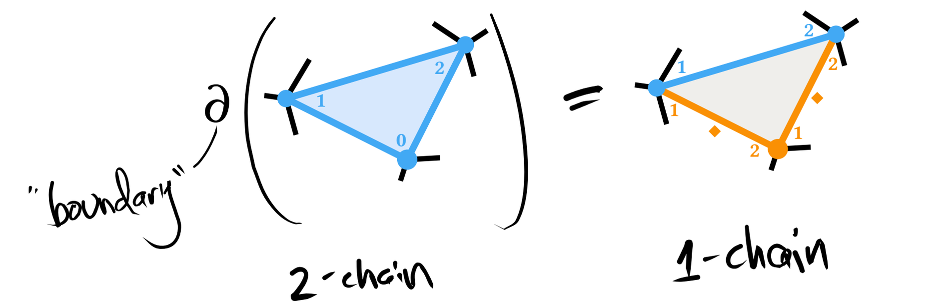


deformation rules:

$\partial C \sim 0$

for any chain C

Recognizing loops algebraically:

◇ The 1-chain C is a sum of closed loops if and only if $\partial C = 0$

<u>Def</u> The n-chain C is an <u>n-cycle</u> if $\partial C = 0$

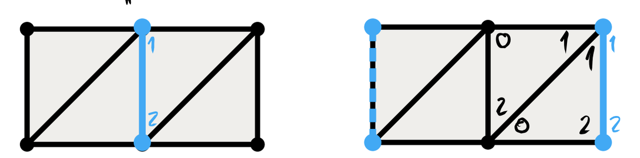
"closed up" shape

◇ Two n-cycles A, B can be deformed into each other if and only if $\boxed{B = A + \partial F}$ for some $(n+1)$-chain F

i.e. can turn A into B using the deformation substitutions coming from $(n+1)$-simplices

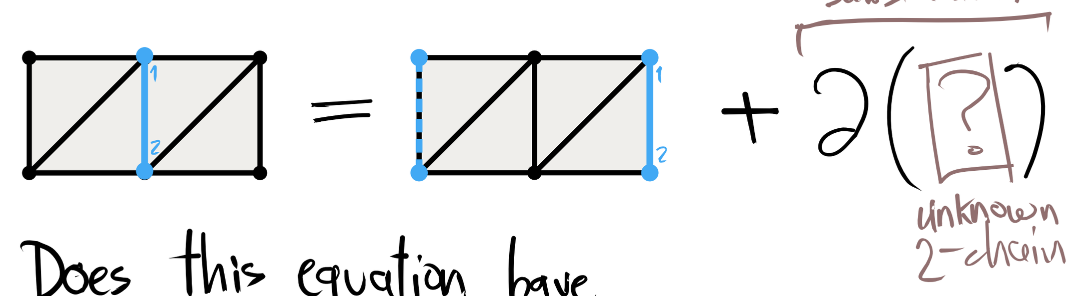
Example

Geometry problem:



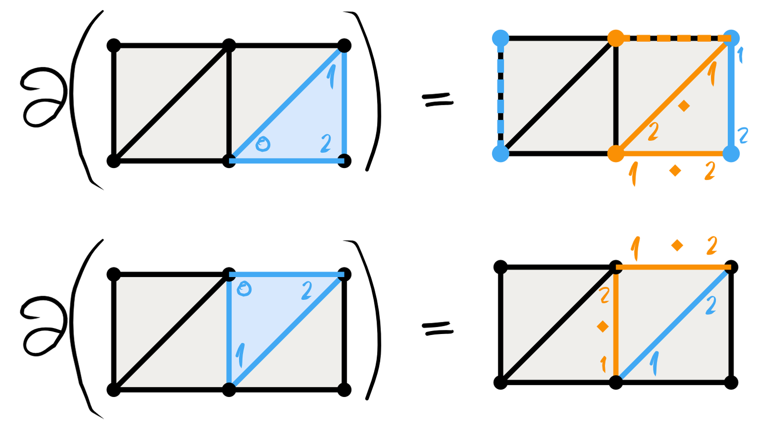
Can these loops be deformed into each other?

Equivalent algebra problem:

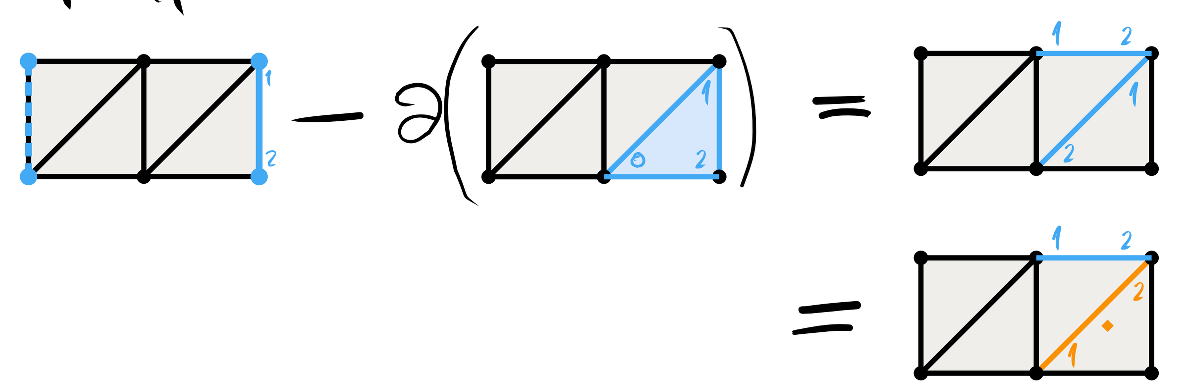


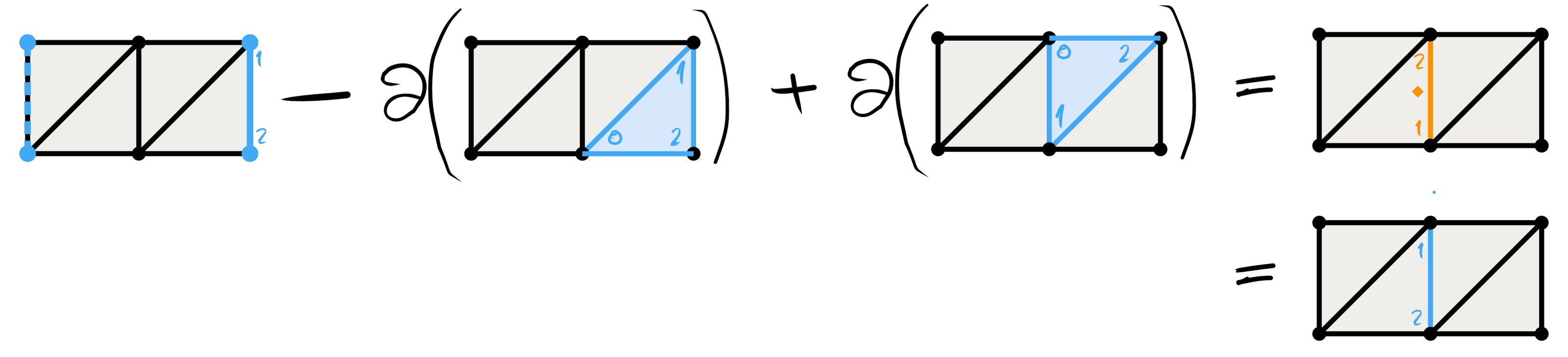
Does this equation have a solution?

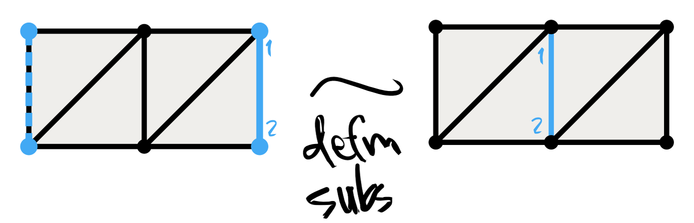
Thérèse built a solution from the deformations



She noticed that

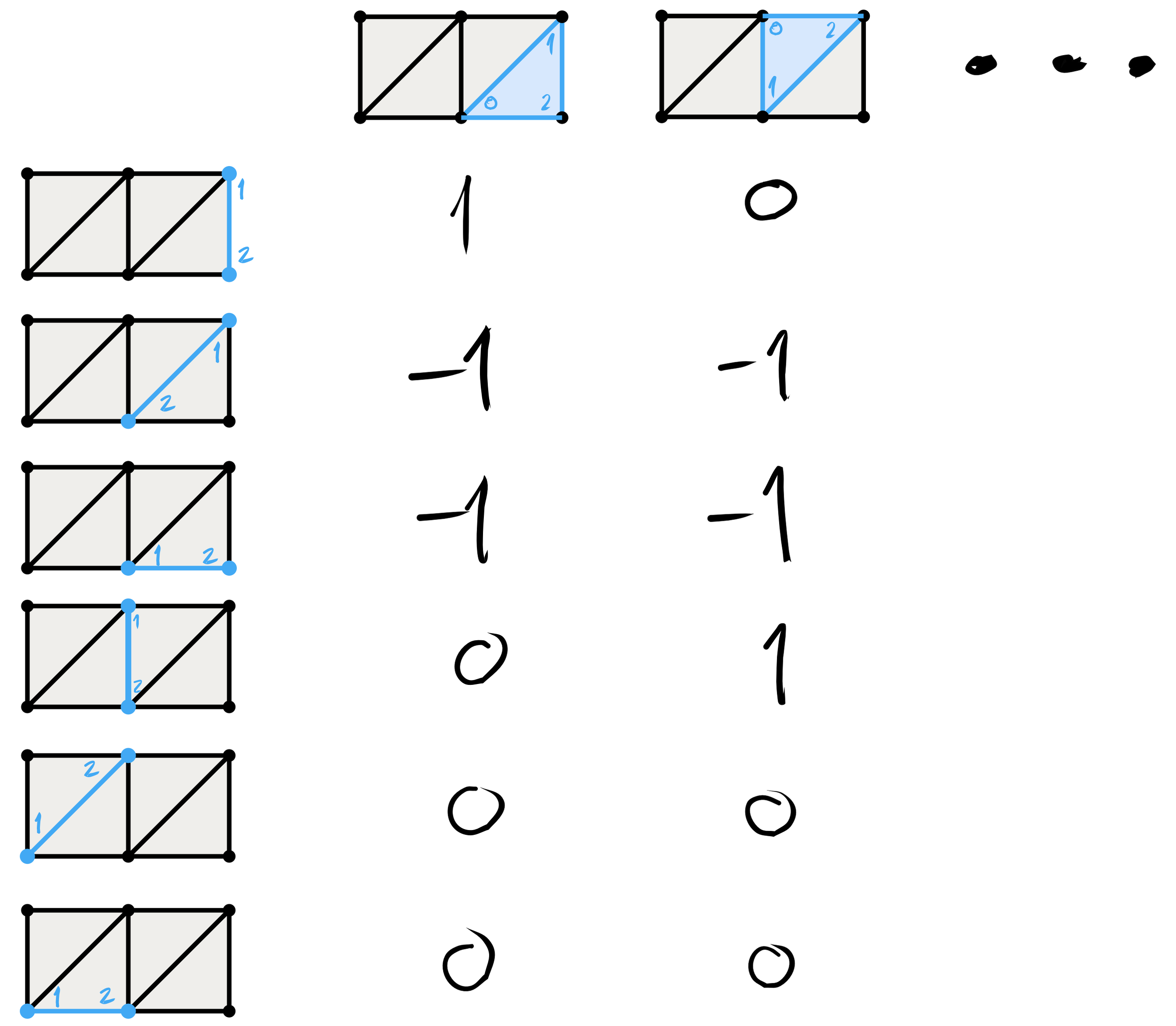




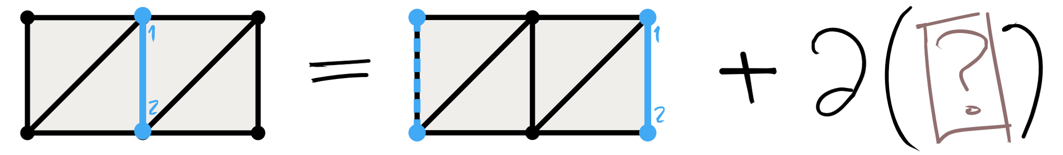


To find a solution more systematically, write the boundary operator as a matrix.

$M_\partial =$



Then the equation we want to solve becomes an integer matrix equation, which we can solve using Gaussian elimination.



$$\begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix} + M_\partial \begin{bmatrix} ? \end{bmatrix}$$

$$\begin{bmatrix}-1\\0\\0\\1\\0\\0\end{bmatrix} = \begin{bmatrix}1 & 0 & \cdots \\ -1 & -1 & \cdots \\ -1 & -1 & \cdots \\ 0 & 1 & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots\end{bmatrix} \begin{bmatrix}-1\\1\\0\\0\end{bmatrix}$$

In this example, the 1-chains have 6 degrees of freedom, and the 1-cycles turn out to have 5. The substitution deformations can move you along 4. Does the last degree of freedom have a geometric interpretation? May be related to the "Dehn twist" operations shown below.