Great Circle of Mysteries: Mathematics, the World, the Mind.

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The universe is built on a plan the profound symmetry of which is somehow present in the inner structure of our intellect.
PAUL VALERY.

Decoding the Mind is impossible without creating a broad (semi)mathematical context allowing one to consistently speak of *mind-like structures*.

But

what kind of mathematics we need to speak about the Mind?

Should we stick to *mathematics of numbers* – the language physicists speak about their World?

Some think that no radical departure from physics is needed. Frances Crick,[1] for instance, believed that much of the mind may be understood in terms of the physiology of the brain:

a person's mental activities are entirely due to the behavior of nerve cells, glial cells, and the atoms, ions, and molecules that make them up and influence them.[2]

Nobody argues, the sole source of your thoughts *is* your brain – this idea has been around for more than 4000 years.[3] But no matter how much you adorn this idea with persuasive words, everything you say about the arrow

$$[\text{BRAIN}] \rightsquigarrow [\text{MIND}]$$

remains metaphoric. No sentence of a kind: *"The mind is caused/produced/generated/constructed* or *determined/controlled/run by the brain"* sheds any light on the nature of this arrow. We can not but accept[4] what Charles Sherrington grudgingly acknowledged nearly a century ago:[5]

As followers of natural science we know nothing of any relation between thoughts and the brain, except as a gross correlation in time and space.

[1] Francis Harry Compton Crick (1916 – 2004), who greatly contributed to the molecular biology, was educated as a physicist.

[2] This is written in Crick's 1994 book *The Astonishing Hypothesis* where he promotes what he believes is a scientific approach to the problem of consciousness.

[3] The oldest document is *Edwin Smith Surgical Papyrus* , dated ≈1500 BCE, that was an incomplete copy of a text from the Old Kingdom of Egypt (circa 2686 - 2181 B.C.E.).

[4] Crick would disagree.

[5] This is written in Sherrington's book *Man on his Nature* based on his Gifford Lectures (1937-1938) where the scientist expounds his philosophical ideas on on man's place in the universe from the point of view of the natural sciences.

But what is wrong with this? What are other relations that you want?

Your own brain, or rather, what we call *ergo-brain*, reconstructs the whole world in all its grandeur from space/time correlations between different events.[6] With a little mathematics, we may try something similar as follows.

Different types of brain injury produce different psychological impairments,[7] and experimental neurophysiology (ideally) delivers a correspondence between the states of mind and collections of the neurons in the brain that are active in the presence of such states. Since the anatomy of the brain is, roughly, the same for all people, this allows an objective comparison of similar collections in different individuals.

For instance, if experiencing a particular color, such as ☻, were universally identifiable by records of excited neurons in the brains of a representative[8] group of people (animals), one would be justified in attributing the "predicate of *existence*" to *the quale* of this color.

More interestingly, the natural combinatorial distance, called *Hamming metric*, between different collectons of neurons in the brain[9] gives us a way to measure distances between states of mind.

If such a distance/metric were a reality, psychology would be equated with "*geometry of the mind(s)*"; and albeit no such metric is available with the current state of knowledge, the very idea of such a distance suggests a possibility of mathematical approach to the study of the mind.

But, on the other hand, it seems there *cannot be* any "mathematics of the Mind": no matter how much you try you can not discern anything "mathematical" in what you consciously perceive as "my Mind": it is too loosely organised with no structurally significant patterns visible in it.

Well..., if you watch soap operas on the screen of a laptop you do not see much structure in the operating system of the computor either. You have to look somewhere else.

> *It is in the admission of ignorance and the admission of uncertainty that there is a hope for the continuous motion... in some direction... .*
>
> Richard Feinmann.

Ergo-Brain Conjecture. There exists a certain elaborate mental entity, we call it *ergo-brain*, that mediate between the electrophysiological dynamics of the brain and the thought processes in the conscious mind.

Ergo-brain is responsible for *deep learning* by humans, in particular for learning mother tongues by children and mathematics by future mathematicians.

Little of the ergo-brain is accessible to introspection. Yet, "ergo-patterns" are recognisable in natural languages and within mathematics.

Ergo-Structures/Ergo-Systems Conjecture . There are particular mathematical, essentially combinatorial, structures, call them *ergo structures*, and a class (mathematical category?) of mathematics objects, called *ergo-systems* that carry within themselves such structures. Ergo-brain is a particular instance of an ergo-system.

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[6] Space-time itself, as it is represented by the ergo-brain, results from such correlations.

[7] This had been already recorded in Surgical Papyrus.

[8] Very likely, the so-defined quale of WHITE for the Inuit in the arctic regions of Greenland would correspond to GREEN for the Pirahã people of Amazonia.

[9] This distance is defined as the number of neurons that belong *only to one* of the two collections.

Our ultimate goal is developing the theory of ergo structures that would bring *mathematical* means for *analysing and synthesising universal learning systems.*[10]

We imagine such a system $\mathcal{LERNER}$ that interacts with *an incoming flow of signals* similarly to a photosynthesizing plant growing in a stream of photons of light or to an amoeba navigating in a see of chemical nutrients and/or of smaller microbes: $\mathcal{LERNER}$ recognizes and selects what is *interesting* for itself in such a flow and uses it for *building* its own structure.

This analogy is not fully far fetched. There is no *significant* difference between human activities and those by amoebas and even by bacteria, well,... on the GRAND SCALE. Say, the probability of finding first million digits of the number $\pi = 3,14159265359....$ "written" at some location of an imaginable UNIVERSE increases by more than a billion-by-billion-by-billion factor if you find a bacterium kind machine feeding on a source of almost amorphous *free energy* at a point within many thousand light years from this location.

Ergo-logic: this is a particular way of thinking that is needed to approach our conjectures.

Ergo-logic sharply contrasts with everything we take for granted about what we are and what happens in our heads. We reject such ideas as

intelligent - rational - intuitive - important,

as far as ergo-brain and ergo-learning are concerned, and replace them by

interesting - curious - funny - informative.

Albeit counterintuitive, the manifestations of this logic are seen in the depth of mathematics and also in *molecular structures* of the live systems uncovered by biologists in the last 50 years.

Also the idea of ergo-brain comes by assessing *limitations of natural selection* in emergence and evolution of human cognition.

The structural patterns we find in the ergo-brain, although being of evolutionary origin, can not be accounted for by the naked survival/selection mechanism, but rather by inevitable constrains on possible ergo-system's architectures. These are, essentially, mathematical constrains, and, seemingly paradoxically, they make ergo-brain more likely to be evolutionary accessible than the "amorphous" human Mind.

And inspired by *the history* of the evolution theory, where the insight by Darwin and Wallace was *not so much in biology per se but in realisation of potentiality of the exponential function*,[11] we search the key to the mystery of the Mind *in mathematics rather than in neurophysiology*:

What stands on the way for relating the world of thoughts in your mind to that of neurones in your brain is disparity of the two structures and incompatibility of the languages describing these structures;[12] amassing data on the

[10] Such a theory may also elucidate the nature of mathematics.

[11] The enormity of the exponential growth of unrestricted populations was obvious to mathematically minded people from the time of antiquity. But this might have been a revelation to the biologists of the 19th century who were not well versed in math – Darwin himself, who has a fine intuitive feeling for large numbers, was unable to correctly evaluate the number of descendents of a coupe of elephants after 500 years.

[12] This is reminiscent of the *collapse of quantum states arrow* that stands for (still unavailable) translation of the "quantum language" to the classical one.

Brain will be of little help.[13] The arrow [BRAIN] $\rightsquigarrow$ [MIND] is as defiant of all attempts to contain it in chains of clever words as its younger sister, the arrow

[MATTER/ENERGY] $\rightsquigarrow$ [LIFE].

One *cannot assert* (as Crick would undoubtedly do) that all life processes are *entirely due* to the interactions between atoms, ions, and molecules that make biological entities.

Of course, physicists disagree: Richard Feynman says in *Six Easy Pieces* of his famous *Lectures on Physics* that

... there is nothing that living things do that cannot be understood from the point of view that they are made of atoms acting according to the laws of physics.

However, the laws of physics are not suspended in a logical vacuum, they are immersed in a mathematical framework. Physics practiced by humans is a *"network of ideas"*[14] within this framework where some "nods" are taken for "laws of physics".

The "spirit of physics" resides in *the combinatorial architecture* of this network that is constrained and directed by many conventions, instructions, assumptions, such as

symmetry, infinitesimal linearity, stability, genericity.

But Life, albeit *constrained* by "physical laws", *excels in violating* "physical conventions and assumptions" – this is what makes Life Life.

Think, for instance, what happens to a 100kg BODY colliding with something tiny, something that weighs less than one billions of a gramm.

Nothing, obviously, but... let BODY be the body of a *predator*. Let your "something" be a few billion molecules that depart from *the scent glands* in the *body* of a potential *pray* and "collide" with the *olfactory epithelium in the nasal cavity* of BODY.

Would you solely rely on the *Law of conservation of momentum* for predicting the time evolution of the distance between BODY and *body* especially if this second *body* happens to be yours?[15]

The idea of "mathematics of the Mind" is not new. "Algebra of thought" was conceived by Leibniz around 1676.

In 1869, William Jevons[16] built a mechanical *Logic Piano*, that, in his words, represented

a mind endowed with powers of thought, but wholly devoid of knowledge.

In 1887, Charles Peirce[17] was asking how much

the business of thinking a machine could possibly be made to perform.

[13] This would be like trying to achieve understanding of *proteins* – of their 3D structures and functions in the cell – by accumulating data on chemosry of *polynuclear acids* – DNA that direct the synthesis of these proteins.

[14] "Idea" may stand for a record of an observation or an experiment as well as a recipe/rule for designing, conducting and interpreting experiments.

[15] Indian leopards (40-80kg) and more rarely tigers (150-300kg) may attack men.

[16] William Stanley Jevons (1835 – 1882) was an economist and logician. His book *A General Mathematical Theory of Political Economy* (1862) was a start of the mathematical method in economics.

[17] Charles Sanders Peirce (1839 – 1914), "the father of pragmatism" and the founder of *semiotic*, was an innovator in mathematical logic philosophy and statistics.

In 1950, this idea was expounded by Alan Turing in the artcle
Computing Machinery and Intelligence where he argues that nothing stands on the way of

BUILDING MACHINES THAT CAN THINK.

But what is *the logical structure* in your mental processes that can be mathematically modelled and implemented on a machine?

The structurally rich *neurophysiology of the brain* is too far removed from what we want to simulate, e.g. the learning process of the mother tongue by a child, while *the flows of your conscious thoughts* are void of interesting structures.

Our suggestion is to switch the focus from *dynamics of the brain* and *logic of thoughts* to

invisible and apparently illogical undercurrents of thoughts

that we collectively call *ergo*.

The core structure of this is determined by *mathematical* necessity of simplicity and universality, while the shape "ergo" takes in the human mind is influenced by the constrains of the neuronal organisation of the brain and by (conjectural) limitations of evolutionary selection.[18]

In *"Memorandum Ergo"* we explain what "ergo" might be and address what we think the correct questions about the Mind are:

Is there enough structural universality in the process of "thinking" to allow a mathematical modeling of this process?

What, conceivably, could serve as MATHEMATICS OF THE ERGO-BRAIN?

[18] Think of the shape of a cucumber grown in a bottle.