Forum

Possible Trends in Mathematics in the Coming Decades

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Here are a few brief remarks on possible trends in mathematics for the coming decades.

1. Classical mathematics is a quest for structural harmony. It began with the realization by ancient Greek geometers that our 3-dimensional continuum possessed a remarkable (rotational and translational) symmetry (groups $O(3)$ and $\mathbb{R}^3$) which permeates the essential properties of the physical world. (We stay intellectually blind to this symmetry no matter how often we encounter and use it in everyday life while generating or experiencing mechanical motion, e.g., walking. This is partly due to noncommutability of $O(3)$, which is hard to grasp.) Then deeper (noncommutative) symmetries were discovered: Lorentz and Poincaré in relativity, gauge groups for elementary particles, Galois symmetry in the algebraic geometry and number theory, etc. And similar mathematics appears once again on a less fundamental level, e.g., in crystals and quasicrystals, in self-similarity for fractals, dynamical systems and statistical mechanics, in monodromies for differential equations, etc.

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The search for symmetries and regularities in the structure of the world will stay at the core of the pure mathematics (and physics). Occasionally (and often unexpectedly) some symmetric patterns discovered by mathematicians will have practical as well as theoretical applications. We saw this happening many times in the past; for example, integral geometry lies at the base of the x-ray tomography (CAT scan), the arithmetic over prime numbers leads to generation of perfect codes, and infinite-dimensional representations of groups suggests a design of large economically efficient networks of a high connectivity.

2. As the body of mathematics grew, it became itself subject to a logical and mathematical analysis. This has led to the creation of mathematical logic and then of the theoretical computer science. The latter is now coming of age. It absorbs ideas from the classical mathematics and benefits from the technological progress in the computer hardware which leads to a practical implementation of theoretically devised algorithms. (Fast Fourier transform and fast multipole algorithm are striking examples of the impact of pure mathematics on numerical methods used every day by engineers.) And the logical computational ideas interact with other fields, such as the quantum computer project, DNA-based molecular design, pattern formation in biology, the dynamics of the brain, etc. One expects that in several decades computer science will develop ideas on even deeper mathematical levels which will be followed by radical progress in the industrial application of computers, e.g., a (long overdue) breakthrough in artificial intelligence and robotics.

3. There is a wide class of problems, typically coming from experimental science (biology, chemistry, geophysics, medical science, etc.), where one has to deal with huge amounts of loosely structured data. Traditional mathematics, probability theory, and mathematical statistics work pretty well when the structure in question is essentially absent. (Paradoxically, the lack of structural organization and of correlations on the local level lead to a high

degree of overall symmetry. Thus the Gauss law emerges in the sums of random variables.) But often we have to encounter structured data where classical probability does not apply. For example, mineralogical formations or microscopic images of living tissues harbor (unknown) correlations which have to be taken into account. (What we ordinarily “see” is not the “true image” but the result of the scattering of some wave: light, x-ray, ultrasound, seismic wave, etc.) More theoretical examples appear in percolation theory, in self-avoiding random walk (modelling long molecular chains in solvents), etc. Such problems, stretching between clean symmetry and pure chaos, await the emergence of a new brand of mathematics. To make progress one needs radical theoretical ideas, as well as new ways of doing mathematics with computers and closer collaboration with scientists in order to match mathematical theories with available experimental data. (The wavelet analysis of signals and images, context-dependent inverse scattering techniques, geometric scale analysis, and x-ray diffraction analysis of large molecules in crystallized form indicate certain possibilities.)

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Both the theoretical and industrial impacts of this development will be enormous. For example, an efficient inverse scattering algorithm would revolutionize medical diagnostics, making ultrasonic devices at least as efficient as current x-ray analysis.

4. As the power of computers approaches the theoretical limit and as we turn to more realistic (and thus more complicated) problems, we face the “curse of dimension”, which stands in the way of successful implementations of numerics in science and engineering. Here one needs a much higher level of mathematical sophistication in computer architecture as well as in computer programming, along with the ideas indicated above in (2) and (3). Successes here may provide theoretical means for performing computations with high power growing arrays of data.

5. We must do a better job of educating and communicating ideas. The volume, depth, and structural complexity of the present body of mathematics make it imperative to find new approaches for communicating mathematical discoveries from one domain to another and drastically improving the accessibility of mathematical ideas to nonmathematicians. As matters stand now, we mathematicians often have little idea of what is going on in science and engineering, while experimental scientists and engineers are in many cases unaware of opportunities offered by progress in pure mathematics. This dangerous imbalance must be restored by bringing more science to the education of mathematicians and by exposing future scientists and engineers to core mathematics. This will require new curricula and a great effort on the part of mathematicians to bring fundamental mathematical techniques and ideas (especially those developed in the last decades) to a broader audience. We shall need for this the creation of a new breed of mathematical professionals able to mediate between pure mathematics and applied science. The cross-fertilization of ideas is crucial for the health of the science and mathematics.

6. We must strengthen financing of mathematical research. As we use more computer power and tighten collaboration with science and industry, we need more resources to support the dynamic state of mathematics. Even so, we shall need significantly less than other branches of science, so that the ratio of return/investment remains highest for mathematics, especially if we make a significant effort to popularize and apply our ideas. So it is important for us to make society well aware of the full potential of mathematical research and of the crucial role of mathematics in near- and long-term industrial development.