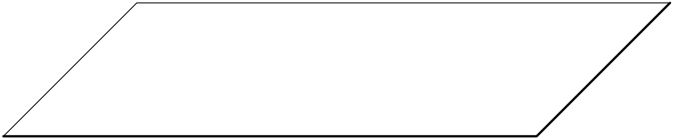
# The Geometry of The Night Sky

## or, An Ape Pointing at The Stars

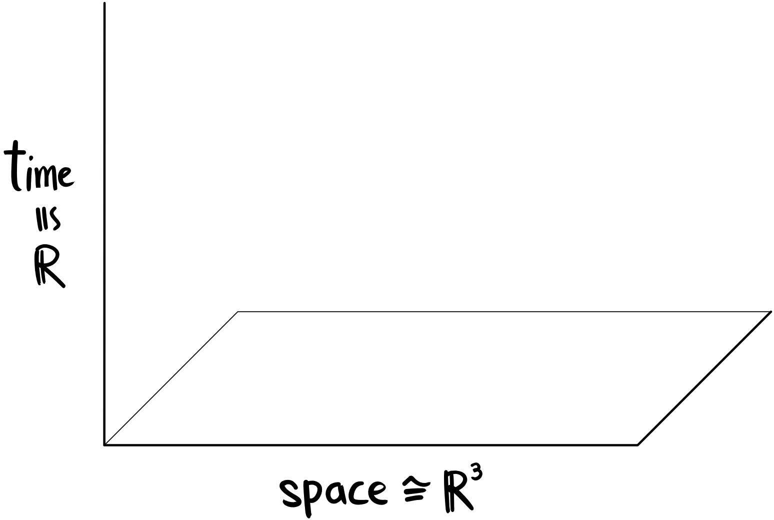
Aaron Fenyes (U.T. Austin)
Joint Math Meetings 2015

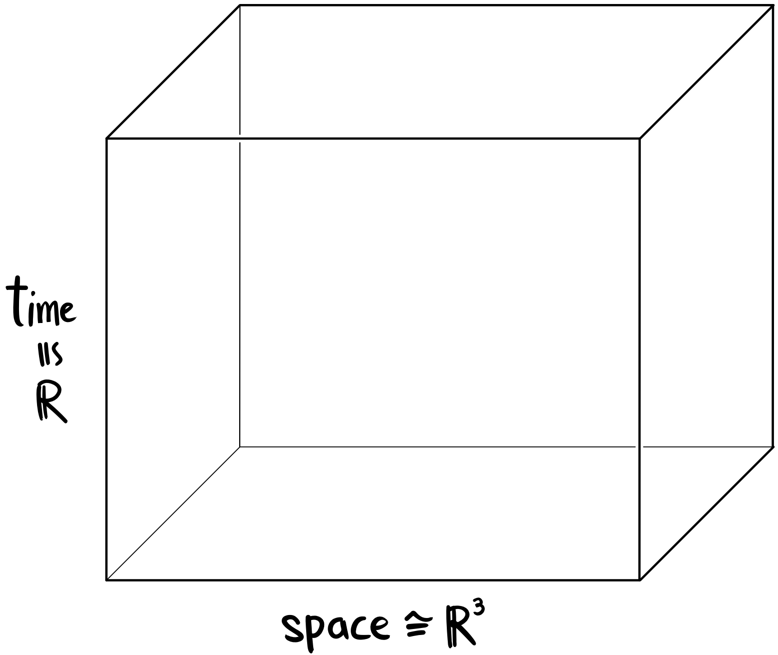
# Part 0

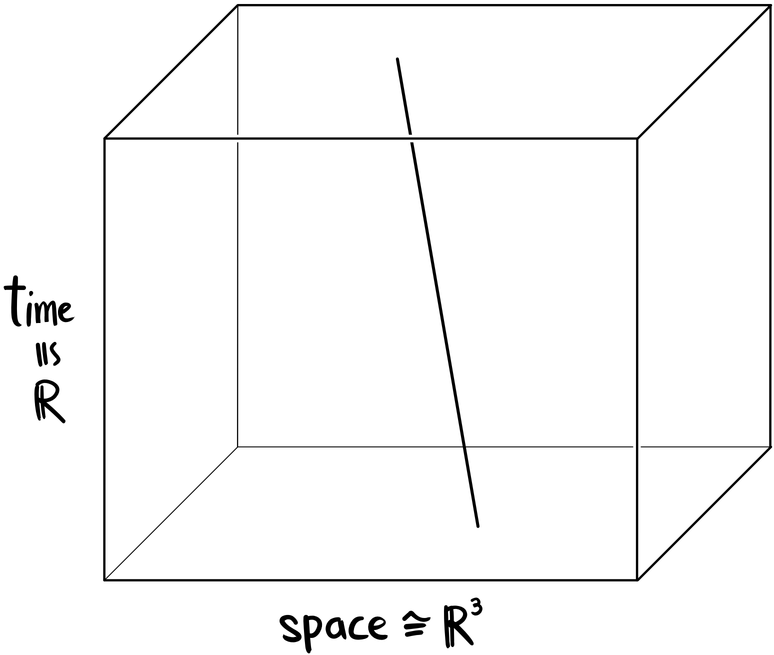
# Flat Spacetime

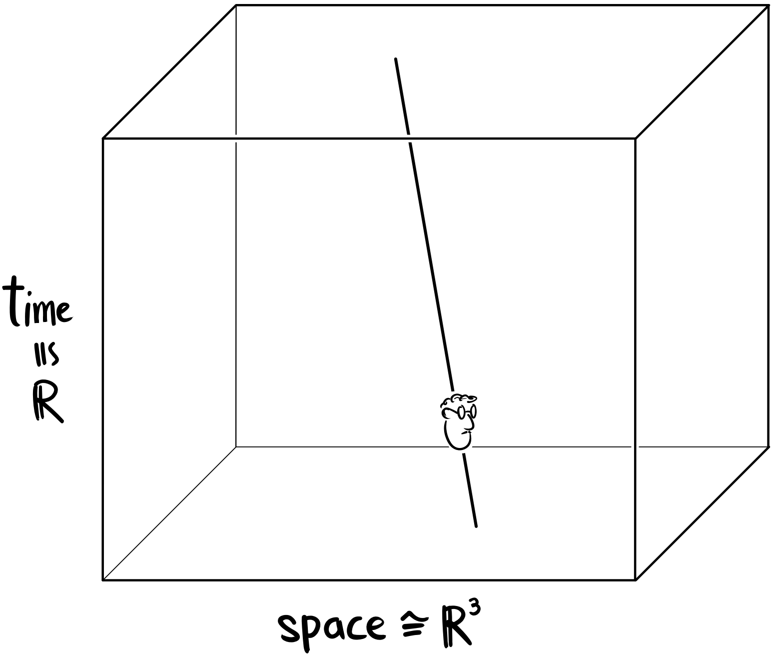


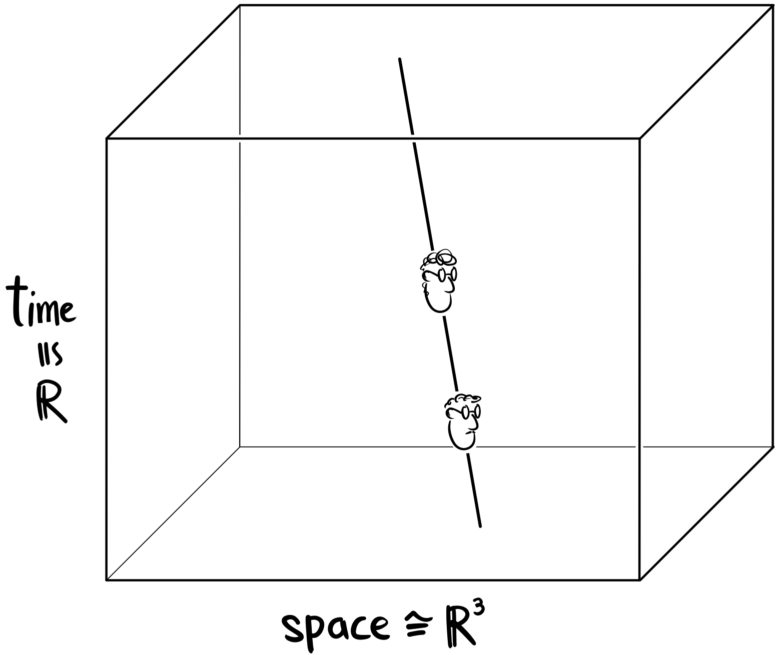
space $\cong \mathbb{R}^3$


time
≅
ℝ
space ≅ ℝ³

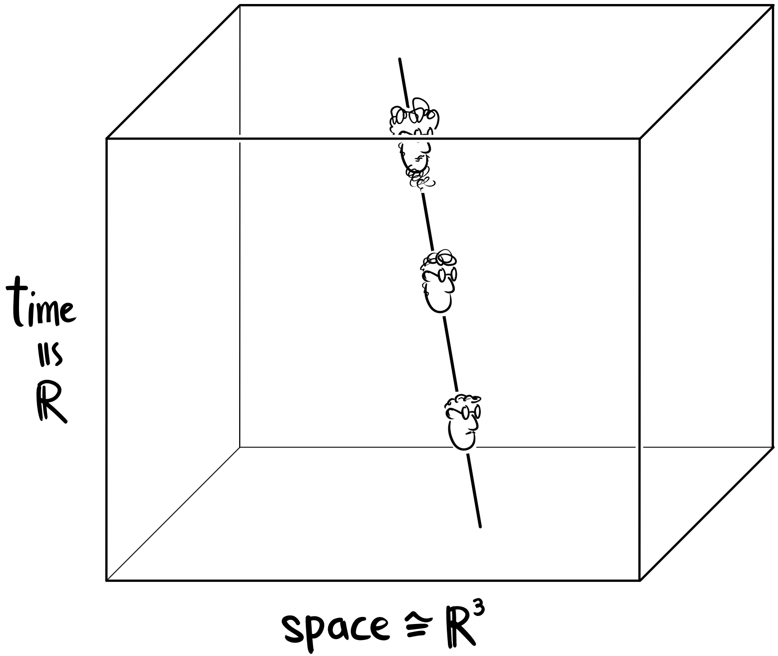

time ≅ ℝ
space ≅ ℝ³

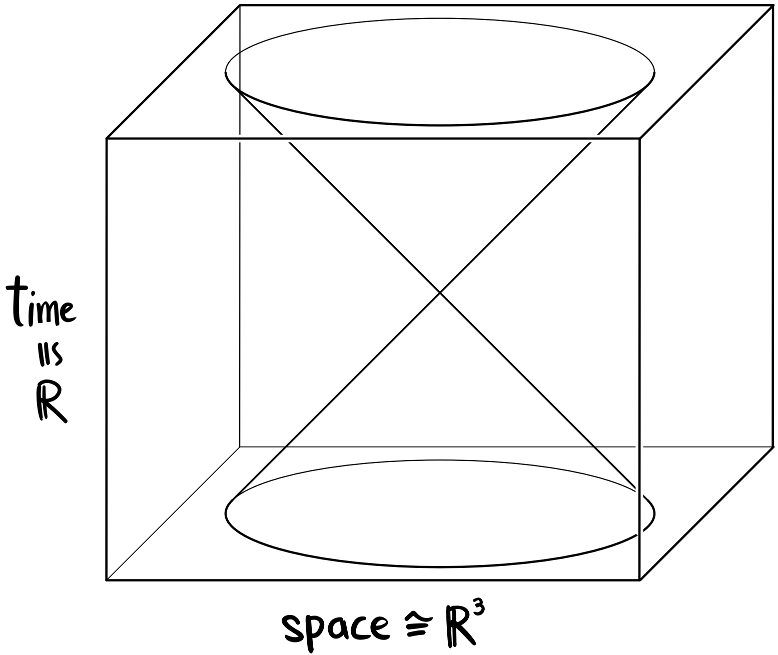

time
≅
ℝ
space ≅ ℝ³

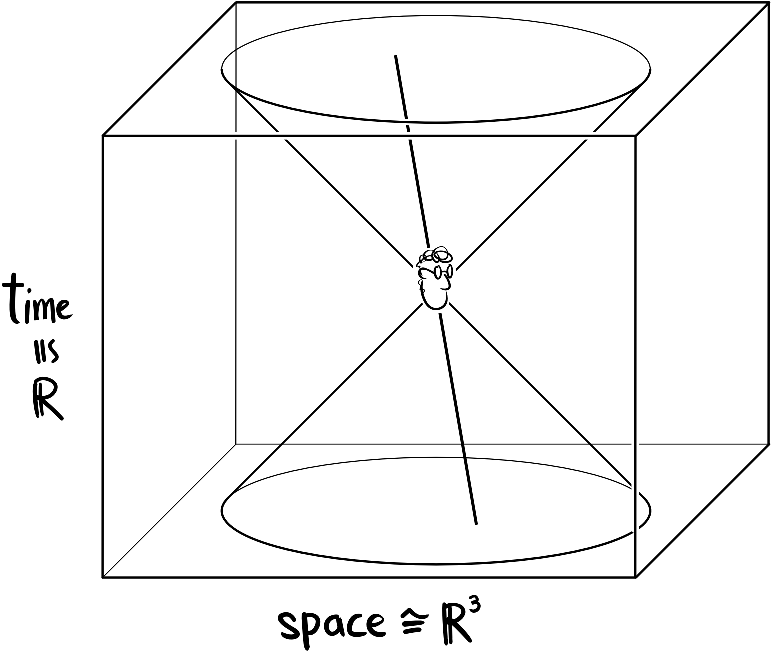

time ≅ ℝ
space ≅ ℝ³

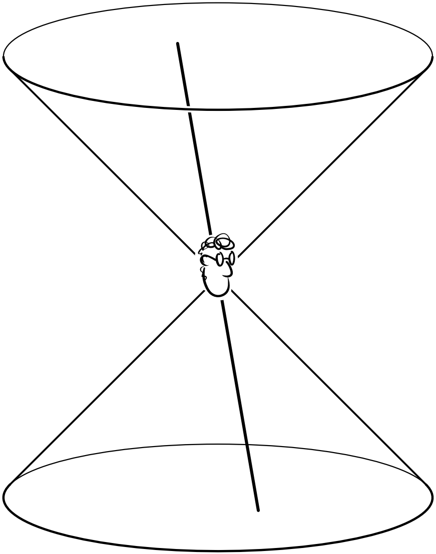


time
≅
R
space ≅ R³


time
≅
ℝ
space ≅ ℝ³

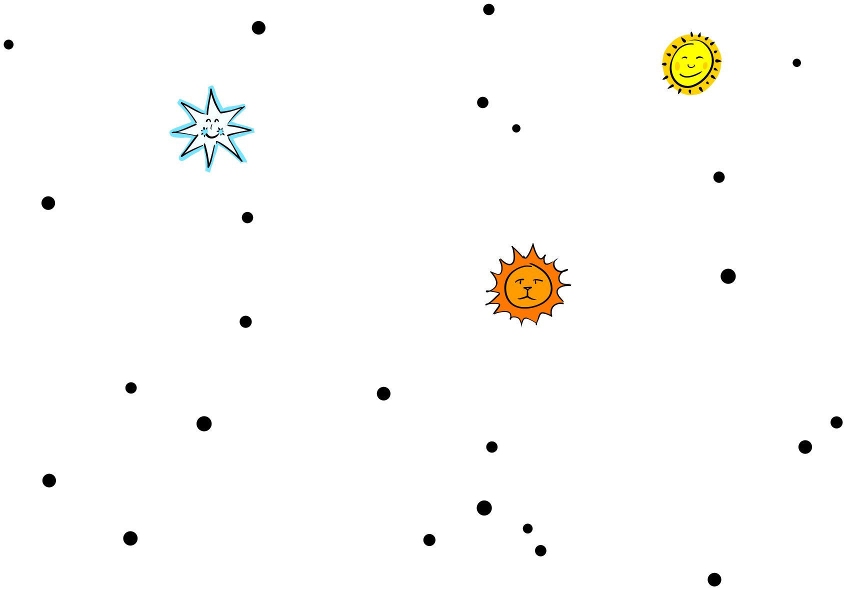

time
≅
ℝ
space ≅ $\mathbb{R}^3$

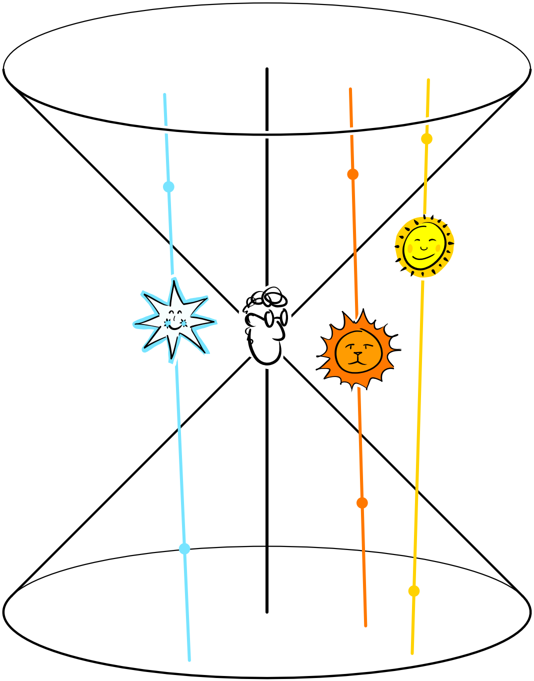

time
≅
ℝ
space ≅ ℝ³

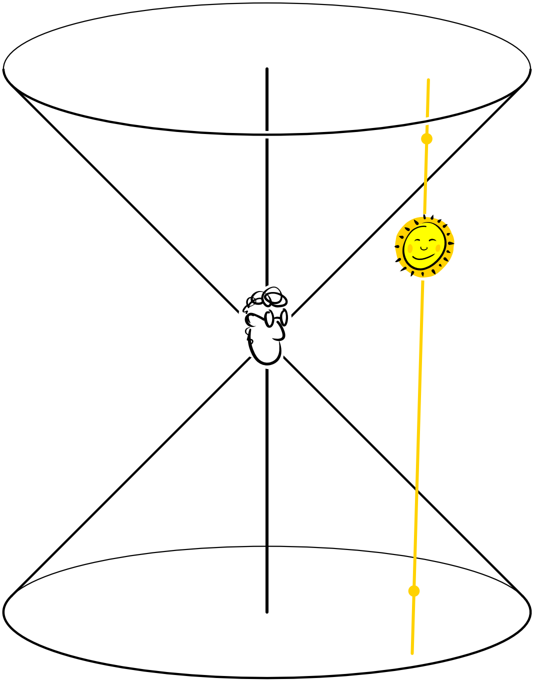


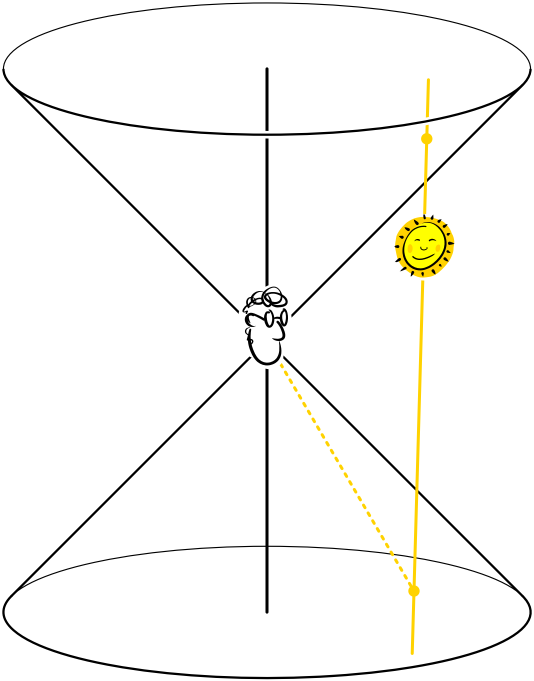
# Part I

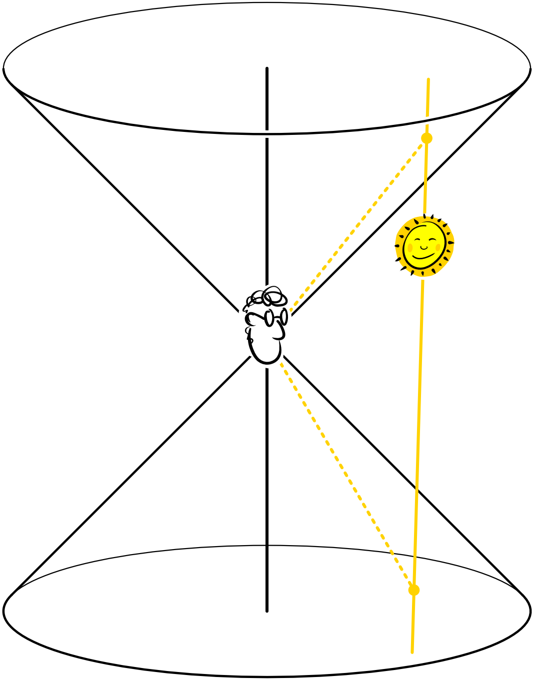
# The Celestial Sphere

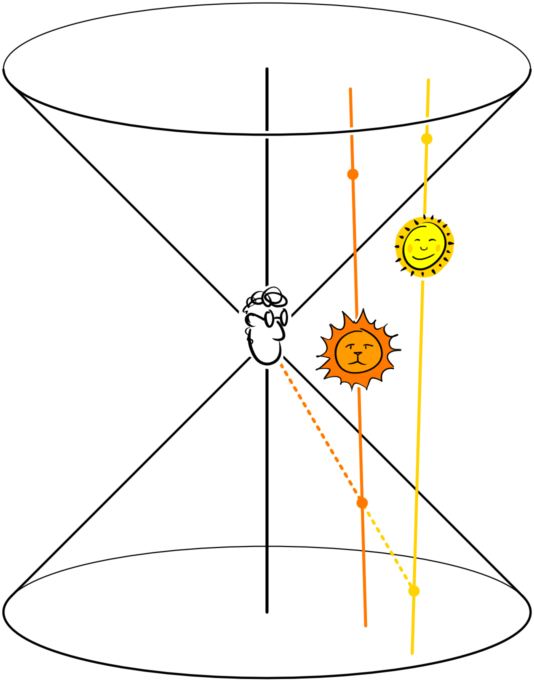


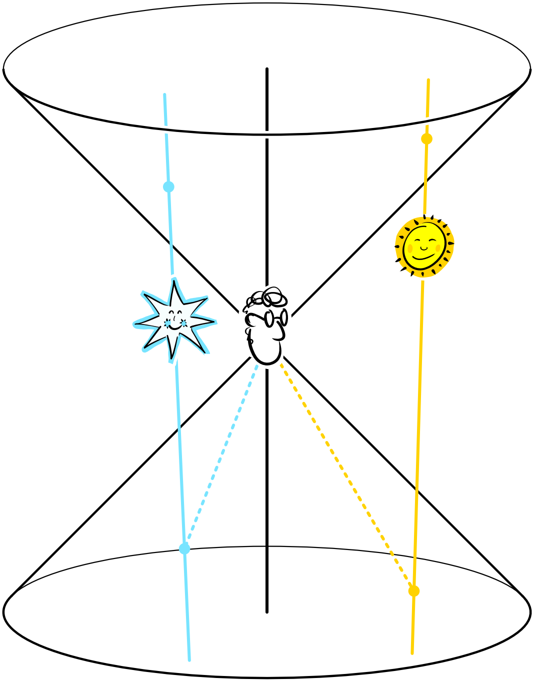


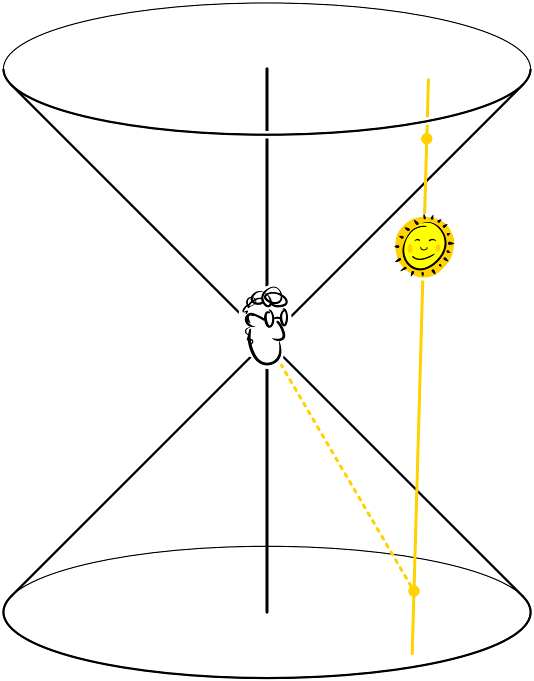


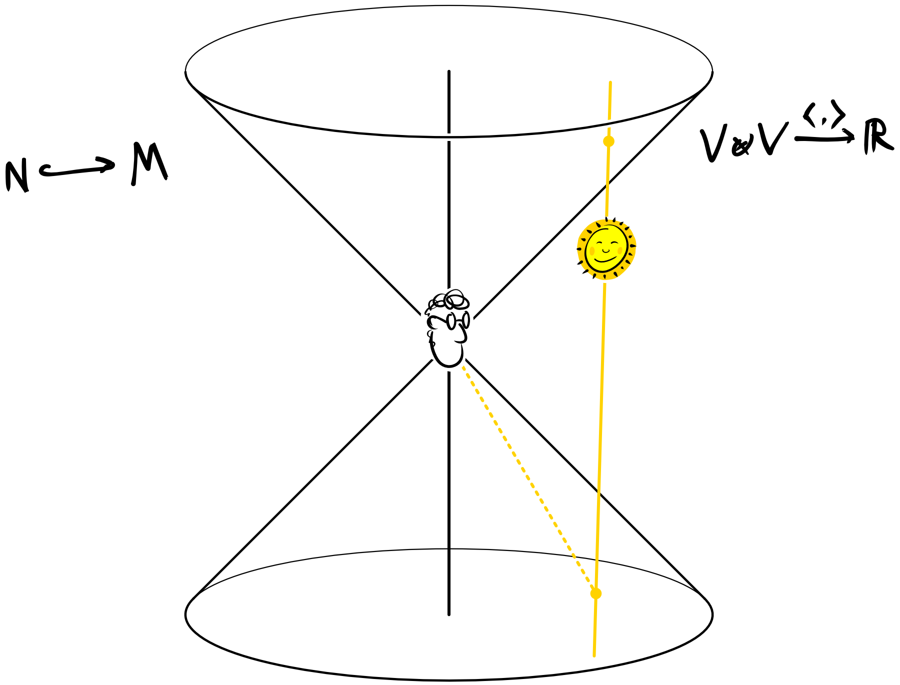


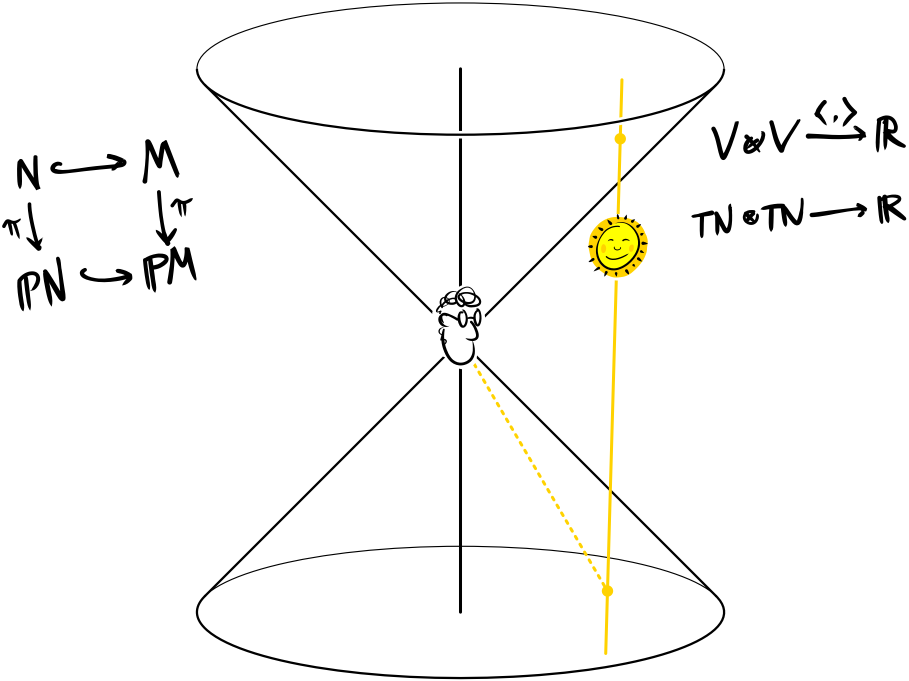









$N \hookrightarrow M$
$V \otimes V \xrightarrow{\langle , \rangle} \mathbb{R}$

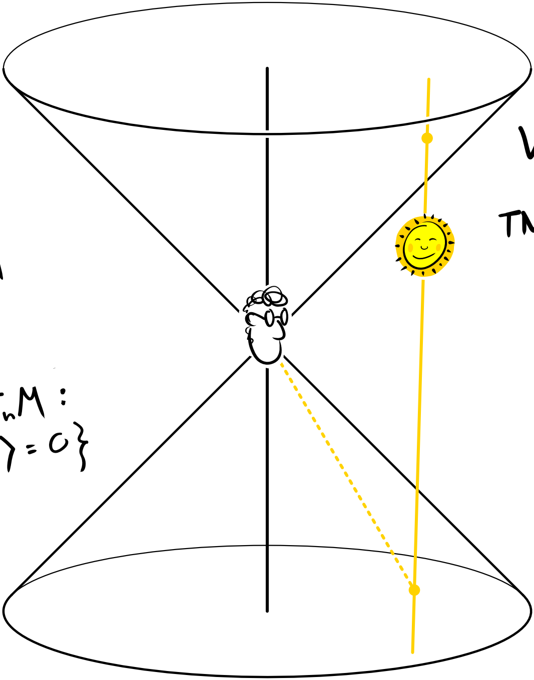


N ↪ M
π ↓ ↓ π
PN ↪ PM
V ⊗ V ⟨ , ⟩→ ℝ
TN ⊗ TN → ℝ

$$\begin{array}{ccc} N & \hookrightarrow & M \\ \downarrow{\scriptstyle \pi} & & \downarrow{\scriptstyle \pi} \\ \mathbb{P}N & \hookrightarrow & \mathbb{P}M \end{array}$$

$n \in N$

$$T_nN = \{ v \in T_nM : \langle v, n \rangle = 0 \}$$



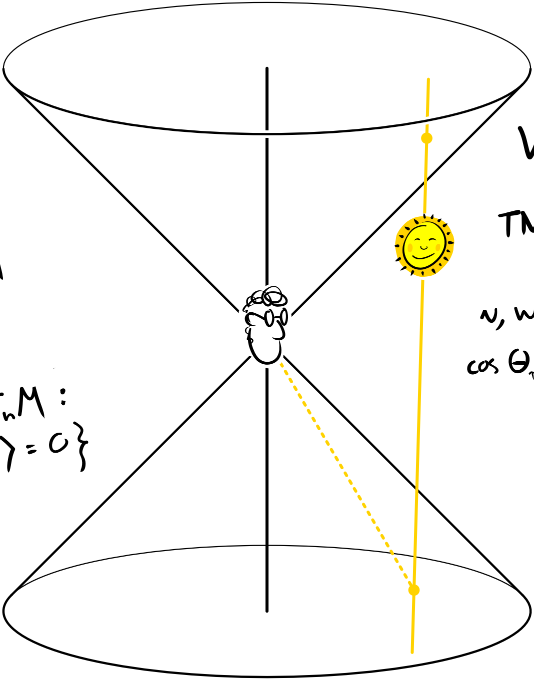
$$V \otimes V \xrightarrow{\langle\, , \rangle} \mathbb{R}$$

$$TN \otimes TN \longrightarrow \mathbb{R}$$

$$\begin{array}{ccc} N & \hookrightarrow & M \\ \downarrow \pi & & \downarrow \pi \\ \mathbb{P}N & \hookrightarrow & \mathbb{P}M \end{array}$$

$n \in N$

$$T_nN = \{ v \in T_nM : \langle v, n \rangle = 0 \}$$



$$V \otimes V \xrightarrow{\langle\, ,\, \rangle} \mathbb{R}$$

$$TN \otimes TN \longrightarrow \mathbb{R}$$

$v, w \in T_nN$

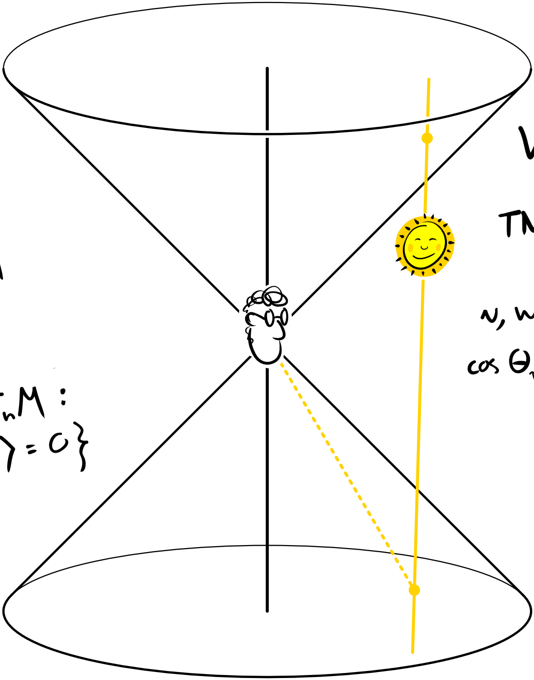
$$\cos \Theta_{\pi_* v, \pi_* w} := \frac{\langle v, w \rangle}{\|v\| \, \|w\|}$$

$$\begin{array}{ccc} N & \hookrightarrow & M \\ \pi \downarrow & & \downarrow \pi \\ \mathbb{P}N & \hookrightarrow & \mathbb{P}M \end{array}$$

$n \in N$

$$T_nN = \{ v \in T_nM : \langle v, n \rangle = 0 \}$$

$$\pi_* v = \pi_* v' \iff v' = v + \alpha n$$



$$V \otimes V \xrightarrow{\langle \, , \rangle} \mathbb{R}$$

$$TN \otimes TN \to \mathbb{R}$$

$v, w \in T_nN$

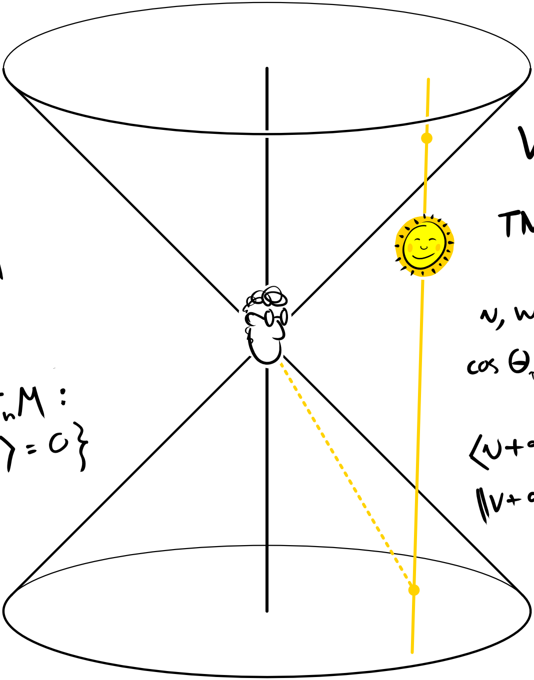
$$\cos \theta_{\pi_* v, \pi_* w} := \frac{\langle v, w \rangle}{\|v\| \, \|w\|}$$

$$\begin{array}{ccc} N & \hookrightarrow & M \\ \downarrow \pi & & \downarrow \pi \\ \mathbb{P}N & \hookrightarrow & \mathbb{P}M \end{array}$$

$n \in N$

$$T_nN = \{ v \in T_nM : \langle v, n \rangle = 0 \}$$

$$\pi_* v = \pi_* v' \iff v' = v + \alpha n$$



$$V \otimes V \xrightarrow{\langle\, , \rangle} \mathbb{R}$$

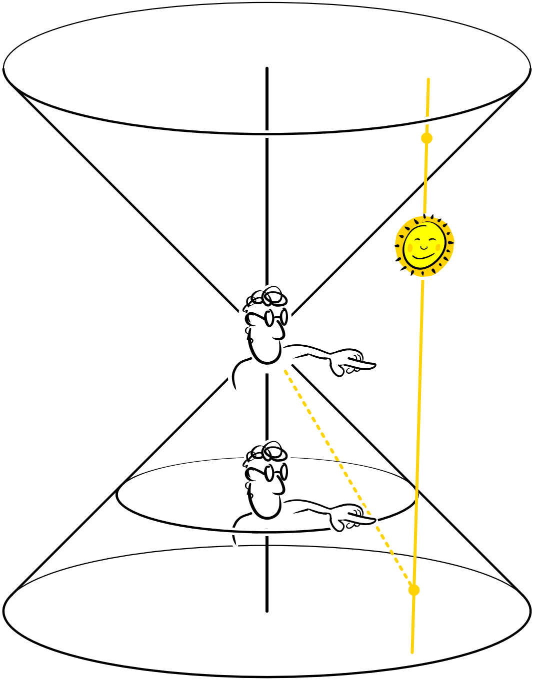
$$TN \otimes TN \longrightarrow \mathbb{R}$$

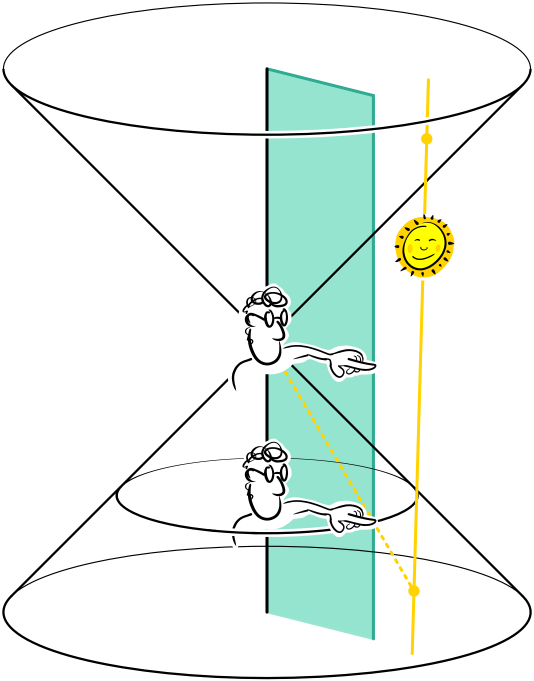
$v, w \in T_nN$

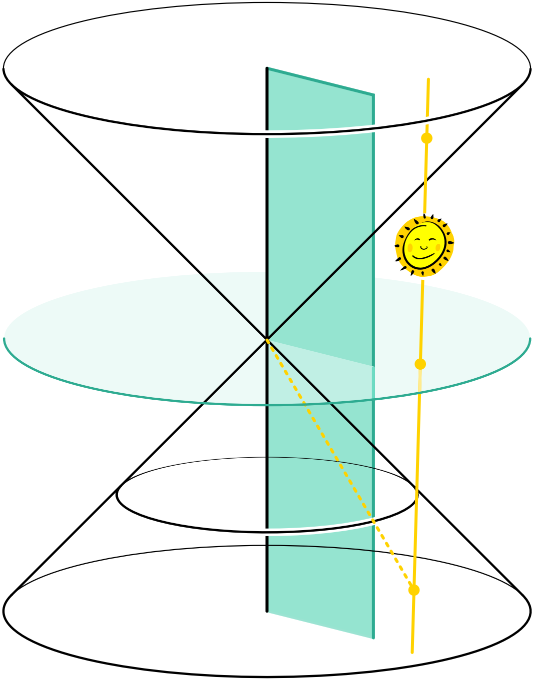
$$\cos \theta_{\pi_* v, \pi_* w} := \frac{\langle v, w \rangle}{\|v\| \, \|w\|}$$

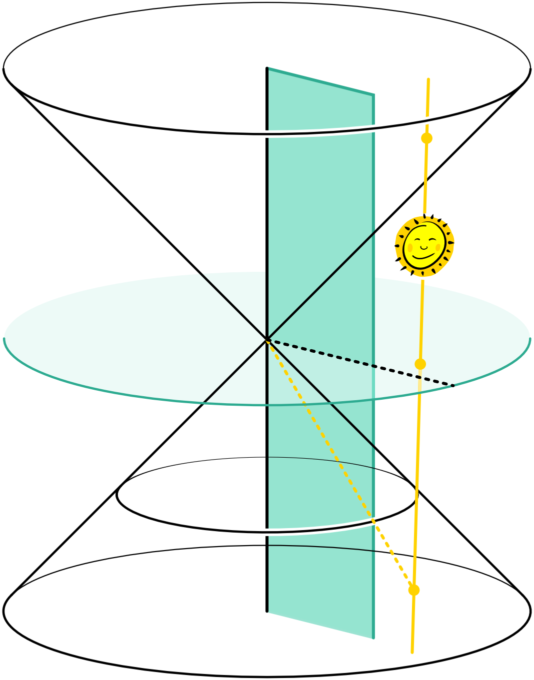
$$\langle v + \alpha n, w \rangle = \langle v, w \rangle$$

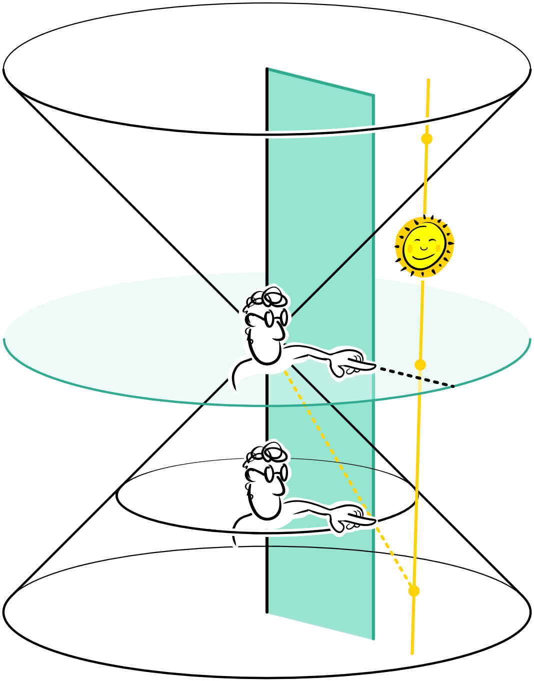
$$\|v + \alpha n\| = \|v\|$$

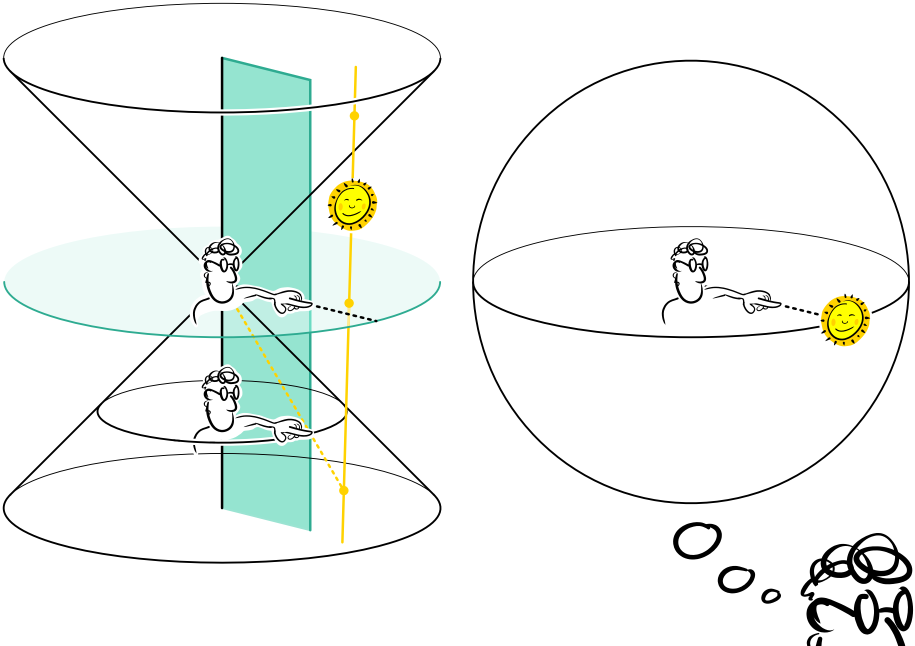






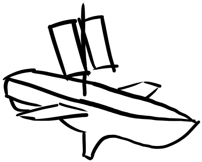


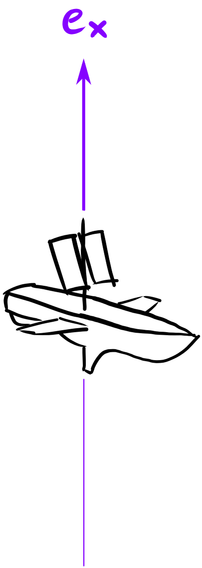


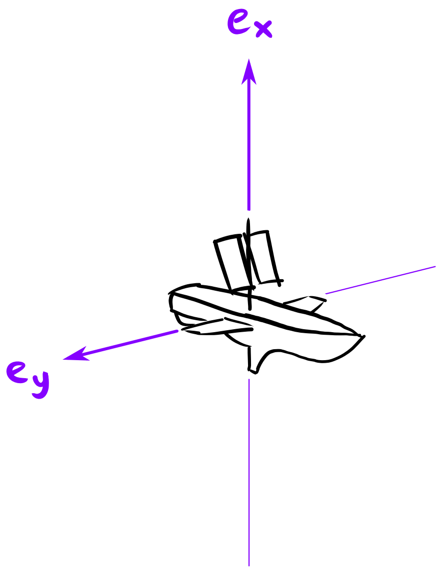


# Part 2

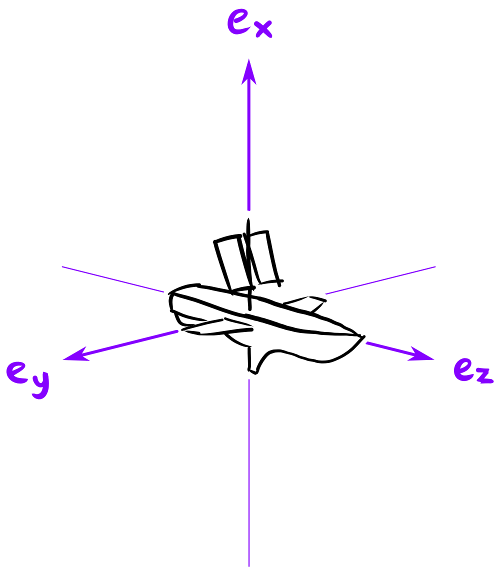
# Lorentz & Möbius



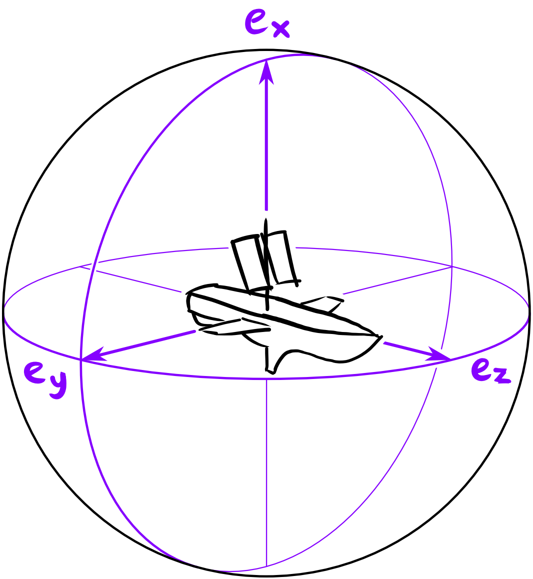

$e_x$



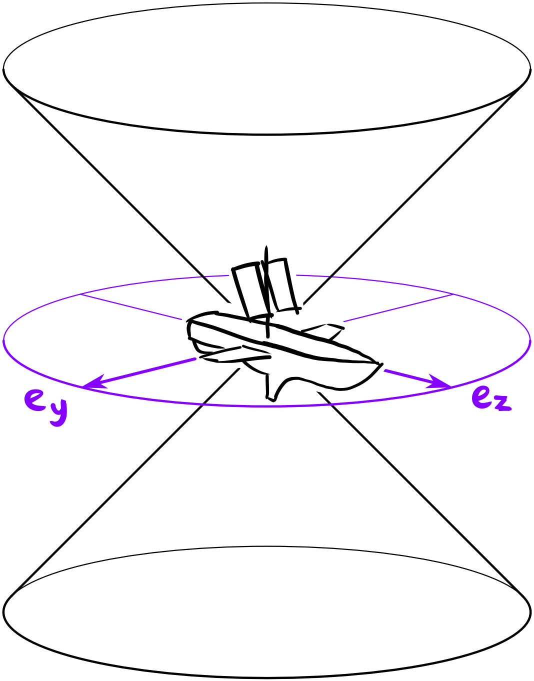
$e_x$
$e_y$



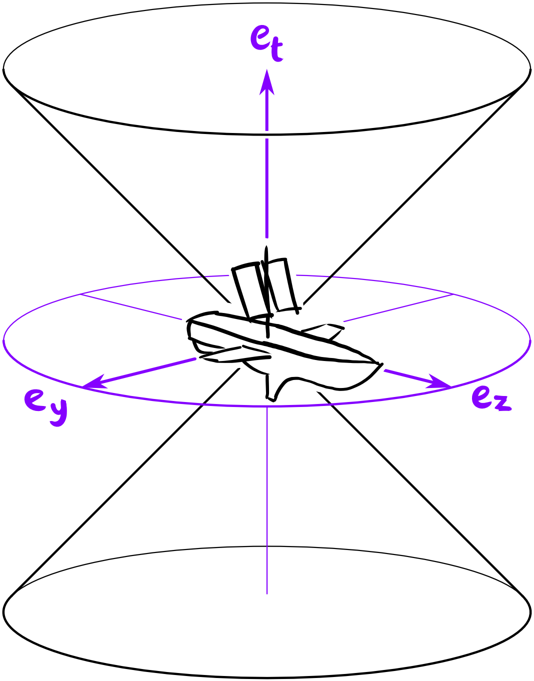
$e_x$
$e_y$
$e_z$

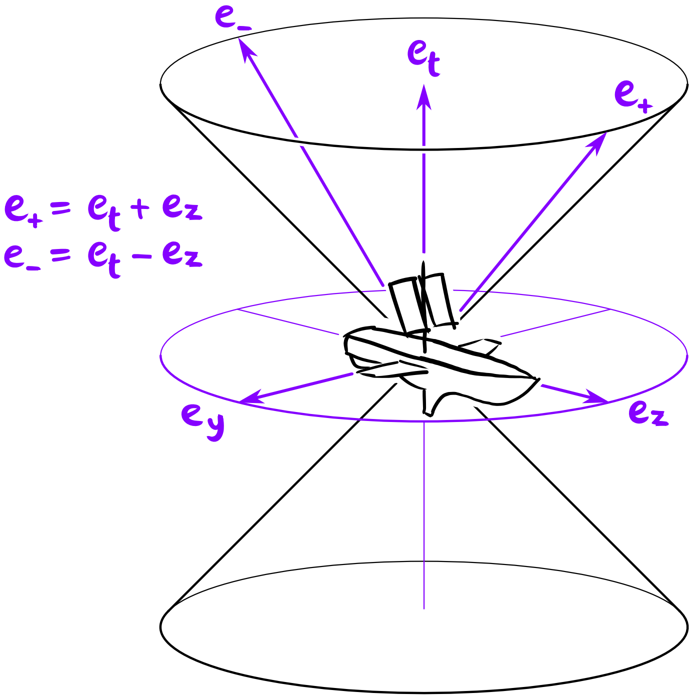


$e_x$
$e_y$
$e_z$

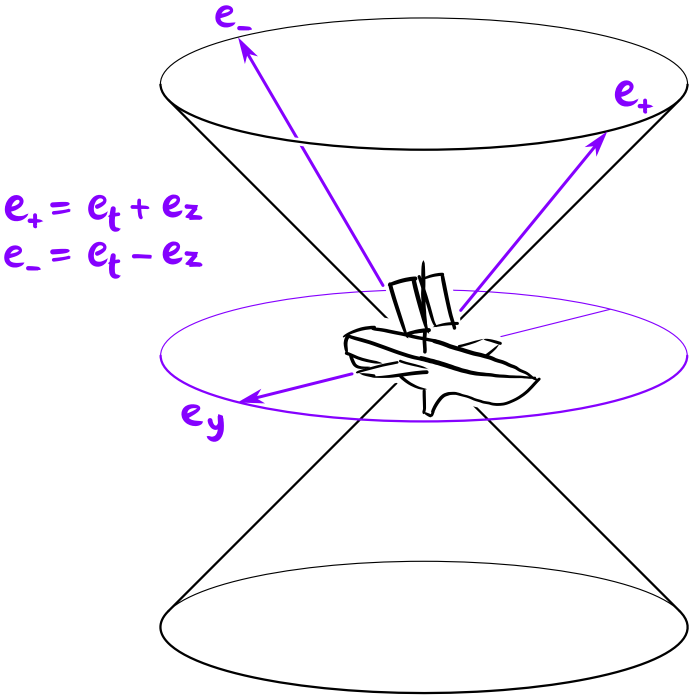


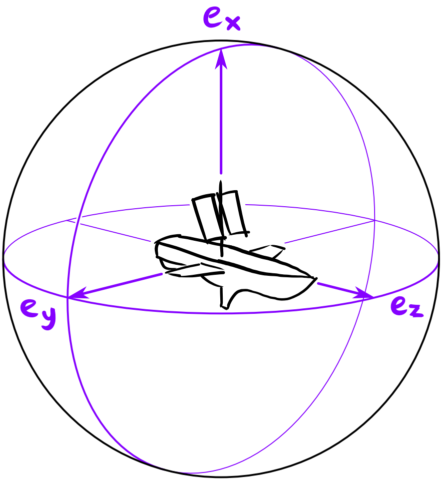
e_y
e_z

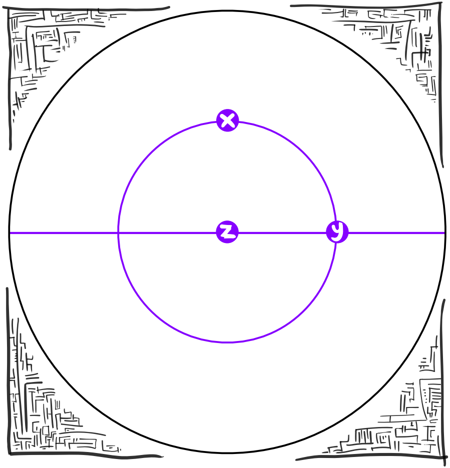

e_t
e_y
e_z

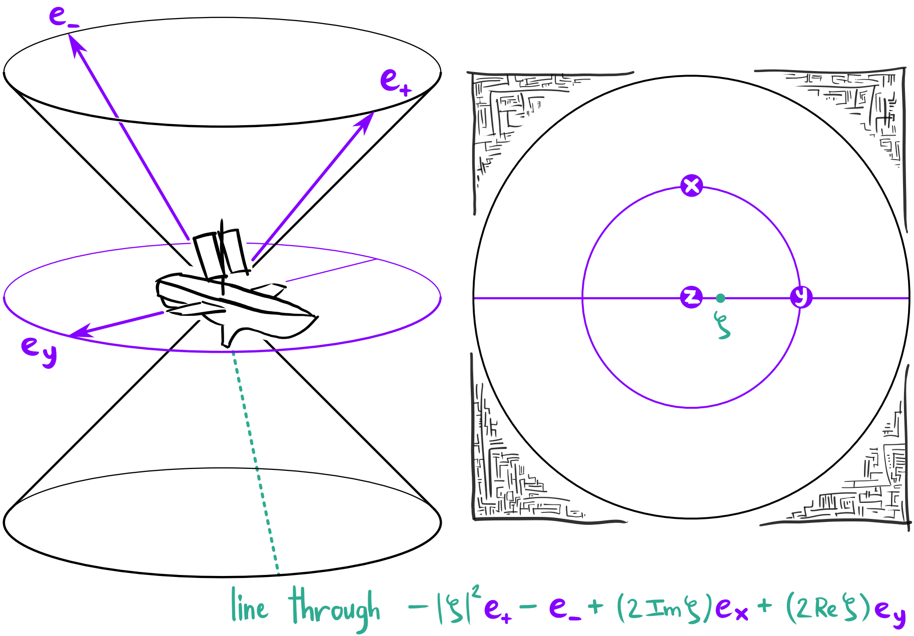


e₋
e_t
e₊
e₊ = e_t + e_z
e₋ = e_t − e_z
e_y
e_z

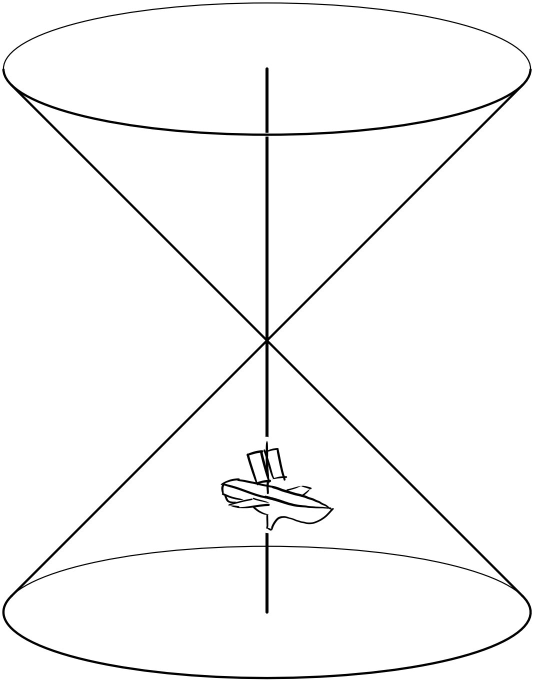

$e_-$
$e_+$
$e_+ = e_t + e_z$
$e_- = e_t - e_z$
$e_y$

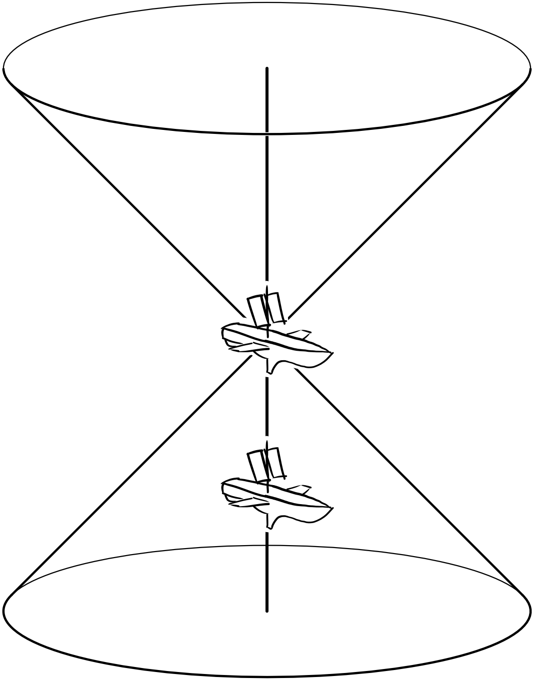

$e_x$
$e_y$
$e_z$

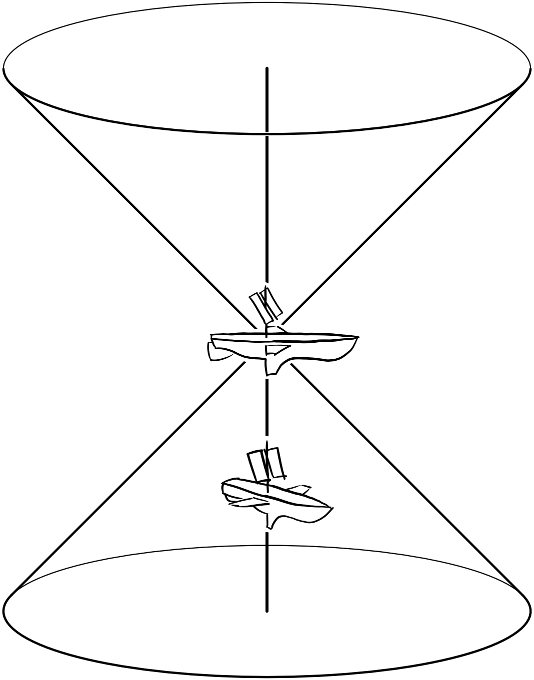

x
z
y

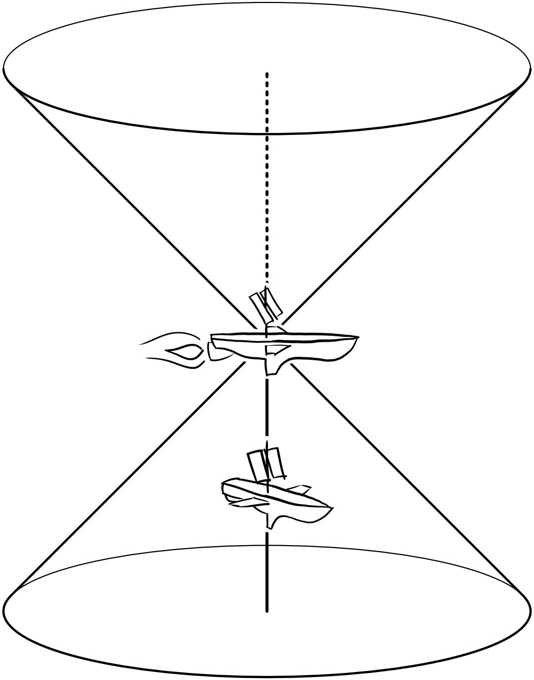


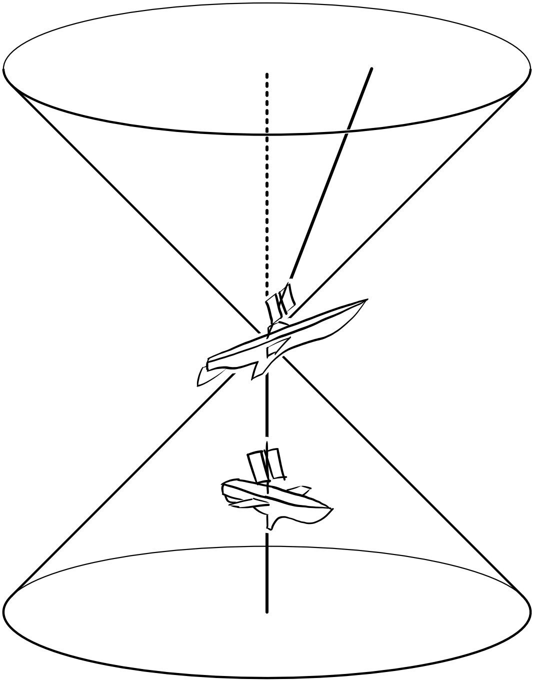
$e_-$
$e_+$
$e_y$
x
z
y
$\xi$
line through $-|\xi|^2 e_+ - e_- + (2\,\mathrm{Im}\,\xi)e_x + (2\,\mathrm{Re}\,\xi)e_y$

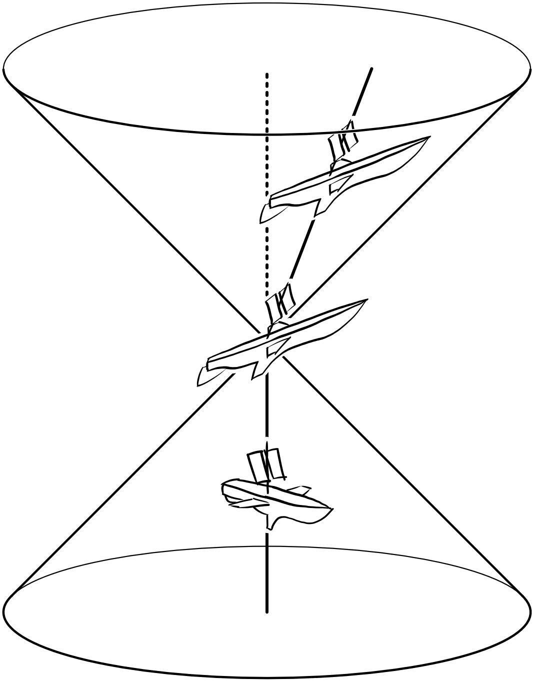


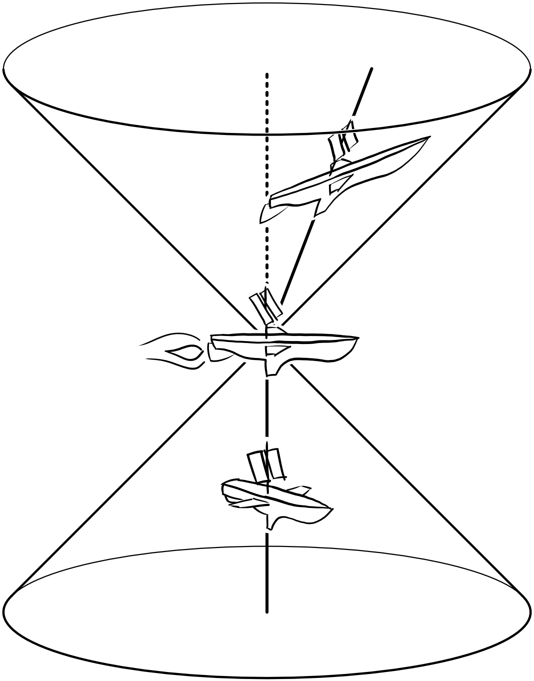


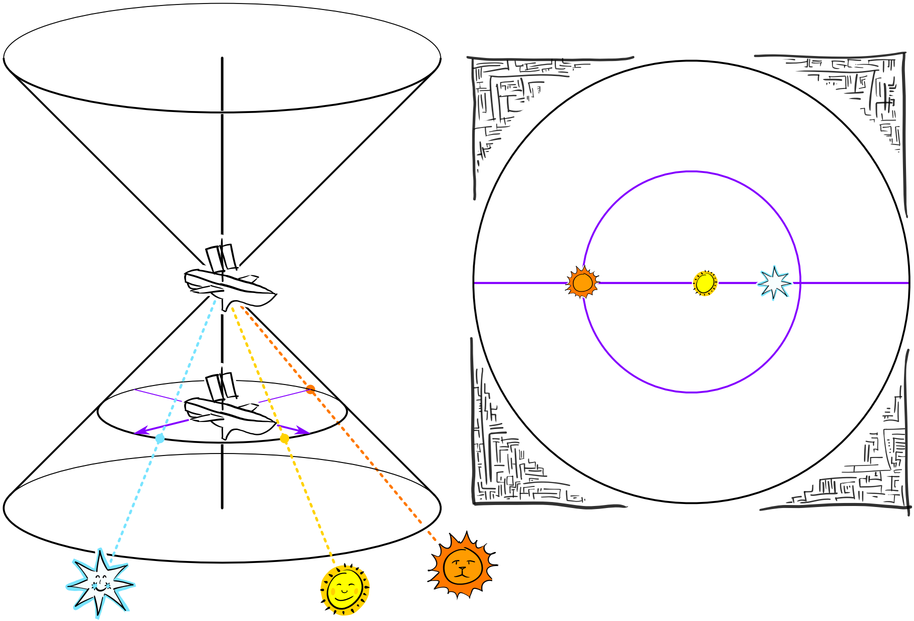


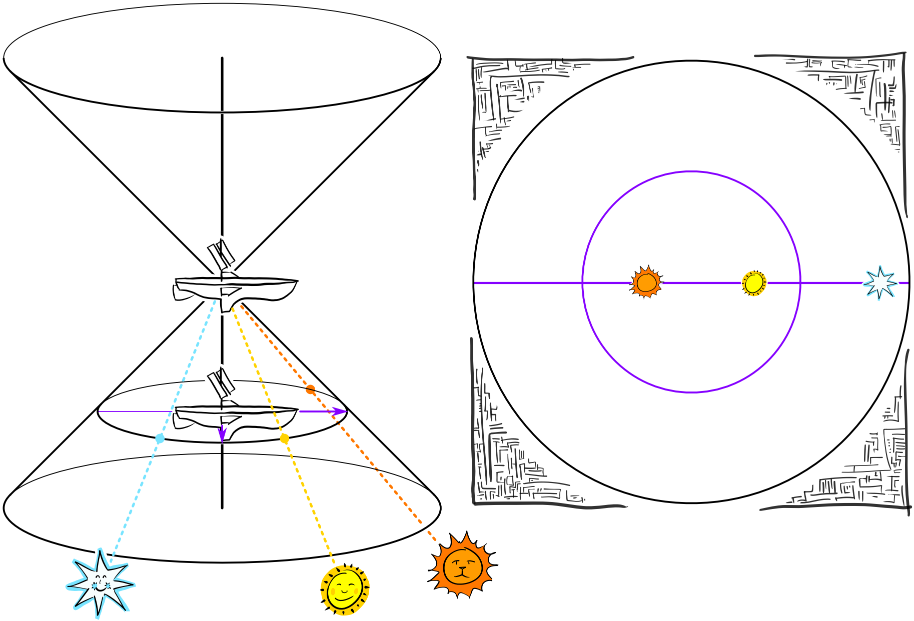


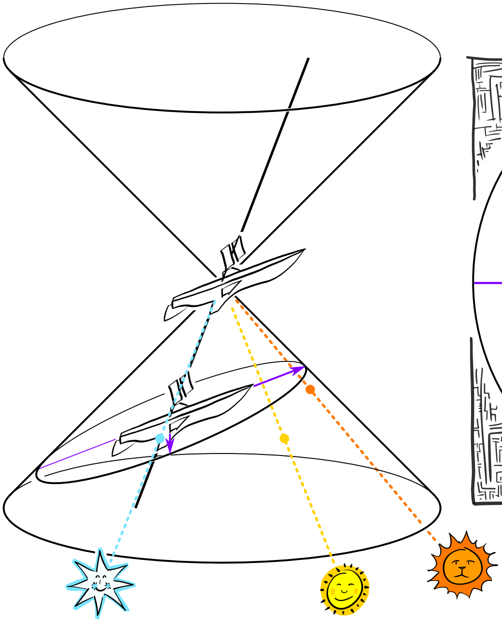


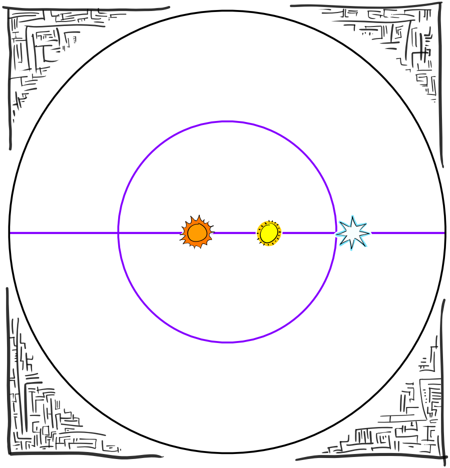


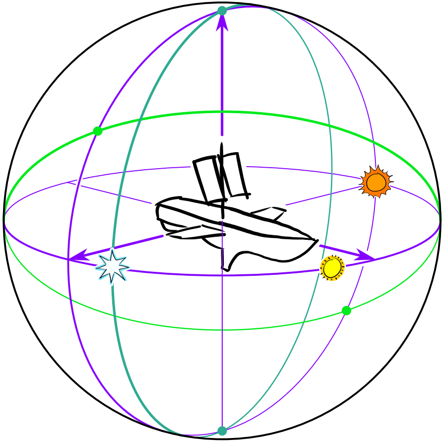


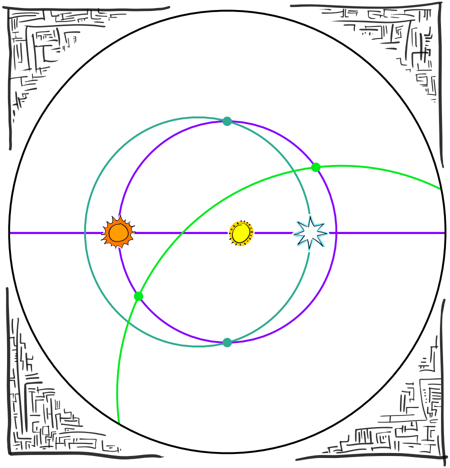


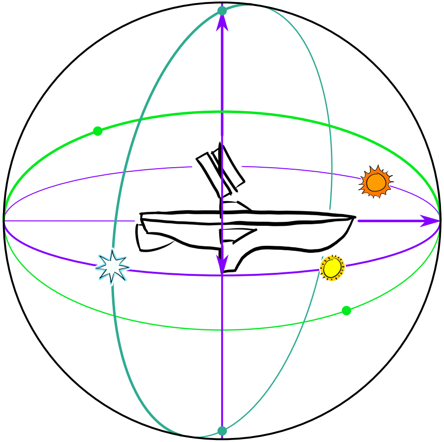


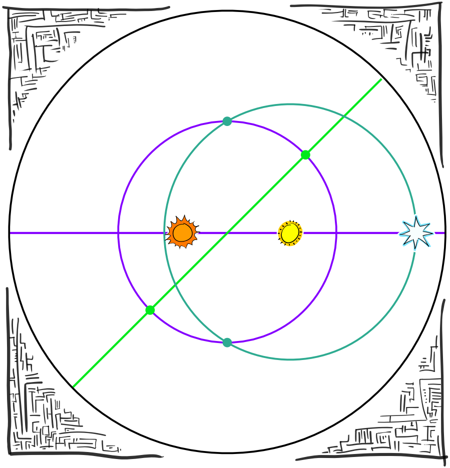


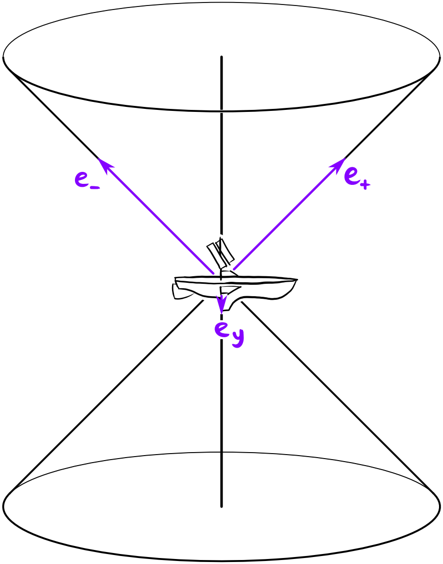












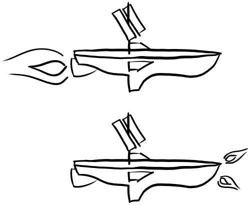

$$e'_+ = \lambda e_+$$

$$e'_- = \frac{1}{\lambda} e_-$$

$$e'_y = e_y$$

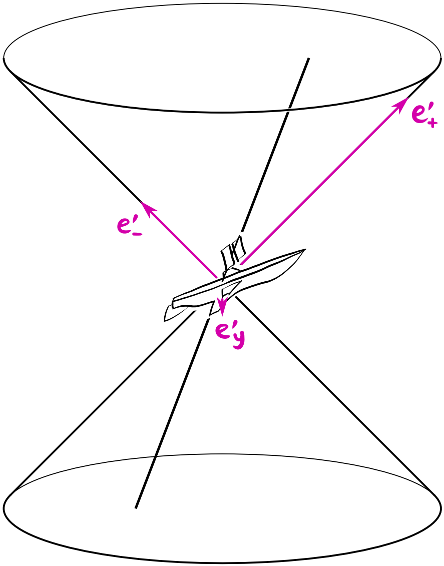
$$e'_x = e_x$$

$$\lambda \in \mathbb{R}_+$$



$\lambda > 1$

$\lambda < 1$



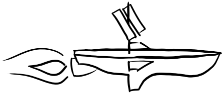

$$e'_+ = \lambda e_+$$

$$e'_- = \frac{1}{\lambda} e_-$$

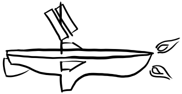
$$e'_y = e_y$$

$$e'_x = e_x$$

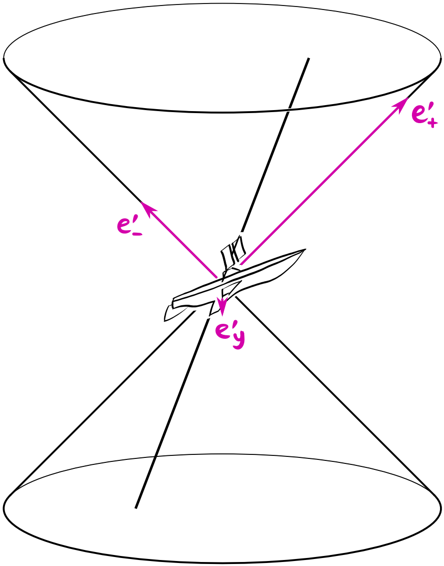
$$\lambda \in \mathbb{R}_+$$



$\lambda > 1$



$\lambda < 1$




$$e'_+ = \lambda e_+$$

$$e'_- = \frac{1}{\lambda} e_-$$

$$e'_y = e_y$$

$$e'_x = e_x$$

$$-|\zeta|^2 e_+ - e_- + (2\,\mathrm{Im}\,\zeta) e_x + (2\,\mathrm{Re}\,\zeta) e_y$$

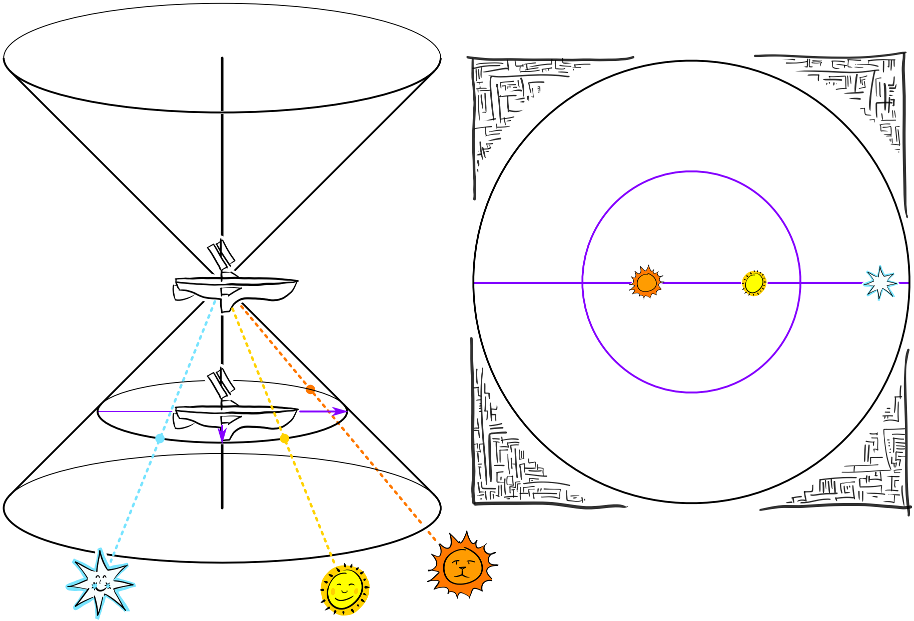
$$\|$$

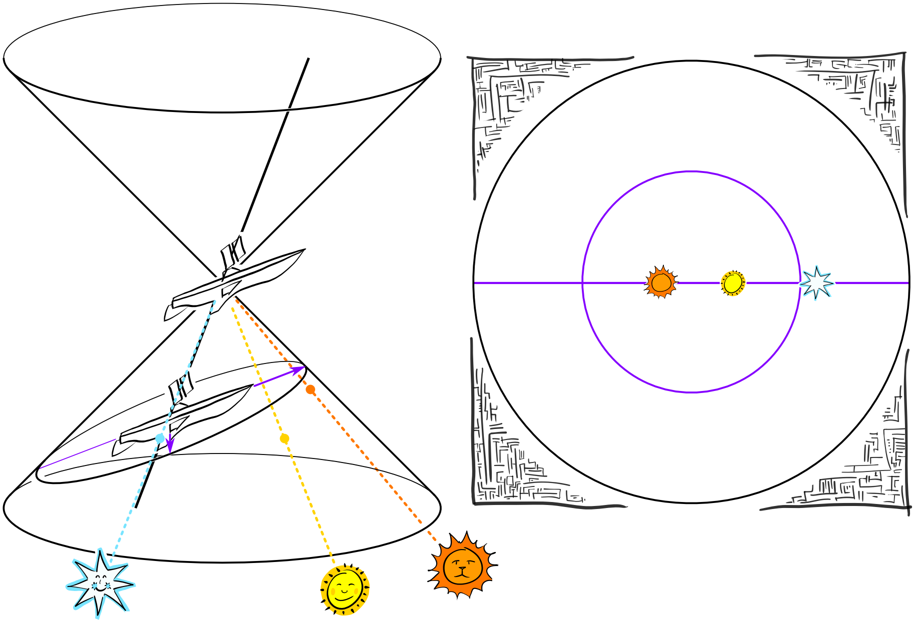
$$-\lambda|\zeta|^2 e'_+ - \frac{1}{\lambda} e'_- + (2\,\mathrm{Im}\,\zeta) e'_x + (2\,\mathrm{Re}\,\zeta) e'_y$$

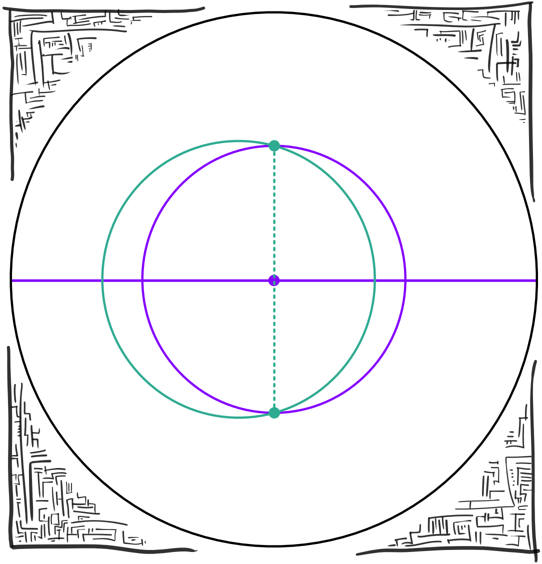
$$\wr$$

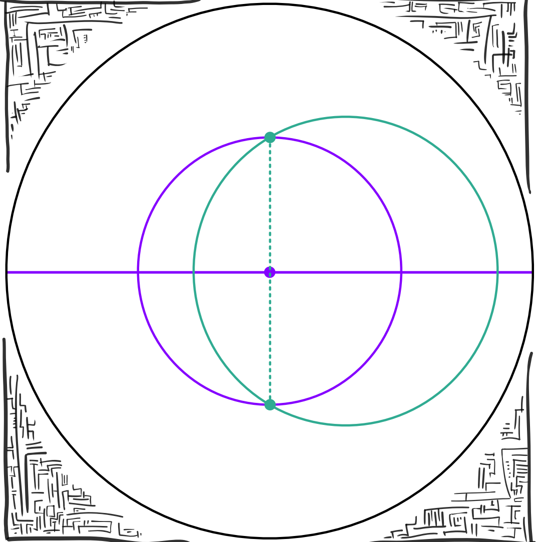
$$-\left|\frac{\zeta}{\lambda}\right|^2 e'_+ - e'_- + \left(2\,\mathrm{Im}\,\frac{\zeta}{\lambda}\right) e'_x + \left(2\,\mathrm{Re}\,\frac{\zeta}{\lambda}\right) e'_y$$

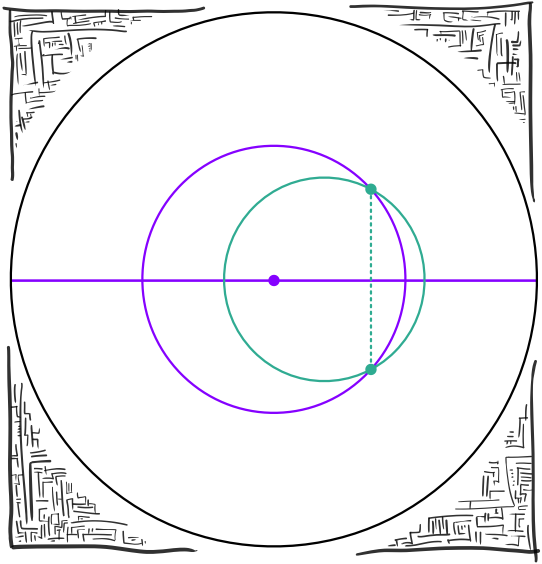
$$\zeta' = \frac{\zeta}{\lambda}$$

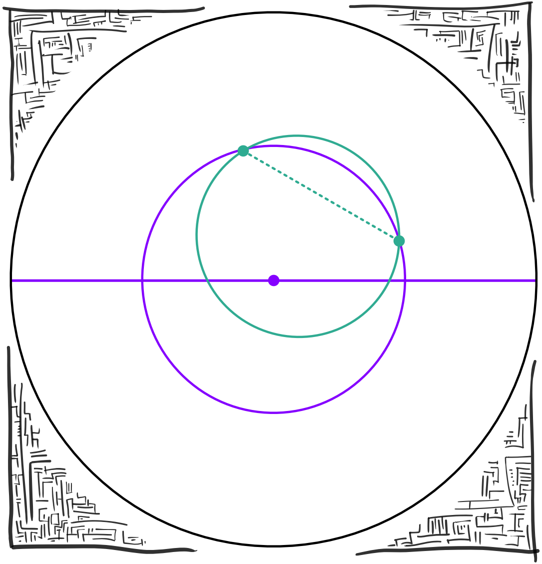












# Part 3

# An Application

Astrolabe by Jean Naze
1553

Photo by Marie-Lan Nguyen

Astrolabe by Jean Naze
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Astrolabe by Jean Naze
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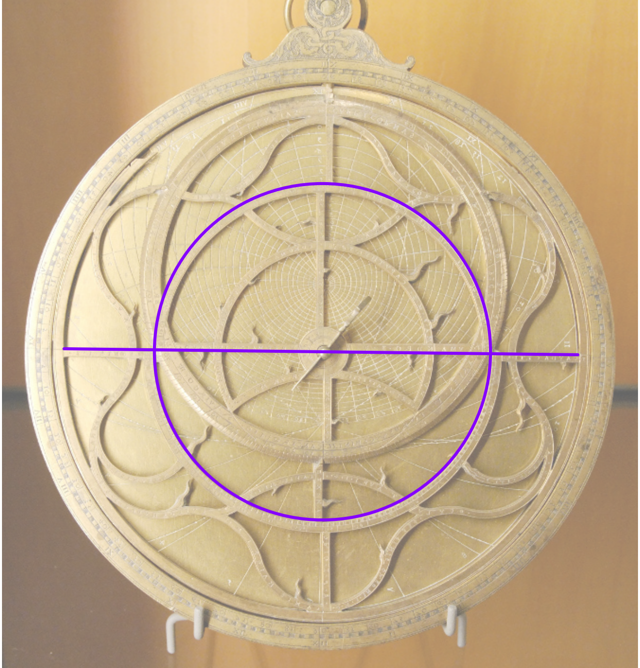
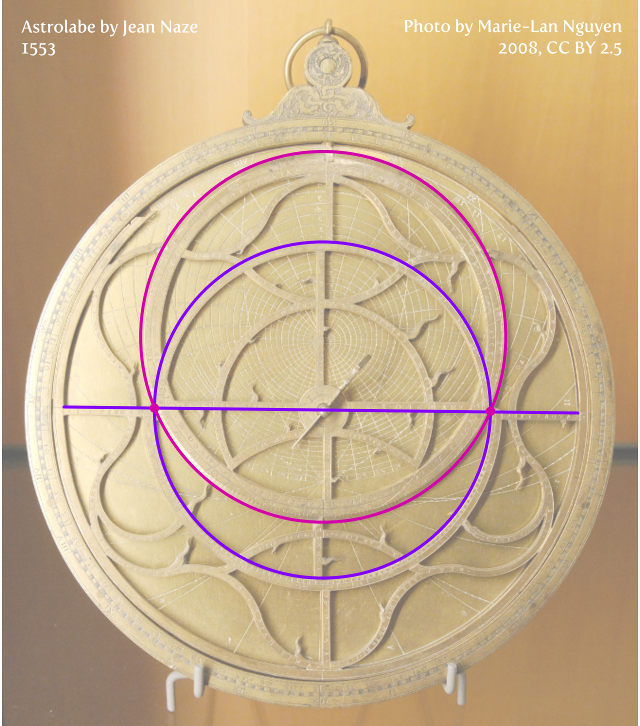
Photo by Marie-Lan Nguyen


Astrolabe by Jean Naze
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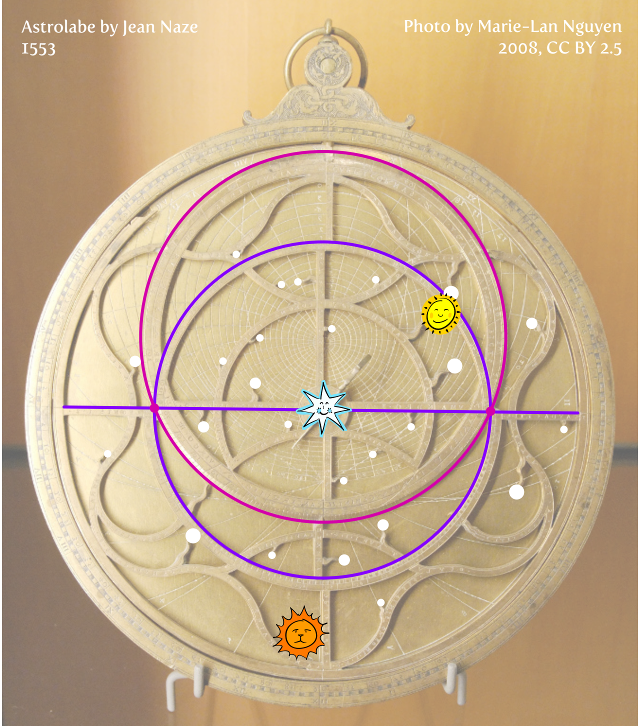
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