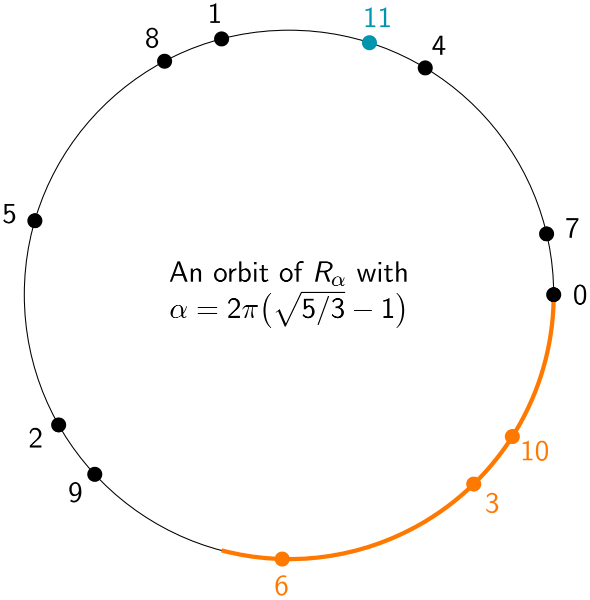

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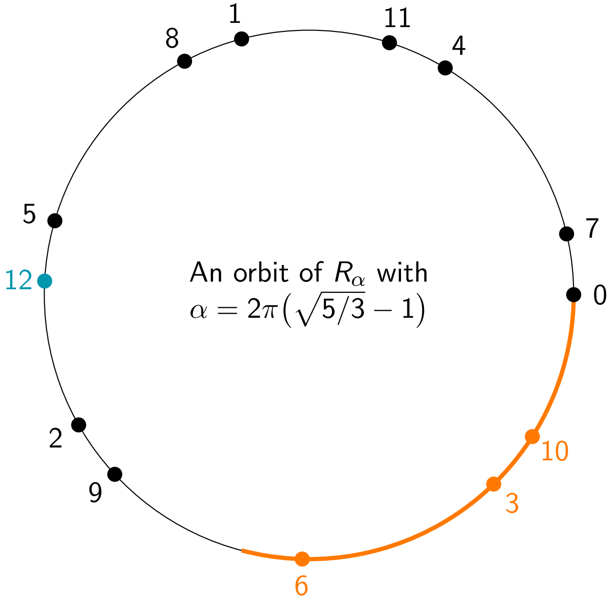

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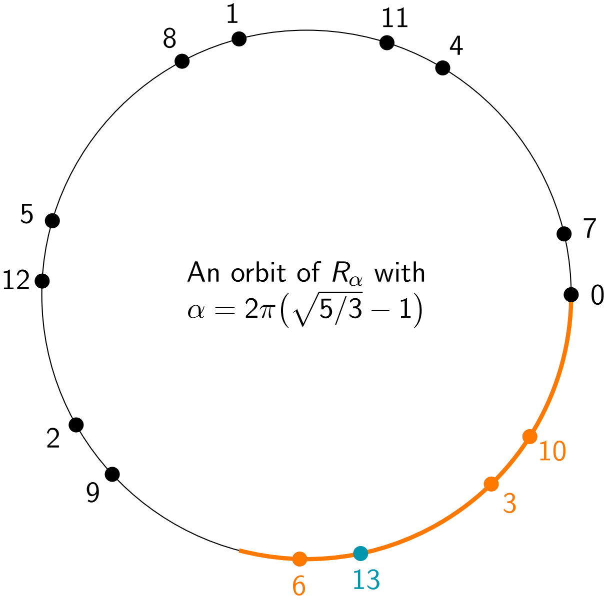

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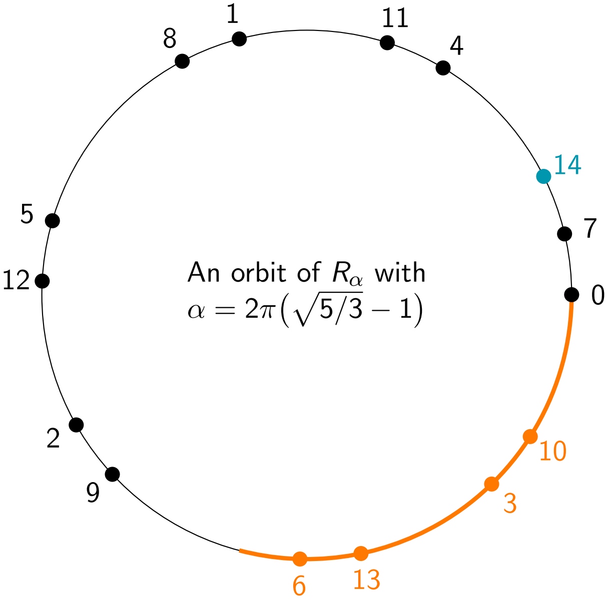

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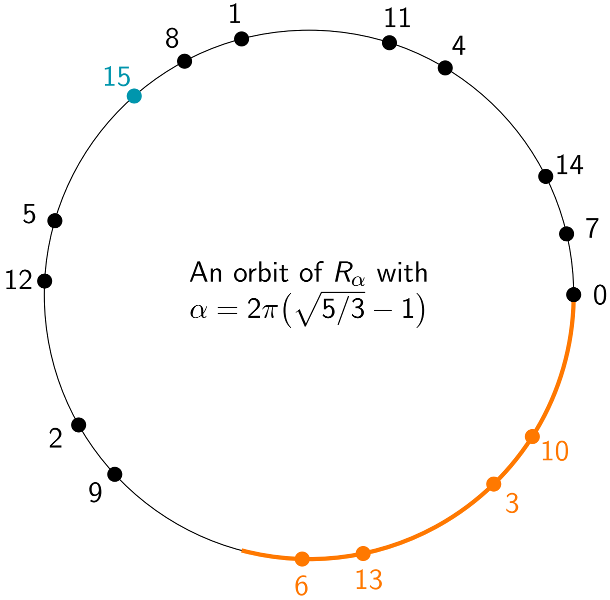
Visualizing rotation map orbits



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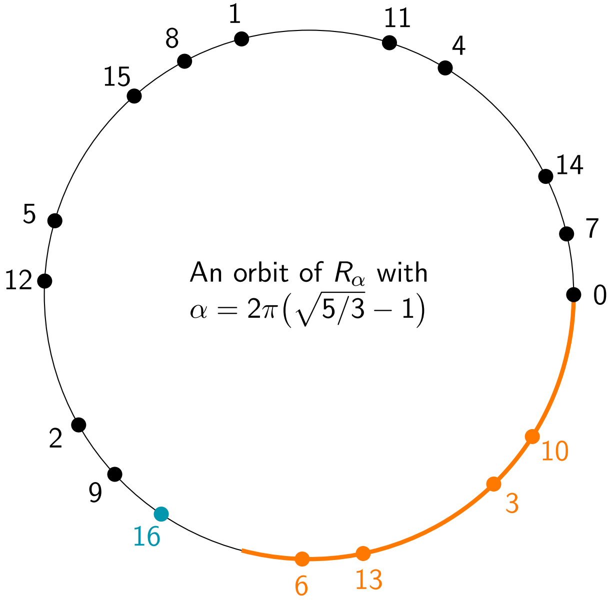

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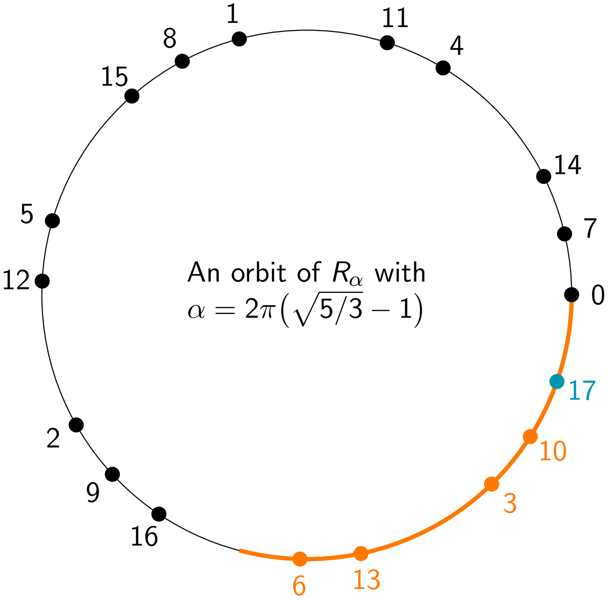

Self-similarity of rotation map itineraries

- Consider $R_\alpha \colon \mathbb{T} \to \mathbb{T}$ with $\alpha = 2\pi\left(\sqrt{5/3} - 1\right)$.
- Define an itinerary function $\tau \colon \mathbb{T} \to \mathbf{2}^{\mathbb{N}}$ by saying

$$\text{the } n\text{th digit of } \tau(\theta) \text{ is } \begin{cases} 1 & \text{if } R_\alpha^n(\theta) \in (0, -\alpha)_{\text{short}} \\ 0 & \text{otherwise.} \end{cases}$$

  We'll call the starting digit of a sequence the 0th digit.
- (Caution: τ is discontinuous at each point whose orbit hits 0.)
- Here are the first few digits of $\tau(0)$.

00010010001001000100100010010010001001000100100010010010001001000100100010010001001001

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0001	001	0001	001	0001	001	0001	001	001	0001	001	0001	001	0001	001	001	0001	001	0001	001	0001	001	0001	001	001
1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	0

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0001 001 0001 001 0001 001 0001 001 001 0001 001 0001 001 0001 001 001 0001 001 0001 001 0001 001 0001 001 001

1 0 1 0 1 0 1 0 0 1 0 1 0 1 0 0 1 0 1 0 1 0 1 0 0

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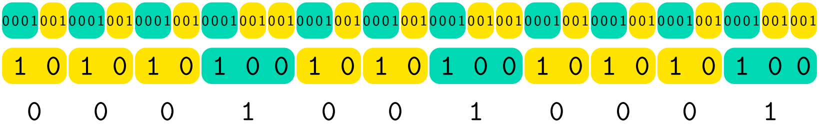

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0001 001 0001 001 0001 001 0001 001 001 0001 001 0001 001 0001 001 001 0001 001 0001 001 0001 001 0001 001 001

1 0 1 0 1 0 1 0 0 1 0 1 0 1 0 0 1 0 1 0 1 0 1 0 0

0 0 0 1 0 0 1 0 0 0 1

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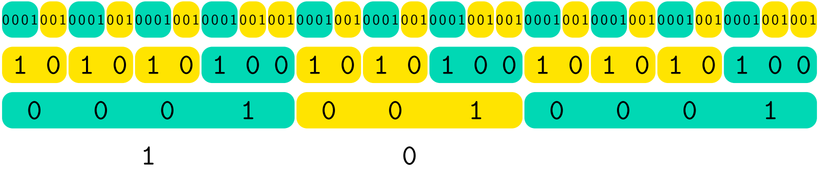

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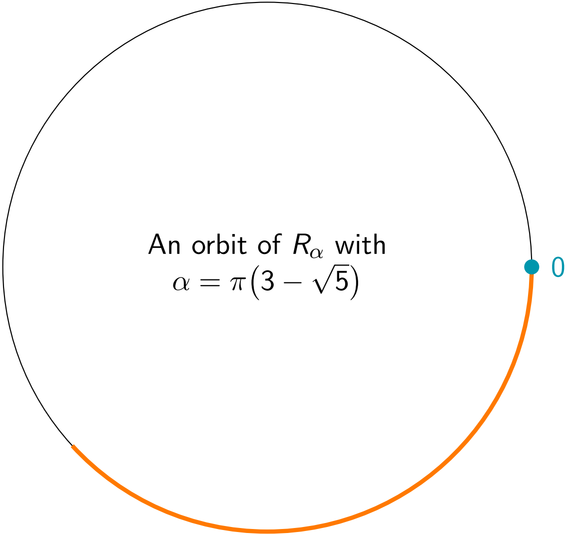
0001 001 0001 001 0001 001 0001 001 001 0001 001 0001 001 0001 001 001 0001 001 0001 001 0001 001 0001 001 001

1 0 1 0 1 0 1 0 0 1 0 1 0 1 0 0 1 0 1 0 1 0 1 0 0

0 0 0 1 0 0 1 0 0 0 1

1 0

Visualizing rotation map orbits



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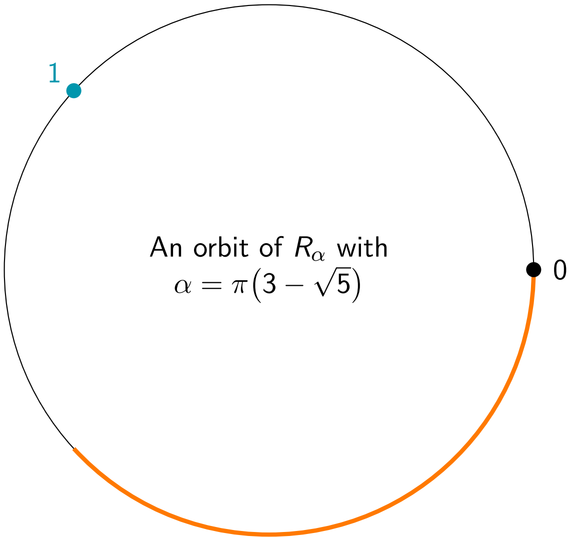

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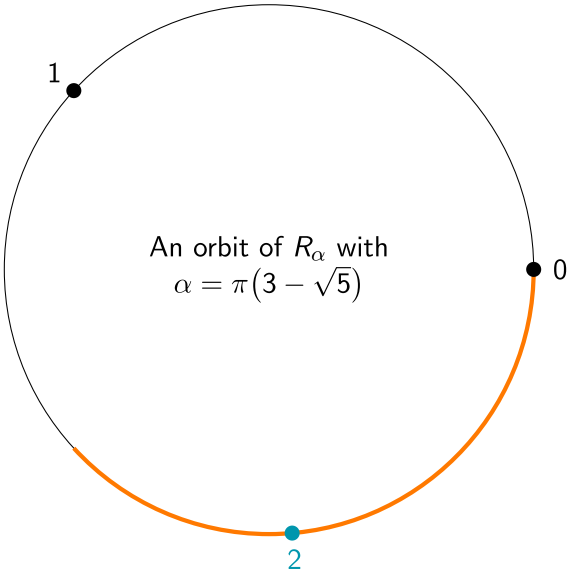

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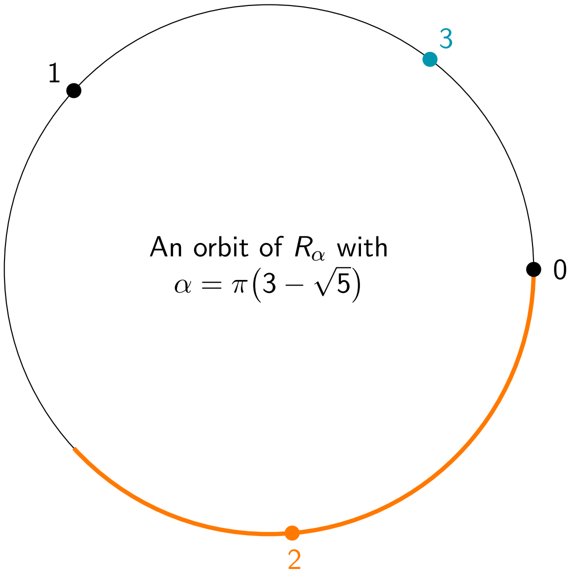

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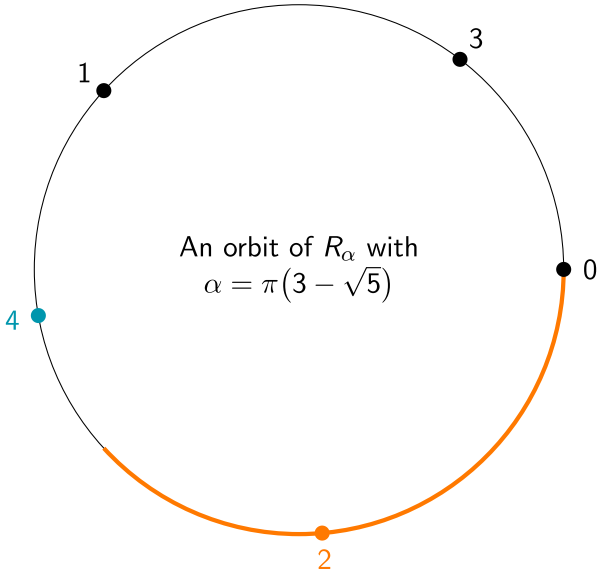

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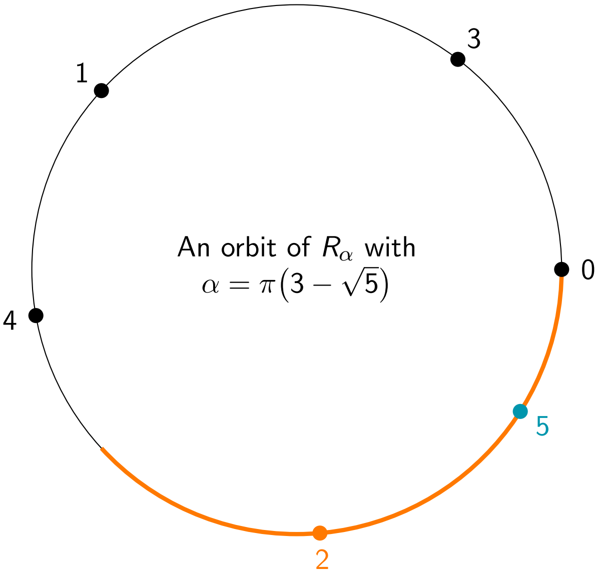
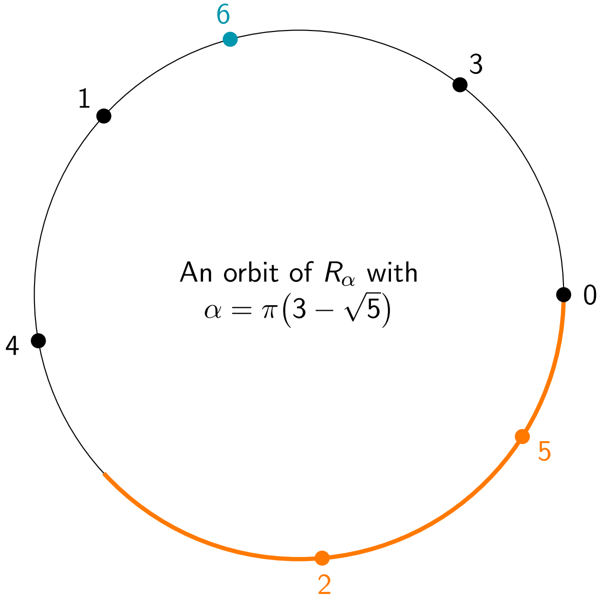


An orbit of R_α with
$\alpha = \pi(3 - \sqrt{5})$

Visualizing rotation map orbits



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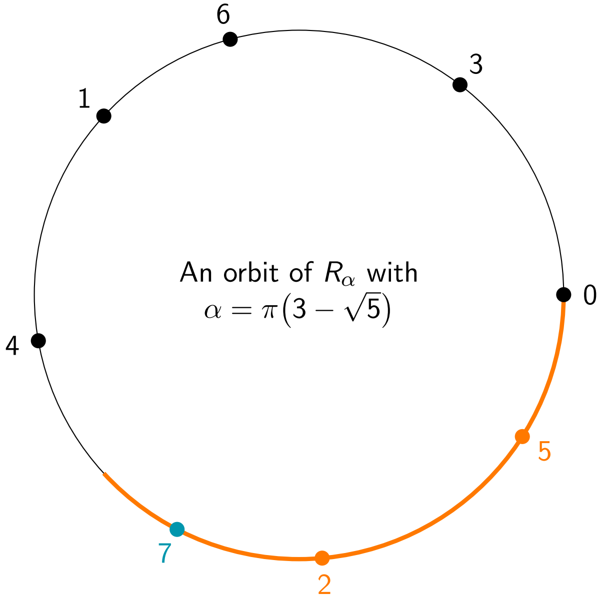
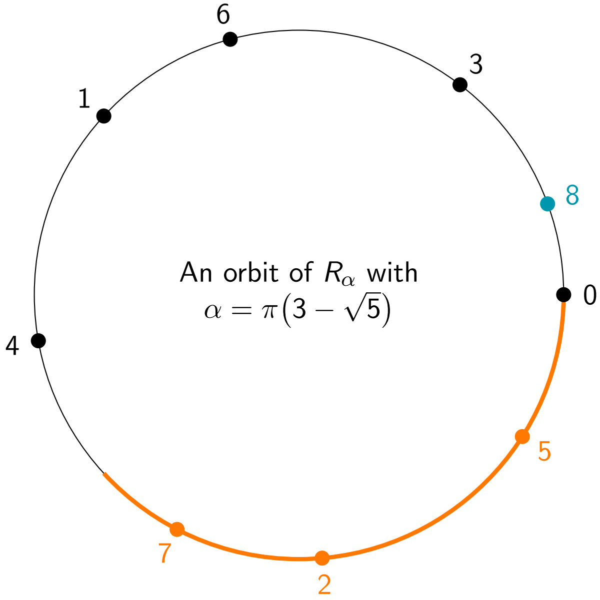


Visualizing rotation map orbits



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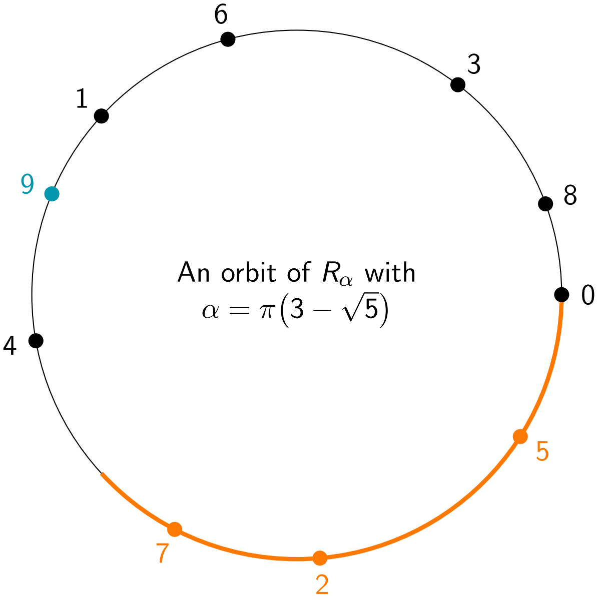

An orbit of R_α with
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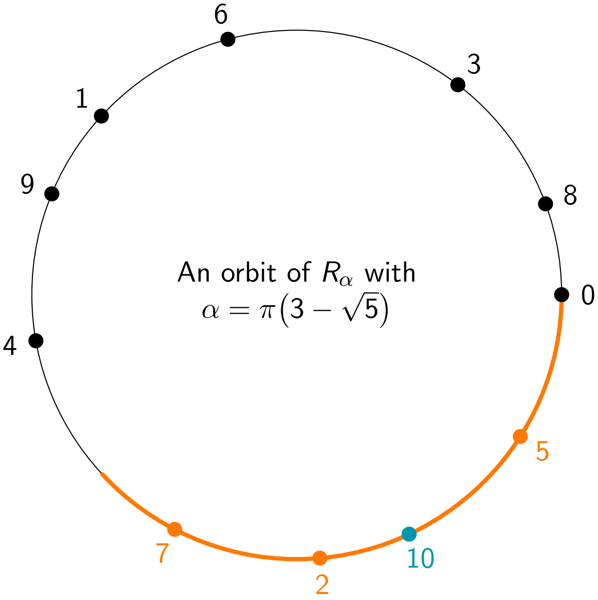
Visualizing rotation map orbits



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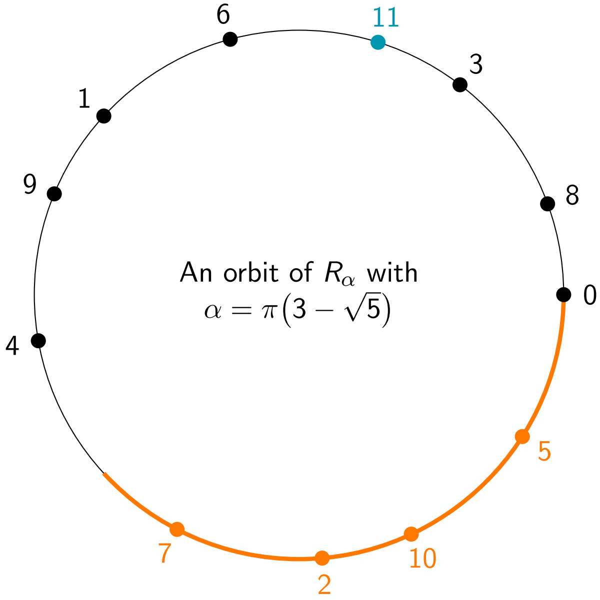

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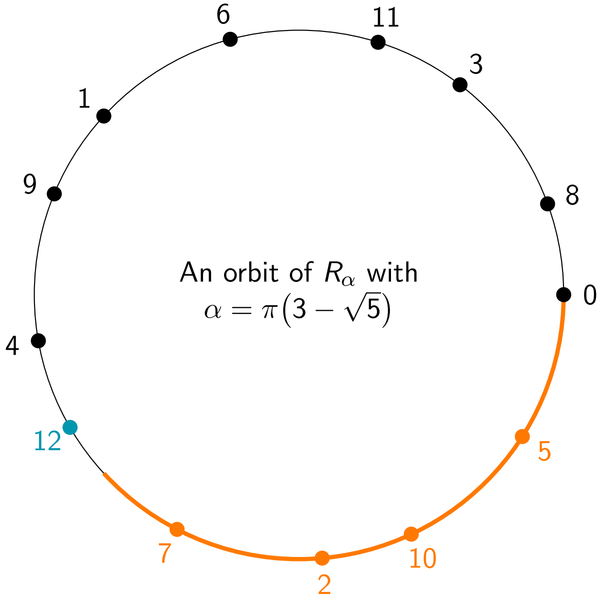

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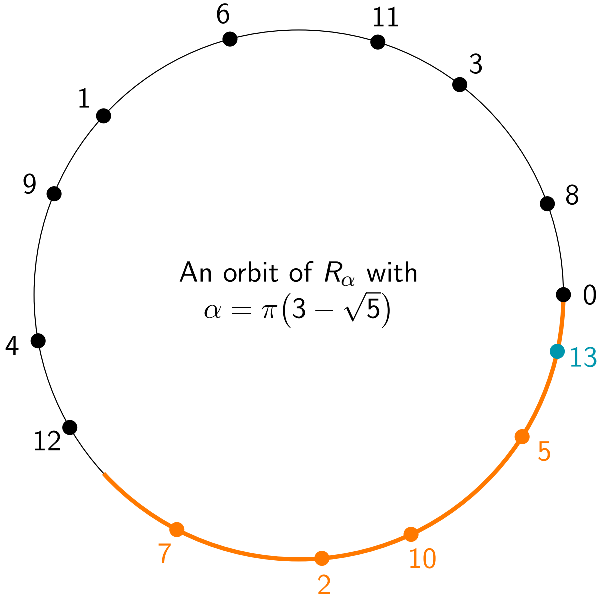

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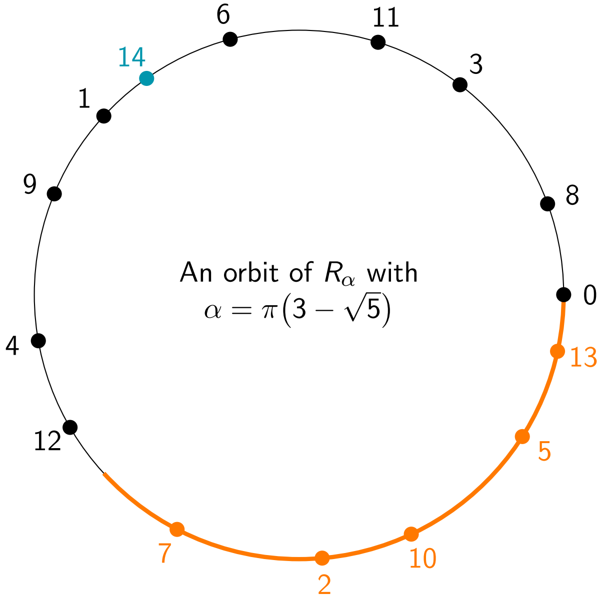

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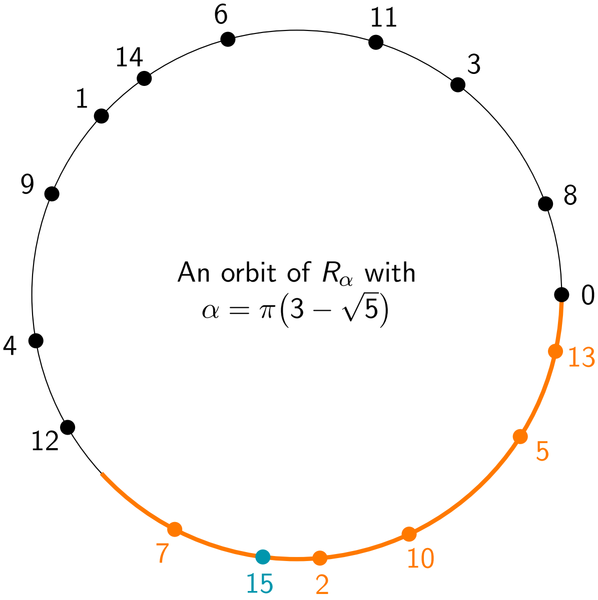

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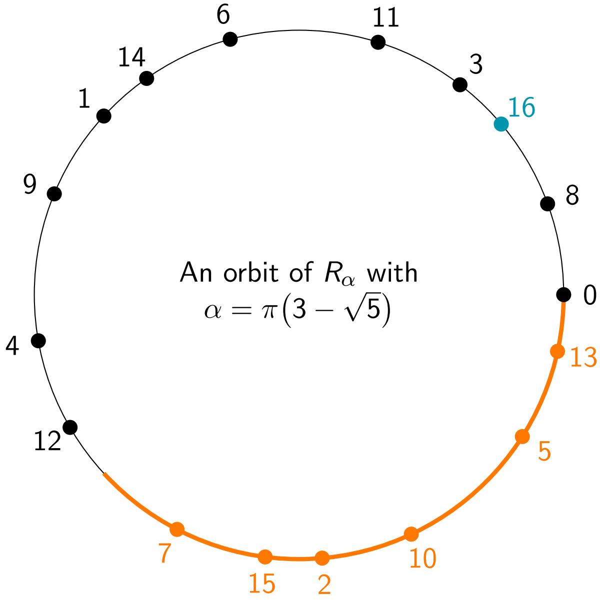

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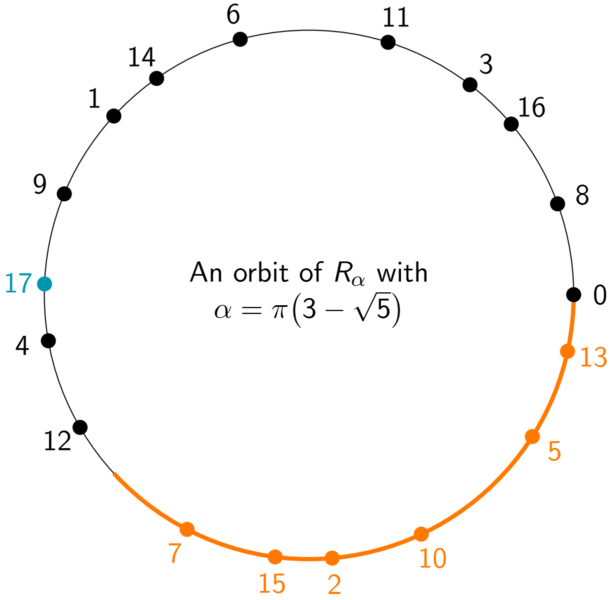

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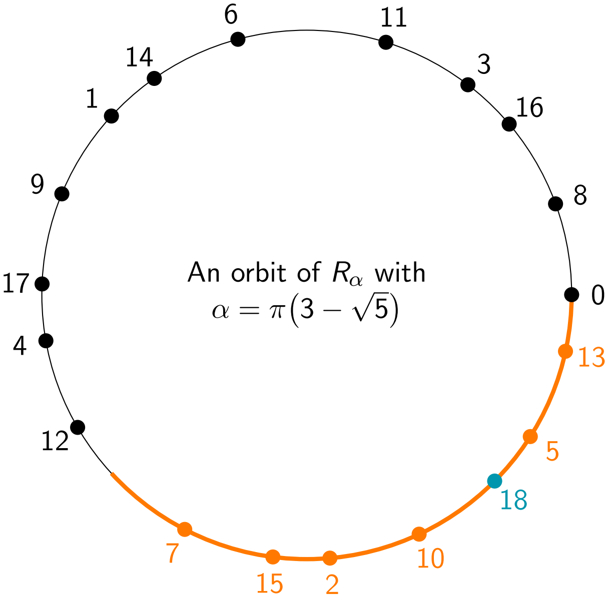

An orbit of R_α with
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Self-similarity of rotation map itineraries

- Consider $R_\alpha \colon \mathbb{T} \to \mathbb{T}$ with $\alpha = \pi\big(3 - \sqrt{5}\big)$.
- Define an itinerary function $\tau \colon \mathbb{T} \to \mathbf{2}^{\mathbb{N}}$ by saying

$$\text{the } n\text{th digit of } \tau(\theta) \text{ is } \begin{cases} 1 & \text{if } R_\alpha^n(\theta) \in (0, -\alpha)_{\mathsf{short}} \\ 0 & \text{otherwise.} \end{cases}$$

  We'll call the starting digit of a sequence the 0th digit.
- (Caution: τ is discontinuous at each point whose orbit hits 0.)
- Here are the first few digits of $\tau(0)$.

0010010100100101001010010010100100101001010010010100101001001010010100100101001001010010100100101001010010010100100101

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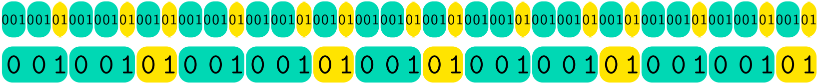

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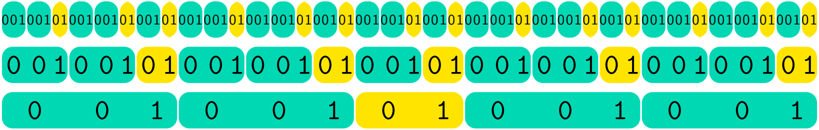

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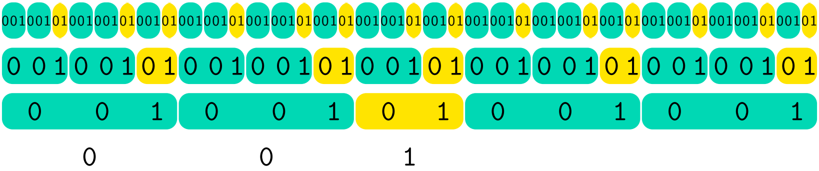

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