Pierre Le Bris

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The model Motivation Finding the limit Usual methods

Some results

Well posedness Large number of particles Proof of the estimate Uniform in time propagation of chaos for the generalized Dyson Brownian motion and 1D Riesz gases.

Pierre Le Bris Joint work with : Arnaud Guillin (LMBP), Pierre Monmarché (LJLL)

LJLL, Sorbonne Université - Paris

Workshop QuAMProcs 09/03/2022

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1D N-particle system in mean field interaction

$$dX_t^i = \sqrt{2\sigma_N} dB_t^i - U'(X_t^i) dt - \frac{1}{N} \sum_{j \neq i} V'(X_t^i - X_t^j) dt,$$

where

- *σ_N* diffusion coefficient,
- (Bⁱ)_i independent Brownian motions,
- *U* confining potential such that either *U'* is Lipschitz continuous or $U'(x) = \lambda x$,

•
$$\exists \alpha \geq 0, \ \forall x \in \mathbb{R}^*, \ V'(x) = -\frac{x}{|x|^{\alpha+1}}.$$

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The (generalized) Dyson Brownian motion

$$dX_t^i = \sqrt{\frac{2\sigma}{N}} dB_t^i - \lambda X_t^i dt + \frac{1}{N} \sum_{j \neq i} \frac{1}{X_t^i - X_t^j} dt.$$

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Question : What happens when $N \to \infty$?

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$$dX_t^i = \sqrt{2\sigma_N} dB_t^i - U'(X_t^i) dt - \frac{1}{N} \sum_{j \neq i} V'(X_t^i - X_t^j) dt.$$

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$$dX_t^i = \sqrt{2\sigma_N} dB_t^i - U'(X_t^i) dt - \frac{1}{N} \sum_{j \neq i} V'(X_t^i - X_t^j) dt.$$

Formally, notice $\frac{1}{N} \sum_{j=1}^{N} V'(X_t^i - X_t^j) = V' * \mu_t^N(X_t^j)$, where $\mu_t^N := \frac{1}{N} \sum_{i=1}^{N} \delta_{X_t^i}$.

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$$dX_t^i = \sqrt{2\sigma_N} dB_t^i - U'(X_t^i) dt - \frac{1}{N} \sum_{j \neq i} V'(X_t^i - X_t^j) dt.$$

Formally, notice $\frac{1}{N} \sum_{j=1}^{N} V'(X_t^j - X_t^j) = V' * \mu_t^N(X_t^j)$, where $\mu_t^N := \frac{1}{N} \sum_{i=1}^{N} \delta_{X_t^i}$. Assuming $\sigma_N \to \sigma$,

$$\begin{cases} dX_t = \sqrt{2\sigma} dB_t - U'(X_t) dt - V' * \bar{\rho}_t(X_t) dt, \\ \bar{\rho}_t = \mathsf{Law}(X_t), \end{cases}$$

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$$dX_t^i = \sqrt{2\sigma_N} dB_t^i - U'(X_t^i) dt - \frac{1}{N} \sum_{j \neq i} V'(X_t^i - X_t^j) dt.$$

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$$\begin{cases} dX_t = \sqrt{2\sigma} dB_t - U'(X_t) dt - V' * \bar{\rho}_t(X_t) dt, \\ \bar{\rho}_t = \mathsf{Law}(X_t), \end{cases}$$

which is linked to

$$\partial_t \bar{\rho}_t = \partial_x \left(\left(U' + V' * \bar{\rho}_t \right) \bar{\rho}_t \right) + \sigma \partial_{xx}^2 \bar{\rho}_t.$$

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Goal : Show $\mu_t^N \to \bar{\rho}_t$.

Some methods :

• Coupling methods (McKean, Sznitman, Eberle ...) :

$$\mathcal{W}_{2}(\mu,\nu)^{2} = \inf_{X \sim \mu, Y \sim \nu} \mathbb{E}\left(\left|X - Y\right|^{2}\right).$$

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$$\mathcal{W}_{2}(\mu,\nu)^{2} = \inf_{X \sim \mu, Y \sim \nu} \mathbb{E}\left(|X - Y|^{2}\right).$$

Let
$$\rho_t^N = Law(X_t^1, ..., X_t^N)$$
, show $\mathcal{W}_2\left(\rho_t^N, \bar{\rho}_t^{\otimes N}\right) \to 0$.

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• Energy/Entropy estimates (Serfaty, Jabin, Wang ...).

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• Coupling methods (McKean, Sznitman, Eberle...) :

$$\mathcal{W}_{2}\left(\mu,\nu
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Let $\rho_t^N = Law(X_t^1, ..., X_t^N)$, show $\mathcal{W}_2\left(\rho_t^N, \bar{\rho}_t^{\otimes N}\right) \to 0$.

- Energy/Entropy estimates (Serfaty, Jabin, Wang ...).
- Tightness (Rogers, Zhi, Cépa, Lépingle...).

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Existence, uniqueness, no collisions

Theorem

Consider $N \ge 2$, and $-\infty < x_1 < ... < x_N < \infty$.

• If $\alpha > 1$, for any $\sigma_N \ge 0$, there exists a unique strong solution $X = (X^1, ..., X^N)$ to the particle system with initial condition $X_0^1 = x_1$, ..., $X_0^N = x_N$, which furthermore satisfies $X_t^1 < ... < X_t^N$ for all $t \ge 0$, \mathbb{P} -a.s.

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• The same result holds for $\alpha = 1$ and $\sigma_N \leq \frac{1}{N}$.

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"Cauchy sequence"

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Lemma

Let $(\mu^N)_{N \in \mathbb{N}}$ be any sequence of independent empirical measures, such that μ_t^N is the empirical measure of the N particle system at time t. Then (for $\lambda > 0$, $\alpha = 1$, $U'(x) = \lambda x$ and $\sigma_N = \frac{1}{N}$), we have for all $t \ge 0$ and all $N, M \ge 1$

$$\mathbb{E}\left(\mathcal{W}_{2}\left(\mu_{t}^{\mathsf{N}},\mu_{t}^{\mathsf{M}}\right)^{2}\right) \leq e^{-2\lambda t}\mathbb{E}\left(\mathcal{W}_{2}\left(\mu_{0}^{\mathsf{N}},\mu_{0}^{\mathsf{M}}\right)^{2}\right) + \frac{C}{N \wedge M}$$

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Remark : The same result holds...

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Remark : The same result holds...

• for U = 0, but no longer uniform in time,

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Remark : The same result holds ...

- for U = 0, but no longer uniform in time,
- for $\alpha \in [1, 2[$, with rate $N^{-\frac{2-\alpha}{\alpha}}$,

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$$\mathbb{E}\left(\mathcal{W}_{2}\left(\mu_{t}^{\mathsf{N}},\mu_{t}^{\mathsf{M}}\right)^{2}\right) \leq \boldsymbol{e}^{-2\lambda t}\mathbb{E}\left(\mathcal{W}_{2}\left(\mu_{0}^{\mathsf{N}},\mu_{0}^{\mathsf{M}}\right)^{2}\right) + \frac{C}{N \wedge M}$$

Remark : The same result holds ...

- for U = 0, but no longer uniform in time,
- for $\alpha \in [1, 2[$, with rate $N^{-\frac{2-\alpha}{\alpha}}$,
- for U' only Lipschitz continuous, but no longer uniform in time,

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Lemma

Let $(\mu^N)_{N \in \mathbb{N}}$ be any sequence of independent empirical measures, such that μ_t^N is the empirical measure of the N particle system at time t. Then (for $\lambda > 0$, $\alpha = 1$, $U'(x) = \lambda x$ and $\sigma_N = \frac{1}{N}$), we have for all $t \ge 0$ and all $N, M \ge 1$

$$\mathbb{E}\left(\mathcal{W}_{2}\left(\mu_{t}^{\mathsf{N}},\mu_{t}^{\mathsf{M}}\right)^{2}\right) \leq \boldsymbol{e}^{-2\lambda t}\mathbb{E}\left(\mathcal{W}_{2}\left(\mu_{0}^{\mathsf{N}},\mu_{0}^{\mathsf{M}}\right)^{2}\right) + \frac{C}{\boldsymbol{N}\wedge\boldsymbol{M}}.$$

Remark : The same result holds ...

- for U = 0, but no longer uniform in time,
- for $\alpha \in [1, 2[$, with rate $N^{-\frac{2-\alpha}{\alpha}}$,
- for U' only Lipschitz continuous, but no longer uniform in time,
- for the supremum, but no longer uniform in time.

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Conclusion

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Using independence, this implies that there exists a (deterministic) $\bar{\rho}_t$ such that

$$\mathbb{E}\left(\mathcal{W}_{2}(\mu_{t}^{N},\bar{\rho}_{t})^{2}\right)\rightarrow0.$$

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Proof of the estimate

Proof of the estimate

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For two sets of points $(x_i)_{i \in \{1,...,N\}}$ and $(y_j)_{j \in \{1,...,N\}}$, with $x_1 \le ... \le x_N$ and $y_1 \le ... \le y_N$, and two measures $\mu = \frac{1}{N} \sum_i \delta_{x_i}$ and $\nu = \frac{1}{N} \sum_i \delta_{y_j}$:

$$W_2(\mu,\nu)^2 = \frac{1}{N}\sum_i |x_i - y_i|^2.$$

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For two sets of points $(x_i)_{i \in \{1,...,N\}}$ and $(y_j)_{j \in \{1,...,N\}}$, with $x_1 \le ... \le x_N$ and $y_1 \le ... \le y_N$, and two measures $\mu = \frac{1}{N} \sum_i \delta_{x_i}$ and $\nu = \frac{1}{N} \sum_j \delta_{y_j}$:

$$\mathcal{W}_2(\mu,\nu)^2 = \frac{1}{N}\sum_i |x_i - y_i|^2.$$

Let

$$-\infty < X_t^1 = \dots = X_t^N < \dots < X_t^{N(M-1)+1} = \dots = X_t^{NM} < \infty$$

$$-\infty < Y_t^1 = \dots = Y_t^M < \dots < Y_t^{M(N-1)+1} = \dots = Y_t^{NM} < \infty .$$

Thus

$$\mu_t^M = \frac{1}{M} \sum_{i=1}^M \delta_{\tilde{X}_t^{i,M}} = \frac{1}{NM} \sum_{i=1}^{NM} \delta_{X_t^i} \quad \text{ and } \quad \mu_t^N = \frac{1}{N} \sum_{i=1}^N \delta_{\tilde{Y}_t^{i,N}} = \frac{1}{NM} \sum_{i=1}^{NM} \delta_{Y_t^i},$$

and

$$\mathcal{W}_{2}\left(\mu_{t}^{N},\mu_{t}^{M}\right)^{2}=\frac{1}{NM}\sum_{i=1}^{NM}\left|X_{t}^{i}-Y_{t}^{i}\right|^{2}$$

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Proof of the estimate-2

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Direct calculations yield :

d

$$\begin{pmatrix} W_2\left(\mu_t^N, \mu_t^M\right)^2 \end{pmatrix}$$

= $-2\lambda W_2\left(\mu_t^N, \mu_t^M\right)^2 dt + 2\sigma \left(\frac{1}{N} + \frac{1}{M}\right) dt + dM_t$
 $- \frac{2}{(NM)^2} \sum_i \left(X_t^i - Y_t^i\right) \sum_j \left(V'(X_t^i - X_t^j) - V'(Y_t^i - Y_t^j)\right) dt.$

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Proof of the estimate-3

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$$\begin{split} \sum_{i} \left(X_{t}^{i} - Y_{t}^{i} \right) \sum_{j} \left(V'(X_{t}^{i} - X_{t}^{j}) - V'(Y_{t}^{i} - Y_{t}^{j}) \right) \\ &= \sum_{i > j} \left(V'(X_{t}^{i} - X_{t}^{j}) - V'(Y_{t}^{i} - Y_{t}^{j}) \right) \left(\left(X_{t}^{i} - Y_{t}^{i} \right) - \left(X_{t}^{j} - Y_{t}^{j} \right) \right) \\ &= \sum_{i > j} \left(V'(X_{t}^{i} - X_{t}^{j}) - V'(Y_{t}^{i} - Y_{t}^{j}) \right) \left(\left(X_{t}^{i} - X_{t}^{j} \right) - \left(Y_{t}^{i} - Y_{t}^{j} \right) \right). \end{split}$$

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Large number of Proof of the estimate $\sum_{i} \left(X_t^i - Y_t^i \right) \sum_{i} \left(V'(X_t^i - X_t^j) - V'(Y_t^i - Y_t^j) \right)$ $=\sum_{i < i} \left(V'(X_t^i - X_t^j) - V'(Y_t^i - Y_t^j) \right) \left(\left(X_t^i - Y_t^j \right) - \left(X_t^j - Y_t^j \right) \right)$ $=\sum_{i>i}\left(V'(X_t^j-X_t^j)-V'(Y_t^j-Y_t^j)\right)\left(\left(X_t^j-X_t^j\right)-\left(Y_t^j-Y_t^j\right)\right).$ $\geq \sum_{i=1}^{k} V'(X_t^i - X_t^j) \left(X_t^i - X_t^j\right) + \sum_{i=1}^{k} V'(Y_t^i - Y_t^j) \left(Y_t^i - Y_t^j\right)$ i > j s.t $Y_{4}^{j} = Y_{4}^{j}$

i > i s.t $X_{4}^{i} = X_{4}^{j}$

$$\geq \sum_{i>j \text{ s.t } Y_t^i = Y_t^j} -1 + \sum_{i>j \text{ s.t } X_t^i = X_t^j} -1$$
$$= -\frac{M(M-1)}{2}N - \frac{N(N-1)}{2}M.$$

Proof of the estimate-3

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Proof of the estimate-3

$$\begin{split} \left(X_{t}^{i}-Y_{t}^{i}\right) &\sum_{j} \left(V'(X_{t}^{i}-X_{t}^{j})-V'(Y_{t}^{i}-Y_{t}^{j})\right) \\ &=\sum_{i>j} \left(V'(X_{t}^{i}-X_{t}^{j})-V'(Y_{t}^{i}-Y_{t}^{j})\right) \left(\left(X_{t}^{i}-Y_{t}^{j}\right)-\left(X_{t}^{j}-Y_{t}^{j}\right)\right) \\ &=\sum_{i>j} \left(V'(X_{t}^{i}-X_{t}^{j})-V'(Y_{t}^{i}-Y_{t}^{j})\right) \left(\left(X_{t}^{i}-X_{t}^{j}\right)-\left(Y_{t}^{i}-Y_{t}^{j}\right)\right) . \\ &\geq \sum_{i>j \text{ s.t } Y_{t}^{i}=Y_{t}^{j}} V'(X_{t}^{i}-X_{t}^{j}) \left(X_{t}^{i}-X_{t}^{j}\right) + \sum_{i>j \text{ s.t } X_{t}^{i}=X_{t}^{j}} V'(Y_{t}^{i}-Y_{t}^{j}) \left(Y_{t}^{i}-Y_{t}^{j}\right) \\ &\geq \sum_{i>j \text{ s.t } Y_{t}^{i}=Y_{t}^{j}} -1 + \sum_{i>j \text{ s.t } X_{t}^{i}=X_{t}^{j}} -1 \\ &= -\frac{M(M-1)}{2}N - \frac{N(N-1)}{2}M. \end{split}$$

Hence

 \sum_{i}

$$\mathbb{E}\left(\mathcal{W}_{2}\left(\mu_{t}^{N},\mu_{t}^{M}\right)^{2}\right) \leq e^{-2\lambda t}\mathbb{E}\left(\mathcal{W}_{2}\left(\mu_{0}^{N},\mu_{0}^{M}\right)^{2}\right) + \frac{C}{N \wedge M}$$

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Proof of the estimate

Thank you

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