A Few Recollections.

January 6, 2011

After a few frustratingly unsuccessful attempts to write my biography, I have arrived at the inevitable conclusion that this is a *logically impossible* task.

Mind you, there are many counterexamples to this "non-existense conjecture". I enjoyed a lot reading the autobiographies in the first Abel's volume. Yet, I think the conjecture is true in a narrow sense, if you separate "mathematician – a human being" from "mathematician – mathematician".

Our non-mathematical lives are, mathematically speaking, not that interesting, unless somebody had a misfortune to live through interesting times or undergo "interesting" personal experiences.

The life of a mathematician is reflected in the ideas we expound in our papers, what else can we add to this? Is there any *non-trivial* "else" to our lives?

Being trivial is our most dreaded pitfall: you say stupid things, not original things, outrageously wrong things – all will be forgotten when the dust settles down. But if you pompously call $a + b = c$ "Theorem" in your paper, you will be forever remembered as "this $a + b$ guy", no matter you prove bloody good theorems afterwords. (Caution: $2 + 2 =_3 4$ is something quite non-trivial, or at least, not quite trivial.)

I was introduced to the idea on September 1st 1960 at the then Leningrad University when our analysis professor Boris Mikhailovich Makarov said to me after our first calculus class – he expressed this in somewhat metaphorical terms – that I should've kept my mouth shut unless I had something non-trivial to say.

Further encouraged by my teachers and fellow students, I tried to follow his advice and, apparently, have succeeded – I hear nothing disrespectful about my mouth for the last 10-20 years. Strangely, this does not make me feel a lot happier.

"Trivial" is relative. Anything grasped as long as two minutes ago seems trivial to a working mathematician. But it may be amusing, looking from afar, to recall personal eureka transition points. (According to Terry Pratchett's Revised Ancient Greek Dictionary, "eureka" translates as "give me a towel".)

Another concept you learn at some point is that of "unsolved problem". David Ruelle has once put it that he sees a problem when he feels annoyed by non-understanding something. Children, like scientists, are good at non-understanding, except that the annoyed ones are their parents bombarded with endless What and Why and When And How and Where and Who.

As your adult personality properly matures without being sidetracked by your scientific or artistic inclinations, you resolve these WWW problems with a single: "This is the stupidest question I have ever heard" – said to a child. (Lipman Bers once boasted to me that he had received this response when he had asked his high school mathematics teacher if there could be two different infinities.)

My parents, were medical doctors rather than mathematicians, actually pathologists; they often discussed, the problems they were encountering during autopsies, with their friends – also pathologists.

One story, I recall, was very funny, at least everybody laughed. My father spent several hours carefully checking and rechecking everything *inside* a body on the dissecting table but was unable to find the cause of death. When he was ready to surrender and shamefully write it off to "the heart failure", the man responsible for moving and cleaning the bodies, said: "Hey, doctor, isn't it funny, the man did not wash his left foot, look at the black marks over there". At a glance my father realized that the course of death was electrocution, apparently, the poor stepped on a high-vlotage wire.

A few comments are in order. By the book, one starts an autopsy with a careful *external* examination of the body before performing dissection. My father, experienced as he was, probably, was absent-minded at the moment: neglecting external inspection strikes as funny to a pathologist as $dx/dy = x/y$ to a mathematician. (Maynard Smith – a great theoretical biologist, complained that editors of biology journals had sometimes "simplified" $dx/dy \to x/y$ in his papers.)

Autopsies have been routinely performed in Russian hospitals. Treating physicians were in a constant dread of the final word by a pathologist like students waiting for results of an examination. Eventually, physicians revolted – the autopsy in most countries if performed, then only rarely and usually on decade-old exhumed corpses – deaths of patients can be safely attributed to "heart failures".

There is an obvious moral to this story for pathologists and mathematicians alike. But the "stupid question" might have escaped you: "What, How and Why is the heart failure when you die?"

It is *not* that the heart just stops – this is what the so called "common sense" would tell you. Actually, stopping and "resetting" the heart is what defibrillators are for — you see them at the airports – they *save* lives, if used promptly – the brain survives a couple of minutes after the oxygen delivery by the blood stops.

What happens to the heart at the critical transition moment before it goes to the finale rest is a change in the dynamics of electric/chemical currents in the heart muscular tissue - a switch to a non-quasiperiodic "chaotic" regime. (A high external voltage can provoke this, but it may also "disperse the chaos".)

Isn't it a "New application of the chaos theory to living systems!?" a bright mathematician may exclaim. Indeed, this is not a bad idea. I bet, there are several articles in Nature with this title. The catch is that biological chaotic systems do not live long, their life spans are even shorter than the half-lifes of such articles – there is no accepted theory of arrhythmia in general and of the ventricular fibrillation (this is what we all will end up with) in particular. The physiology of the heart and mathematics at the bottom of it are not that trivial. And, probably, the true "stupid child question" has not been asked yet.

Biology in general and medicine in particular are full of annoying nearly mathematical puzzles. At 5 you ask:

Can four elephants beat a whale in a fight?

Twenty years later you come up with:

How, in principle, a humble bacterium, a tiny virus, e.g. HIV whose all "knowledge of the world" is written down in four letters on a 9749-long string of RNA, outsmart all of humanity with terabytes (10^{12}) of "information" stored in our individual synaptic memories and as much in our libraries?

What is the virus knows we don't? How many bits have we to add to (to erase from?) our knowledge banks to beat 9749?

My second story needs a little preamble. There are several innocuous reactions turning water-like solutions into red ones looking like blood.

Something more amusing you get of a mixture of potassium permanganate with concentrated sulfuric acid,

$$6KMnO_4 + 9H_2SO_4 \to 6MnSO_4 + 3K_2SO_4 + 9H_2O + 5O_3.$$

The O_3 (ozone) vapor will ignite paper soaked with alcohol; with some luck, an explosion throws sulfuric acid into your eyes.

According to the basic chemistry safety rule, you *first* produce artificial blood, place a large bowl B with it in front of you and *only then* proceed with mixing $KMnO_4$ and H_2SO_4 in a test-tube T making sure that B is *strictly on the line* between T and your eyes.

When T explodes, the bowl in between protects your eyes from the sulfuric acid, while the bloody contents of B picturesquely splash all over your face.

This happened to me at a demonstration of "miracles of chemistry" at our high school when I was about 13. The audience was duly impressed, especially our chemistry professor. But myself, I missed the best of the show as I could not see my face all in "blood" with no mirror near at hand.

I had no idea, of course, why the damn thing had exploded (some readers might have already guessed what was wrong in the above protocol), but afterwards, our chemistry teacher – Ivan Ivanovich Taranenko said to me that it was he who had make the mistake: accidentally, when I started, I was about to mix $KMnO_4$ and H_2SO_4 in a flat dish, but Ivan Ivanovich suggested to use a test-tube instead. The heat escape as well as the escape of the gaseous product were limited in the relatively narrow test-tube and the explosion followed.

At the time, I was not much impressed by my teacher's honesty, I assumed this was an ordinary human behaviour.

Then I found out how psychologically difficult it was to emulate even a minor version of this, e.g. by *properly* acknowledging an influence of somebody's remark on your own theorem. For example, writing my early paper on Banach conjecture, I convincingly persuaded myself, that the advice by Dima Fuks to look at the homotopy groups of the classical groups for evaluating dimensions of their k-classifying spaces, was too trivial to deserve being mentioned.

I am afraid, I accumulated a score of such "unmentionable" remarks and many of may colleagues told me of similar painful fights with themselves they have had while resolving the "acknowledgement problem". But others could not see there any problem at all. Probably, honesty comes naturally to certain people and some see no difficulty because they have never tried to be honest.

When I lived in Russia, the main output of the Soviet radio transmitters was the white (it always felt grayish to me) noise. (2-7 years in prison was an alternative to the official point of view that no such thing as "white noise" existed. But undaunted Soviet admirers in the West admitted its existence and suggested plausible explanations for it, where the most convincing one was preventing flying sauces from landing on Soviet agricultural fields with little green men hungry for the tasty green crops.)

This "white noise" did not cover the FM (40-50 Mhz) and television (around 70 Mhz) frequencies being unneeded for an obvious reason. But one evening TV-jamming began. People in the apartment house where we lived were opening doors and worriedly looking at each other. They did not dare to ask aloud what they thought was happening but "yes, it is" was transparent in everybody's eyes.

Of course, there were no secrets in the family and my mother hurried to tell me the news. I was triumphant: the first (and the last) time in my life something made by my hands worked! This "something" - a small radio transmitter I assembled – was supposed to generate 42 Mhz. But who cares for 40% error, the very fact it functioned made me bubble with pride.

My involvement into make-it-yourself-radio-something was influenced by my close friend, Lev Slutsman, with whom we went through the high school and the math department at the University together. The mathematics of the electricity laws was for him something real, something he felt with his fingers, devices made by him worked. His was a quite special and rare facet of mathematical gift – mathematics in the bones as much as in the head. (Lev now works in US and authors a multitude of patents on algorithms for testing large communication networks and something else of this kind.)

There was another boy, Dima Smirnov, in our high school class with a similar, albeit not with apparently mathematically colored, ability. Dima was the worst, the laziest student in the class, he hardly managed to graduate.

Once, we were supposed to do something at home and to bring it to the class. Many boys, myself included, brought up models of gliders, which we had assembled from standard pieces bought in a store along with an instruction of how to make it.

The teachers evaluated our creations according to how pretty they looked. Mine was the second dirtiest, Dima's was four times as dirty and fully asymmetrical. Obviously, he was too lazy even to read the instruction. His was the only glider that glided.

Neither the teachers nor the fellow students were impressed by Dima's glider. We felt embarrassed. It looked unjust, incongruous, completely absurd, that this ugly thing soaked with oil and covered by smudges of dirty glue could so gracefully glide a dozen of meters in the air, while all beautifully assembled clean ones were heading straight down to the ground despite all efforts to propel them horizontally. (After graduation, Dima entered the physics department at the University and became a very successful experimental physicist.)

I met later on two experimental physicists in US and in France (whose names I forgot since it was not so long ago). One of them was working on quantum computers and the second one was making nano-devices, if I recall correctly, with the atomic force microscope – a device for "touching atoms" rather than for "looking" at them. All mathematics of quantum mechanics, at least all I have ever heard of, including representation of C^*-algebras, for instance, was at their fingertips.

What level of mathematics do you need to sustain in a scientific community that so much would percolate to somebody's fingers!?

Learning and understanding mathematics is difficult, both by reading articles and/or by talking to people. (Actually, not so much by talking but by listening – "You can not learn much with your mouth open" – Dennis Sullivan used to say to me.)

Rarely, something you read will inspire you right on the spot, but I remember an exception – Tony Phillips' 1966-paper in Topology on the existence of submersions.

We studied earlier, at the students' seminar run by our professor Vladimir Abramovich Rokhlin, the immersion theory of Smale and Hirsch. I thought I had a fair idea of what was going on.

The fact that submersions, something quite opposite to immersions, had, however, followed in steps of Smale-Hirsch was a revelation to me. It took me about a year to understand what was on the bottom of this similarity.

Something else written by Tony, a private letter to me, also kept me puzzled for quite awhile. This letter contained a couple of pages of incomprehensible mathematics, starting with something like:

... an involutive gromomorphism $G : SU \to US$ of admissible type... T transforms $MG \to SB$...

I could not understand a single sentence in it. But when I showed this to my friend, an analyst Volodia Eidlin, he asked me: "What is a gromomorphism?"

"You mean homomorphism" – I replied – "There is no such thing as gromomorphism". ("Homomorphism" is spelled and pronounced as "gomomorphism" in Russian.)

"Do you ever read anything as it is written?" – he was annoyed – "This is "gromomorphism", black on white."

"Must be a misspel..." – I mumbled, but then it dawned on me. Tony's was an encoded message. He was suggesting I would immigrate from the Soviet Union to US and invited me to SUNY at Stony Brook where he worked. (We met with Tony when he visited to Russia a year earlier. His visit was brief, but long enough to learn the basic conspiracy survival rules in Soviet Russia.)

Several years later I followed his suggestion. When I arrived at Stony Brook, I enjoyed Tony's hospitality as well as that of the whole mathematics department at SUNY.

The only problem I had with people was "culture shock". Everybody was very kind to me and offering their help in overcoming this mysterious "culture shock". As I could not figure out what this shock was all about and not wanting to disappoint anybody I had to invent a few shocking stories about how I missed white bears skating on the streets of Leningrad in the darkness of polar nights and a cosy family iceberg in our seller where we kept perishable foods.

When you read a book or an article you may come across something that the author had no idea of putting in there. When you listen to a mathematician you often learn something he/she might expect you knew beforehand, something obvious from his/her perspective, something one would not dare to put on paper.

One of such "obvious" things I learned from Dima Kazhdan who remarked to me at a visit from Moscow, that Kurosh subgroup freedom theorem follows from the fact the covering of a graph is again a graph: dimension is invariant under coverings.

Until that moment the group theory was to me a slippery formalism impossible to hold steady in my hand. But with this remark everything started slowly falling into place; very slowly – it took me about 20 years afterwords to express some other fragments of the group theory in the geometric language.

I am certain there are lots of "omitted in view of their triviality" remarks nobody ever said to me, something basic and simple I've never understood.

This equally applies to non-mathematics, you can not learn everything from the books. Only rare authors – I recall seeing this in writings by Richard Feynman (QED), Charles Cantor (The Science and Technology Behind the Human Genome Project) and Maxim Frank-Kamenetskii (Unraveling DNA) – have the clarity of mind as well as the courage to pinpoint something essential that is obvious to the initiated and with no way to guess by an outsider.

A particular frustrating instance of this happened to me with learning the French language, rather than math or science. Armed with several textbooks, tapes, etc, I dutifully followed the phonetic rules and trained myself to read aloud *ent*'s at the ends of the verbs as in *ils parl**ent***. Much later, after I've lived for ten years in Paris and have already acquired a full automatism speaking "French", I came across a textbook published in 1972 in Quebec where the author – Gilbert Taggart explained, along with lots of other things which were too late for me to learn, that this *ent* was not meant for reading.

I kept asking myself why this was kept secret in most other textbooks and eventually realized how stupid the question was: *everybody* knew this, no single person apart from myself have ever uttered "ils parl**ent**" no matter how much I tried to find one in Paris (Wouldn't it be different in Quebec?)

As I mentioned, what you learn from a mathematics paper may, especially after some time, diverges from what the author had in mind. But something opposite – kind of convergence may also occur. This once happened to me... with a help by a burglar.

When I started studying Nash's 1956 and 1966 papers (it was at Rokhlin's seminar ≈1968), his proof has stricken me as convincing as lifting oneself by the hair. Under a pressure by Rokhlin, I plodded on, and, eventually, got the gist of it: It was a seemingly circular "fixed point by iteration" argument, where the iterated maps were *forced* to contraction by adjusting the norms in the spaces involved at each step of the iteration process. The final result poped up at the end of a lengthy but a straightforward computation, which, miraculously, did lift you in the air by the hair.

I wrote an abstract version of Nash's theorem in a 1972 paper where I isolated the iteration process in the space of norms and where a part of Nash's argument was absorbed by definitions.

But when I tried to reproduce this in my book on partial differential relations, I found out that the price for the "correct formalization" was non-readability – I had to write everything anew.

It was a hard job, I was relieved when it was over and I gave the manuscript to our typist at SUNY, it was about 1979, when I still was at Stony Brook.

Next week, the secretary office was burglarized and my manuscript disappeared along with a couple of typewriters. I had to write everything again for the third time.

Trying to reconstruct the proof and being unable to do this, I found out that my "formalization by definitions" was incomplete and my argument, as stated in

1972 was invalid (for non-compact manifolds). When I simplified everything up and wrote down the proof with a meticulous care, I realized that it was almost line for line the same as in the 1956 paper by Nash - his reasoning turned out to be a stable fixed point in the "space of ideas"! (I was neither the first nor the last to generalize/simplify/improve Nash, but his proof remains unrivaled.)

What are our ideas – "From creation to decay; Like the bubbles on a river; Sparkling, bursting, borne away" (Shelley).

Is mathematics invented or discovered?

Even if we had ever learned the answers we would be as dissatisfied as an ancient geographer if his straightforward question: – "Does the Earth rest on a whale or on the backs of four elephants?"– were befuddled by:

"Nothing exists except atoms in the void; everything else is opinion". (Leucippus? Democritus? Lucretius?)

We, mathematicians, are an equally long way from asking the right questions.