Brief Research Statement & Curriculum Vitae

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A - Selected research activities.

A.1 - Overview.

This section contains a brief review of some of my research activities. The following themes will be discussed.

The convex Plateau problem

We provide a general solution to the non-linear Plateau problem for locally strictly convex hypersurfaces immersed into Cartan-Hadamard manifolds.

The asymptotic Plateau problem

In [23]*, Labourie defines the asymptotic Plateau problem for constant extrinsic curvature surfaces in Cartan-Hadamard manifolds. We present a complete solution of this problem.*

Singular perturbation constructions

We use a singular perturbation argument to construct the first known examples of translating solitons of the mean curvature flow in $\mathbb{R}^3$ *of finite topology and of arbitrary finite genus.*

Finite type k-surfaces

We develop a theory of finite-type k-surfaces in $\mathbb{H}^3$. We study the relations between certain naturally-defined functions on the moduli space $\mathcal{S}_k$ of this class of surfaces. We use these functions to study, on the one hand, the geometric structure of individual k-surfaces and, on the other, the local structure of these moduli spaces.

A.2 - The convex Plateau problem.

Summary: *We provide a general solution to the non-linear Plateau problem for locally strictly convex hypersurfaces immersed into Cartan-Hadamard manifolds.*

The Plateau problem, which is one of the oldest problems of submanifold theory, can be formulated in the following manner. Let $X := X^{m+1}$ be a riemannian manifold, let $\Gamma := \Gamma^{m-1}$ be a compact, codimension 2 submanifold immersed in X, let K be a curvature function (such as, for example, mean curvature, extrinsic curvature, etc.), and let k be a positive real number. We say that a compact hypersurface Σ immersed in X is a solution of the **Plateau problem** (Γ, K, k) whenever its boundary $\partial\Sigma$ coincides with Γ and its K-curvature is constant and equal to k. A Plateau problem is said to be **non-parametric** whenever the required solutions are graphs over a certain domain, and it is said to be **parametric** otherwise. In what follows, we will only be concerned with the parametric case, which is the most general.

Even though the best-known version of the Plateau problem concerns surfaces of constant mean curvature, there is no shortage of curvature functions for which the Plateau problem can be formulated. Amongst these functions, there is a sub-family, which we refer to as **convex curvature functions**, which yields Plateau problems more tractable to existing PDE techniques. One well-known example of a convex curvature function is **extrinsic curvature**,

$$K_m := \mathrm{Det}(A)^{1/m}, \tag{A.1}$$

where A here denotes the Weingarten operator of the hypersurface. Others are given by the **curvature quotients**

$$K_{m,n} := (\sigma_m(A)/\sigma_n(A))^{\frac{1}{m-n}}, \tag{A.2}$$

where $\sigma_0, \cdots, \sigma_m$ here denote the elementary homogeneous polynomials, uniquely defined by the relation

$$\mathrm{Det}(\mathrm{Id} + tA) = \sum_{i=0}^{m} t^i \sigma_i(A). \tag{A.3}$$

A Plateau problem is said to be **convex** whenever it concerns a convex curvature function.

Starting with the foundational work [5] of Caffarelli-Nirenberg-Spruck, a large number of results have been obtained for convex Plateau problems in $\mathbb{R}^{m+1}$ and $\mathbb{H}^{m+1}$, as well as in certain other elementary spaces, such as cartesian products, and certain twisted products (see, for example, [14], [15], [16], [33] and [35]).

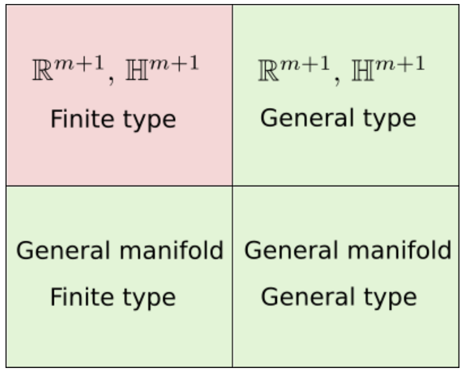

Figure 1 - A schematic and approximate representation of the aims of my work on the Plateau problem for convex curvature functions. Existing results, which only concerned problems in space-forms and curvature functions of infinite type (as well as certain particular cases of functions of finite type) are represented by the upper-left box.

However, despite all these results, there were two important limitations of this theory which stood out, and which my research sought to overcome (see Figure 1).

The first limitation concerned the ambient spaces studied. Indeed, the most general results known at the time rested heavily on the geometric structure of space forms, to the point of excluding generalisations to other spaces. Indeed, even in ambient spaces as simple as the cartesian product $\mathbb{H}^m \times \mathbb{R}$ and complex hyperbolic space $\mathbb{CH}^m$, the general Plateau problem remained open.

The second limitation was more subtle, and concerned the asymptotic properties of curvature functions. In order to describe this limitation correctly, it is worth defining more precisely what we understand by curvature functions. We thus define a curvature function to be a smooth function K of the principle curvatures of the hypersurface which satisfies a certain list of properties that naturally arise from geometric and analytic considerations.[1] In the convex case, we are interested in concave functions defined over the cone

$$\Lambda_m^+ := \{(x_1, \cdots, x_m) \mid x_i > 0 \ \forall i\}\,. \tag{A.4}$$

In particular, it follows from concavity that the limit

$$K_\infty(x_1, \cdots, x_{m-1}) := \operatorname*{Lim}_{t \to +\infty} K(x_1, \cdots, x_{m-1}, t) \tag{A.5}$$

exists at every point of Λ_{m-1}^+ and is either everywhere finite, or everywhere infinite. We say that a curvature function is of **finite type** or of **infinite type** depending on the value of this limit.

The family of curvature functions of finite type includes many natural curvature functions such as, for example, the special lagrangian curvature (see ref. **P24**) and the curvature quotients mentioned above. However, finite type curvature functions introduce an extra layer of complexity, and for this reason, were not treated in the initial work [5] of Caffarelli-Nirenberg-Spruck. Furthermore, in all subsequent work, they have only been studied partially and in certain exceptional cases. In summary, convex curvature functions of finite type have presented a hard and subtle problem, which was still yet to be studied in all generality.

In ref. **P9**, we overcome these two limitations. On the one hand, we provide a general solution to the convex Plateau problem for hypersurfaces in arbitrary Cartan-Hadamard manifolds and, on the other, we completely solve the problem of curvature functions of finite type. This work rests on three innovations. Firstly, we develop a purely geometric manner of building the barrier functions introduced in [5]. Second, we obtain new a priori estimates for locally strictly convex immersions of non-trivial boundary in general riemannian manifolds. Finally, we develop a new topological degree theory of immersed hypersurfaces. In addition, the arguments developed in ref. **P9** in fact apply equally well to hypersurfaces in general riemannian manifolds, although, in the general case, it becomes necessary to take into account the global geometry of the ambient space.

[1] These conditions, as well as their motivations, are explicitely described in the introduction to ref. **P9**.

Open problems: To address the case of non-convex curvature functions. To study the problem in semi-riemannian manifolds. The case $K = \sigma_2$ is of particular interest, since it is related to the scalar curvature of the hypersurface.

A.3 - The asymptotic Plateau problem.

Summary: *In [23], Labourie defines the asymptotic Plateau problem for constant extrinsic curvature surfaces in Cartan-Hadamard manifolds. We present a complete solution of this problem.*

Complete surfaces of constant extrinsic curvature immersed in space-forms have been of great interest to geometers ever since they were introduced by Gauss in his famous 1827 paper. At first, the results of Hopf, Volkov-Vladimirova, Sasaki and Hilbert suggested the existence of an almost trivial classification of these objects. However, it quickly became clear in each of the three cases not considered by these results - namely, surfaces of constant curvature equal to -1 in $\mathbb{S}^3$, surfaces of vanishing curvature in $\mathbb{H}^3$, and surfaces of constant curvature equal to $0 < k < 1$ in $\mathbb{H}^3$, a rich theory may be uncovered which, furthermore, has deep applications in many fields of modern mathematics.

Labourie's work concerns the third case mentioned above, namely, that of immersed surfaces in $\mathbb{H}^3$ of constant extrinsic curvature equal to $0 < k < 1$. In a series of papers (see, for example, [19], [20], [21] and [22]), Labourie establishes a straightforward relation between constant extrinsic curvature surfaces on the one hand and pseudo-holomorphic curves in the unitary bundle on the other. In this manner, he lays the basis of a theory of constant extrinsic curvature surfaces which today has applications in many different fields, such as, for example, hyperbolic geometry, general relativity, Teichmüller theory, etc. (see, for example, [2], [3], [4] and [31]).

A central element of Labourie's theory is what he calls the "asymptotic Plateau problem". In order to state it correctly, it is first necessary to introduce some definitions. Let X be a Cartan-Hadamard manifold. An **immersed surface** in X is a pair (S, e), where S is a topological surface and $e : S \to X$ is an immersion. Let ν_e denote its unit normal vector field; let I_e, II_e and III_e denote its three fundamental forms; let A_e denote its Weingarten operator; and let $K_e := \mathrm{Det}(A_e)$ denote its extrinsic curvature. We say that a surface is **locally strictly convex** (LSC) whenever II_e is everywhere positive definite; and we say that it is **quasicomplete** whenever it is complete with respect to the metric $\mathrm{I}_e + \mathrm{III}_e$. Observe, in particular, that every complete surface is trivially quasicomplete. For $0 < k < 1$, we say that (S, e) is a k**-surface** whenever it is LSC, quasicomplete, and of constant extrinsic curvature equal to k.

We also require the concept of assymptotic Gauss map, which is defined as follows. We first denote the unit sphere bundle of X by UX and its ideal boundary by $\partial_\infty X$. The **horizon map** of X is the function $f : UX \to \partial_\infty X$ which sends every vector ξ of UX to the unique ideal point towards which it points. The **assymptotic Gauss map** of (S, e) is then the function $\phi_e : S \to \partial_\infty \mathbb{H}^3$ defined by

$$\phi_e := h \circ \nu_e. \tag{A.6}$$

When (S, e) is a k-surface (and, more generally, when (S, e) is LSC), this function is a local homeomorphism.

Labourie's asymptotic Plateau problem concerns the unique prescription of k-surfaces by their asymptotic Gauss maps. Let S be a surface, let $\phi : S \to \partial_\infty X$ be a local homeomorphism, and let $0 < k < 1$ be a real number. For $0 < k < 1$, we say that an immersion $e : S \to X$ is a solution of the **asymptotic Plateau problem** (S, ϕ, k) whenever (S, e) is a k-surface with asymptotic Gauss map equal to ϕ. Observe, in particular, that this problem identifies geometric objects in X with topological objects in its ideal boundary. It may thus be considered as a manifestation of the holographic principle of the AdS-CFT correspondence of theoretical physics. In [23], Labourie proves a series of partial existence and uniqueness results for asymptotic Plateau problems. However, the general case has remained open.

In ref. **S1** we present a complete solution of this asymptotic Plateau problem. We determine necessary and sufficient conditions for the existence of solutions and we show that these solutions, whenever they exist, are always unique. In addition, we work in a framework that is more general than that studied by Labourie in two ways. Firstly, whilst Labourie only treats the case of ambient manifolds of bounded geometry (that is, which satisfy certain geometric restrictions at infinity), we only require pinched curvature of the ambient space. Next, whilst Labourie is only concerned with the case of 3-dimensional ambient spaces, upon using the concept of special Lagrangian curvature (which we introduced in ref. **P24**) we show that the problem has a complete solution in ambient spaces of arbitrary dimension.

Open problems: To study the dynamical properties of the space of k-hypersurfaces, following [23]. In particular, the study of certain entropies defined in terms of the space of k-surfaces - following recent ideas of Calegari-Marques-Neves (see [6]) - appears promising. The case of surfaces of constant intrinsic curvature as well as the case of semi-riemannian ambient spaces also strike us as interesting.

A.4 - Construction of surfaces by singular perturbation.

Summary: *We use a singular perturbation argument to construct the first known examples of translating solitons of the mean curvature flow in $\mathbb{R}^3$ of finite topology and of arbitrary finite genus.*

We say that an embedded hypersurface Σ in $\mathbb{R}^{m+1}$ is a **translating soliton** of the mean curvature flow whenever its evolution by this flow is given by translations of the ambient space. The problem of classification of such solitons is of interest for two reasons. Firstly, they provide non-trivial examples of mean curvature flows which are defined for all time, that is, so-called "eternal" mean curvature flows. Secondly, they provide infinitesimal models for the singularities that may arise from limits of more general mean curvature flows.

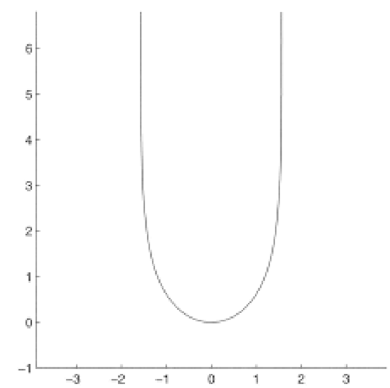

Figure 2 - The "grim reaper" curve is, up to rigid motions and dilatations, the graph of the logarithm of the secant. The evolution of this curve under mean curvature flow is given by upwards translation parallel to the y-axis.

Every affine hyperplane is trivially a translating soliton, where the direction of translation is parallel to the hyperplane. The simplest non-trivial example of a translating soliton is the so-called "grim-reaper" curve in $\mathbb{R}^2$ (see Figure 2). Next, the cartesian product of this curve with a straight line yields a translating soliton in $\mathbb{R}^3$. Finally, upon desingularising the union of this latter soliton with a hyperplane about a Sherk tower, Nguyen obtains in [28] and [29] examples of translating solitons of infinite topology, which she calls "translating tridents" (see Figure 3).

Despite the many examples of translating solitons that had been constructed, there were no known examples of translating solitons in $\mathbb{R}^3$ of non-trivial **finite** topology. In ref. **S5**, independentally of, but simultaneously with [7], we construct the first known examples of finite genus translating solitons of the mean curvature flow. More precisely, upon desingularising the union of three catenoidal ends about a Costa-Hoffman-Meeks (CHM) surface, we obtain translating solitons with three ends and of genus equal to g, for all $g \in \mathbb{N}$. We observe that the construction of [7] is weaker, since it only holds for $g \gg 1$.

The use of CHM surfaces in desingularisation constructions presents a subtle problem. Indeed, although this class of surfaces has already been used in previous constructions (such as, for example, [18] and [27]), in each of these cases, the authors make use of asymptotic properties that are not satisfied by translating solitons. For this reason, it was necessary to determine explicit estimates for operators defined over these surfaces. This in turn leads us to a fundamental difficulty, namely, the low order of symmetry of CHM surfaces.

In order to understand the relation between symmetry and norm estimates, it is instructive to first review the case of bounded harmonic functions defined over the semi-cylinder $C^+ := \mathbb{S}^1\times]0,\infty[$. Indeed,

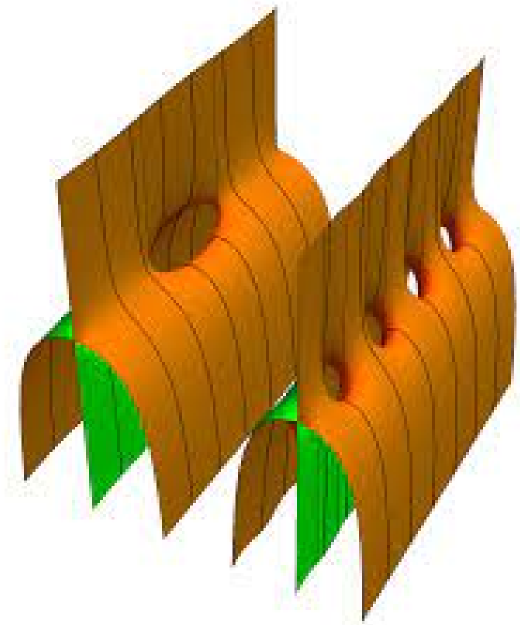

Figure 3 - Nguyen's "translating tridents" are examples of mean curvature flow solitons of non-trivial topology. Their evolutions under the mean curvature flow are are given by downwards translations parallel to the z=axis.

upon applying the technique of separation of variables, we show that every such function has the form

$$f(\theta,t) = a_0 + \sum_{m=1}^{\infty}\big(a_m\cos(m\theta) + b_m\sin(m\theta)\big)e^{-mt}. \tag{A.7}$$

In particular, when f is symmetric by rotation by $2\pi/M$ about the t-axis, the coefficients a_m and b_m all vanish for $0 < m < M$ and the difference $(f - a_0)$ decreases exponentially with e^{-Mt}. This phenomenon, which is the archetype of arguments used to derive norms of operators and functions out of symmetries, trivially cannot be applied when no symmetries are available!

In our work, we get round the lack of symmetry in our construction upon observing that the difficulties that we encounter in fact arise from the loss of information which takes place when functional norms are used to express properties of functions. This phenomenon is well illustrated by the C^2 norm. Indeed, when the second derivative of a function f is large, its C^2 norm provides no information concerning the size of its *first* derivative. We address this loss of information by introducing a well-chosen combination of Hölder and Sobolev norms, which we call the **hybrid norm**. We show that this hybrid norm efficiently summarises the asymptotic behaviour of our construction, and thus allows us to obtain the estimates we require in order to construct mean curvature flow solitons by desingularisation around CHM surfaces.

Open problems: To construct helicoidal mean curvature flow solitons of non-trivial topology. The techniques that we have developed here may also be applied to other constructions involving CHM surfaces.

A.5 - Finite type k-surfaces (c.f. Section A.3).

Summary: *We develop a theory of finite-type k-surfaces in $\mathbb{H}^3$. We study the relations between certain naturally-defined functions on the moduli space $\mathcal{S}_k$ of this class of surfaces. We use these functions to study, on the one hand, the geometric structure of individual k-surfaces and, on the other, the local structure of these moduli spaces.*

We say that a k-surface (S, e) is of **finite type** whenever it is complete (as opposed to quasi-complete) and of finite area. The study of this class of surfaces was begin in my PhD thesis. We denote by $\mathcal{D}_k$ the space of k-surfaces of finite type in $\mathbb{H}^3$, where two k-surfaces are identified whenever they are equivalent up to reparametrisation. We first show how finite-type k-surfaces are related to pointed ramified coverings of the Riemann sphere $\hat{\mathbb{C}}$. Indeed, we recall that a **pointed ramified covering** of $\hat{\mathbb{C}}$ is a triple (Σ, P, ϕ), where Σ is a closed Riemann surface, $P \subseteq S$ a finite subset, and $\phi : \Sigma \to \hat{\mathbb{C}}$ is a non-constant holomorphic function whose ramification points are contained in P. We denote by $\mathcal{R}$ the space of pointed ramified coverings of $\hat{\mathbb{C}}$, where two elements are identified whenever they are equivalent up to biholomorphism. We show in ref. **P6** that the spaces $\mathcal{S}_k$ and $\mathcal{R}$ are naturally homeomorphic with respect to their natural topologies.

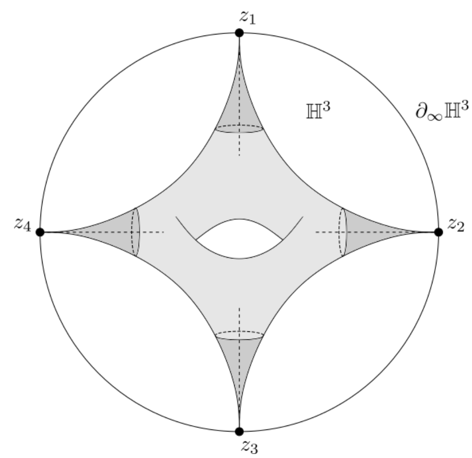

Figure 4 – Every k-surface decomposes into a compact part and a finite collection of "k-ends".

The homeomorphism $\Phi_k : \mathcal{S}_k \to \mathbb{R}$ is constructed in the following manner. First, we prove in ref. **P30** a **structure theorem** which states that every finite-type k-surface naturally decomposes into a compact component (S_0, e) and a finite collection $(S_1, e), \cdots, (S_m, e)$ of "ends", each of which is asymptotic to a cylinder of exponentially decreasing radius around a geodesic ray. We call ends of this type "***k*-ends**" (see Figure 4). Observe that every k-end terminates in an ideal point of $\partial_\infty\mathbb{H}^3$, which we call its **extremity**. We now recall that $\partial_\infty\mathbb{H}^3$ naturally identifies with $\hat{\mathbb{C}}$. Thus, since the asymptotic Gauss map ϕ_e of e is a local homeomorphism, it defines a canonical holomorphic structure over S, which we denote by $\phi_e^*\hat{\mathbb{C}}$. It follows from the structure theorem that $(S, \phi_e^*\hat{\mathbb{C}})$ is biholomorphic to the complement of a finite subset P of a closed Riemann surface Σ, and that ϕ_e extends to a holomorphic function from this surface into $\hat{\mathbb{C}}$. The triplet (Σ, P, ϕ_e) is a pointed ramified covering of $\hat{\mathbb{C}}$. We denote

$$\Phi_k(S, e) := (\Sigma, P, \phi_e), \tag{A.8}$$

and in ref. **P6**, we show that, for all $0 < k < 1$, $\Phi_k : \mathcal{S}_k \to \mathcal{R}$ is a homeomorphism.

In ref. **S3**, we deepen our understanding of finite type k-surfaces. We first indentify functions and geometric objects that are related to one-another by a Schläfli-type formula. Indeed, an asymptotic study of the k-ends allows us to show that each k-end possesses a well-defined axis, which we call its **Steiner geodesic**. Whilst one of the extremities of this geodesic trivially coincides with the extremity of the k-end, the other extremity trivially defines a different point of $\partial_\infty\mathbb{H}^3$, which we call the **Steiner point** of the k-end. We show that every k-surface has a well-defined volume (even in the non-embedded case), as well as a **renormalised energy**, which is well-defined up to a quadratic function of the extremities.

We obtain a Schläfli type formula which relates the extremities, the volume, the Steiner points and the renormalised energy, and we find two interesting applications. First, upon applying this formula to the Killing fields of the ambient space, we obtain a sequence of **balancing conditions** satisfied by the extremities and the Steiner points. In particular, these balancing conditions allow us to determine the Steiner points of certain finite type k-surfaces with large numbers of symmetries (see Figure 5). Next, the Schläfli formula allows us to show that the extremities and the Steiner points together define lagrangian immersions of the strata of $\mathcal{S}_k$ into certain open Kähler manifolds. In particular, this shows that the extremities and Steiner points define conjugate variables over $\mathcal{S}_k$.

Open problems: To study the second derivatives of these functionals. To study the equivariant case. To study the case of ambiant manifolds of non-constant curvature. To study the higher-dimensional case. To study the relationships between the functionals studied here and holomorphic functionals over $\mathcal{R}$.

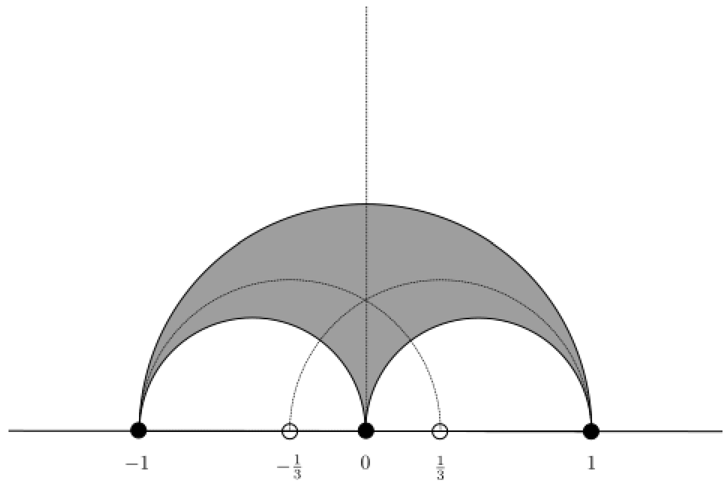

Figure 5 - Steiner geodesics and Steiner points - An embedded k-surface with 3 extremities at -1, 0 and 1 is obtained by "fattening" the convex hull of these three points in $\mathbb{H}^3$. In this case, by symmetry, the Steiner geodesics are independent of k and their respective Steiner points are situated at $1/3$, ∞ and $-1/3$.

B - Administrative activities.

B.1 - Administrative activities internal to UFRJ.

1 - Head of Department, Mathematics Department, Mathematics Institute, UFRJ, 12/02/2019–07/12/2020.
2 - Vice Head of Department, Mathematics Department, Mathematics Institute, UFRJ, 19/11/2014–12/02/2019.
3 - President of the Subcommission CPA-CCMN (Internal UFRJ Evaluation Commission), 23/06/2020–31/12/2021.
4 - Substitute president of the CPA (Internal UFRJ Evaluation Commission), 23/06/2020–31/12/2021.
5 - Participating member of the CCMN-UFRJ Working group on the academic impact of the COVID pandemic, 23/06/2020–31/12/2021.
6 - Participating member of the Commission for the Purchase of Videoconferencing Software, 29/09/2020–31/12/2020.

B.2 - Hiring committees.

1 - Public call for Adjunct Professors, UFRJ, 2021
2 - Public call for Adjunct Professors, UFF-Niteroi, 2019
3 - Public call for Adjunct Professors, UFF-Niteroi, 2016

B.3 - Events and seminars.

1 - Pangolin seminar, organised jointly with Sébastien Alvarez (Universidad de la República), François Fillastre (Université de Cergy-Pontoise) and Andrea Seppi (Université Grenoble-Alpes).
2 - First geometry meeting of the Instituto de Matemática UFRJ, organised jointly with Maria Fernanda Elbert, 2 day event, 2021
3 - GDAR Mini Workshop, IM-UFRJ, 1 day event, 2018
4 - Spring School: Geometry and Physics, 1 week event, 2008

B.4 - Peer review.

1 - 2019, Calc. Var. PDEs
2 - 2018, Calc. Var. PDEs
3 - 2018, Analysis and PDEs
4 - 2018, Journal of Differential Geometry
5 - 2018, Geometriae Dedicata
6 - 2017, Annales de l'Institut Fourier
7 - 2016, Duke Mathematical Journal
8 - 2016, Journal of Geometric Analysis
9 - 2016, Publicacions Matematiques
10 - 2015, Journal of the European Mathematical Society
11 - 2015, Mathematische Zeitschrift
12 - 2015, Reports@SCM (periódico electronico da socidade matematica de Catalunha)
13 - 2014, Journal of Geometric Analysis
14 - 2013, Ensaios Matematicos
15 - 2013, Geometria Dedicata

C - Examination platform.

In order to address needs arising as a consequence of the COVID-19 pandemic, in September 2020 I developed an online platform for the remote application of randomized exams. This platform, designed together with Prof. Helena Lopes of the Mathematics Institute of the UFRJ, was built to meet the following specifications.

(1) To be programmable with a degree of versatility sufficient for the realization of randomised tests having the level of sophistication required for calculus courses;

(2) To be compatible with all major internet browsers; and

(3) To address the specific needs of lower-income brasilian students with unreliable internet access.

In addition, due to its modularity and computational simplicity, the platform is highly versatile, allowing its rapid implementation without the need for a dedicated IT team. The platform was used by the Mathematics Department throughout the period of the COVID-19 pandemic and received positive feedback from its users. Screen shots are shown in Figures 6 and 7.

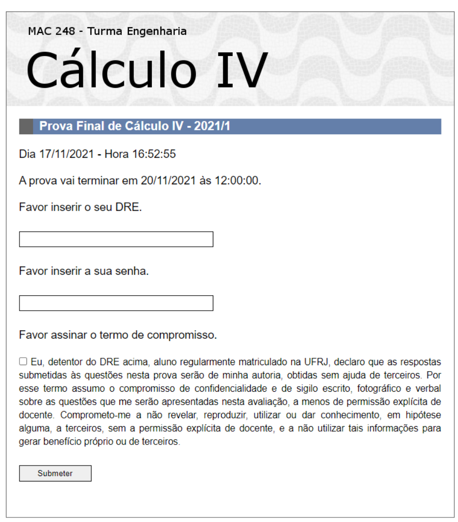

Figure 6 – The access page to the examination platform.

D - Teaching experience and orientations.

D.1 - Teaching.

1 - "An introduction to Gromov-Witten Invariants", Max Planck Institute, Leipzig, Alemanha, 8 hours, 04/2007-06/2007
2 - "Symplectic Gromov-Witten Invariants", Max Planck Institute, Bonn, Alemanha, 2 hours, 2008
3 - "An introduction to Morse/Floer Homology", Centro de Recerca Matemática, Barcelona, Espanha, 10 hours, 2012
4 - Cálculo 1, Undergraduate UFRJ, 2 classes of 60 hours, 2013-1
5 - Cálculo 4, Undergraduate UFRJ, 1 classes of 40 hours, 2013-2
6 - Tópicos em Geometria, Graduate UFRJ, 1 class of 40 hours, 2014-1
7 - Cálculo 1, Undergraduate UFRJ, 2 classes of 60 hours, 2014-2
8 - Cálculo 4, Undergraduate UFRJ, 1 class of 40 hours, 2015-1
9 - Geometria Diferencial, Graduate UFRJ, 1 class of 40 hours, 2015-1
10 - Cálculo 4, Undergraduate UFRJ, 1 class of 40 hours, 2015-2
11 - Cálculo 4, Undergraduate UFRJ, 1 class of 40 hours, 2016-1
12 - Cálculo 4, Undergraduate UFRJ, 1 class of 40 hours, 2016-2
13 - Análise Geométrica, Graduate UFRJ, 1 class of 40 hours, 2016-2
14 - Cálculo 4, Undergraduate UFRJ, 1 class of 40 hours, 2017-1
15 - Geometria Diferencial, Undergraduate UFRJ, 1 class of 40 hours, 2017-2
16 - Cálculo 4, Undergraduate UFRJ, 1 class of 40 hours, 2018-1
17 - Topologia Diferencial, Graduate UFRJ, 2018-1
18 - Cálculo 4, Undergraduate UFRJ, 1 class of 40 hours, 2018-2
19 - Topologia Diferencial, Graduate UFRJ, 1 class of 40 hours, 2019-1
20 - Geometria Riemannaina, Graduate UFRJ, 1 class of 40 hours, 2019-2
21 - Geometria Diferencial, Graduate UFRJ, 1 class of 40 hours, 2020-1
Classes available online at http://im.ufrj.br/~moriarty/GD_2020_1/index.php
22 - Geometria Riemanniana, Graduate UFRJ, 1 class of 40 hours, 2020-4 (PLE)
Classes available online at http://im.ufrj.br/~moriarty/GD_PLE/index.php
23 - Geometria Diferencial, Graduate UFRJ, 1 class of 40 hours, 2021-1
Classes available online at http://im.ufrj.br/~moriarty/GD_2021_1/index.php
24 - Análise Geométrical, Graduate UFRJ, 1 class of 40 hours, 2021-1
Classes available online at http://im.ufrj.br/~moriarty/AG_2021_1/index.php
25 - Cálculo 4, Undergraduate UFRJ, 1 class of 40 hours, 2021-1
Classes available online at
https://www.youtube.com/playlist?list=PLgiGE2bd21CrnKRL1E8JGv5QXQ4VLV8VY

D.2 - Orientations.

1 - Dennis Leonardo Becerra Hernandez, Master's Thesis entitled "Espaços homogêneos e grupos de Lie", defendida 10/10/2017
2 - Claudia Veronica Salas Magaño, PhD Thesis entitled, "Sobre fluxos de curvatura média eternais em perturbações de $\mathbb{S}^3$", defendida 02/06/2020
3 - Pedro Henrique Birindiba Batista, PhD Orientation, in progress.
4 - Ian Mateus Brito Perreira, Master's Orientation, in progress.
5 - Lejzer Javier Castro Tapia, Master's Orientation, in progress.

E - Presentations.

E.1 - Seminars.

1 - The Kulkarni-Pinkall form and locally strictly convex immersions in $\mathbb{H}^3$, Seminario de Geometría, Universidad de Granada, Spain, 2021
2 - On the Weyl problem in Mikowski space, Differential Geometry Seminar, Technische Universitaet Wien, Austria, 2021

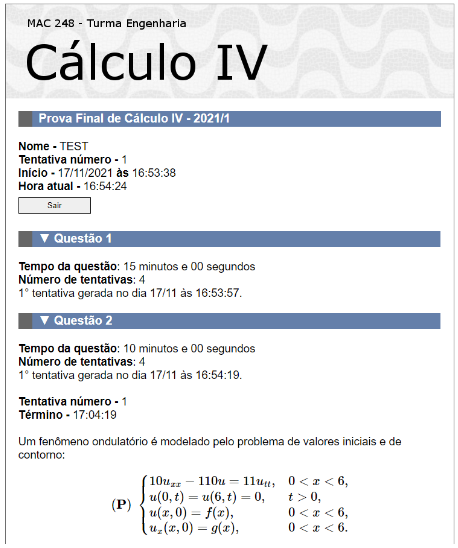

Figure 7 - The examination platform after user access with a question visualised.

3 - On the asymptotic structure of finite-type k-surfaces in 3-dimensional hyperbolic space, Pangolin Seminar, 2020

4 - On eternal forced mean curvature flows of tori in perturbations of the unit sphere, Pangolin Seminar, 2020

5 - Complete embedded minimal surfaces of Costa-Hoffman-Meeks type in 3-dimensional hyperbolic space, Seminário de Geometria Diferencial, Universidade de Brasilia, Brasil, 2019

6 - On the asymptotic geometry of finite-type k-surfaces in three-dimensional hyperbolic space, Geometry Seminar, University of Leicester, UK, 2019

7 - On an integrable system with boundary, Séminaire GEDP, Université de Cergy-Pontoise, France, 2019

8 - Superfícies mínimas de gênero finito no espaço hiperbólico, Seminario de Geometria Diferencial, UFF, Brasil, 2018

9 - On the Morse index of higher dimensional free boundary minimal annuli, Geometry Seminar, PUC-Rio, Brasil, 2018

10 - On the Morse index of higher dimensional free boundary minimal annuli, Geometry Seminar, University College Cork, Ireland, 2018

11 - Special Lagrangian Curvature, Seminário de Geometria, Universidade de São Paulo, Brasil, 2015

12 - Perturbing the Costa Surface, Geometry Seminar, Universidad Autónoma de Barcelona, Spain, 2015

13 - Cauchy surfaces of constant scalar curvature in Minkowski spacetimes, Seminário de Geometria Diferencial, IMPA, Brasil, 2015

14 - On complete finite area surfaces of constant extrinsic curvature in 3-dimensional hyperbolic space, Seminário de Geometria Diferencial, IMPA, Brasil, 2014

15 - Bifurcation for solutions of the Allen-Cahn equation, Seminário de Geometria, Universidade de Brasilia, Brasil, 2014

16 - Orbifolds in an infinite-dimensional setting, Seminário de Geometria Diferencial, IMPA, Brasil, 2013

17 - Orbifolds in an infinite-dimensional setting (part II), Seminário de Geometria Diferencial, IMPA, Brasil, 2013

18 - The Plateau problem for convex curvature functions, Séminaire de Géométrie, Université Paul Sabatier, France, 2012

19 - Le problème de Plateau pour les fonctions de courbure convexes, Séminaire de théorie spectrale et géométrie, Université Grenoble-Alpes, France, 2012

20 - Compacité des sphères plongées à courbure gaussienne constante, Séminaire de Géométrie, Université Paris VII, France, 2011

21 - Théorie de degré des hypersurfaces immergées, Séminaire de Géométrie, Université Paris VII, France, 2011

22 - The Euler characteristic and the generalised Minkowski problem, Geometry Seminar, Universidad Autonoma de Madrid, Spain, 2011
23 - Immersed spheres of constant Gaussian curvature in three dimensional manifolds, Geometry Seminar, Universität Heidelberg, Germany, 2011
24 - Compactness for CMC surfaces, Seminário de Géométria, UFF, Brasil, 2011
25 - Geometric barrier techniques and the non-linear Plateau problem, Seminari d'Equacions en Derivadas Parcials i Aplicacions, Universitat Politècnica de Catalunya, Spain, 2011
26 - The non-linear Dirichlet problem in Hadamard manifolds, Séminaire de Géométrie, Université Paris VII, France, 2010
27 - Le problème de Dirichlet non-linéaire dans des variétés d'Hadamard, Sminaire GEDP, Université Cergy-Pontoise, France, 2010
28 - The non-linear Plateau problem in Hadamard manifolds, Oberseminar Analysis, Geometrie und Physik, Freie Universität Berlin, Germany, 2010
29 - The non-linear Dirichlet problem in negatively curved manifolds, Oberseminar Geometrie, Max Planck Institute for Mathematics in the Sciences, Germany, 2010
30 - The non-linear Plateau problem in Hadamard manifolds, Seminário de Geometria Diferencial, IMPA, Brasil, 2010
31 - Degree theory for immersed hypersurfaces, Seminário de Geometria Diferencial, IMPA, Brasil, 2010
32 - The Plateau problem for general convex curvature functions, Seminário de Geometria Diferencial, IMPA, Brasil, 2010
33 - Barrier techniques in hypersurface theory, Seminário Análise-EDP, UFRJ, Brasil, 2010
34 - Barrier techniques and the non-linear Plateau problem, Seminario de Geometría, Universidad de Granada, Spain, 2010
35 - Degree theory of immersed submanifolds, Seminario de Geometría, Universidad de Granada, Spain, 2010
36 - k-surfaces à points, Séminaire de Géométrie, Université de Nantes, France, 2008
37 - Feuilletages à courbure spéciale lagrangienne constante des variétés hyperboliques, 2008
38 - Constant curvature foliations of hyperbolic ends, Seminar Geometrie, Ludwig Maximilians Universität, Germany, 2008
39 - Constant curvature foliations of hyperbolic ends, Seminar Geometrie, Wastfälische Wilhelms Universität, Germany, 2008
40 - Feuilletages à courbure spéciale lagrangienne constante, Séminaire de Géométrie, Université de Cergy-Pontoise, France, 2008
41 - k-surfaces à points, Séminaire de Geométrie, Université Paris VII, France, 2007
42 - Pointed k-surfaces, Seminar Geometrie, Max Planck Institute for Mathematics in the Sciences, Germany, 2007
43 - Problèmes de Plateau equivariants, Séminaire d'Analyse, Université de Cergy-Pontoise, France, 2006
44 - Equivariant Plateau problems, Seminar Symplektische Geometrie, Universität Leipzig, Germany, 2006
45 - Problèmes de Plateau equivariants, Séminaire de géométrie ergodique, École Polytechnique, France, 2006
46 - Problèmes spéciaux legendriens, Séminaire de géométrie, Université Claude Bernard, France, 2005
47 - Problèmes spéciaux legendriens, Séminaire de groupes et géométrie, Université Paul Sabatier, France, 2005
48 - Special legendrian problems, Seminar Geometrie, Max Planck Institute for Mathematics in the Sciences, Germany, 2005
49 - Problèmes de Plateau equivariants, Séminaire de théorie spectrale et géométrie, Université Grenoble-Alpes, France, 2005
50 - Equivariant Plateau problems, Seminar Geometrie, Max Planck Institute for Mathematics in the Sciences, Germany, 2005 2005
51 - Problèmes de Plateau equivariants, Séminaire de géométrie symplectique, École Polytechnique, France, 2005

E.2 - Conferences.

1 - Finite-type k-surfaces, in "Geometry, groups and dynamics", conference in honour of François Labourie, Cargèse, France, 2022
2 - Milking the cow: getting the most out of functional norms, in "Grupo de Trabalho GD 2019", UFF, Brasil, 2019
3 - Constant mean curvature annuli and the sinh-Gordon equation, in "32^o Colóquio Brasileiro de Matemática", IMPA, Brasil, 2019
4 - On eternal mean curvature flows of 2-tori in the 3-sphere, in "1^o Joint Meeting Brazil-France in Mathematics", IMPA, Brasil, 2019
5 - On the elliptic sinh-Gordon equation with Durham boundary conditions, in "Minimal surfaces: integrable systems and visualisation", University of Leicester, UK, 2019
6 - On the Morse index of higher dimensional free boundary minimal catenoids, in "II^o Workshop de Geometria Diferencial", UFF, Brasil, 2018
7 - On the Morse index of higher dimensional free boundary minimal annuli, in "VIII^o Workshop de Geometria Diferencial", UFA, Brasil, 2018
8 - Morse homology and problems of prescribed mean curvature, in "Geometric analysis, metric geometry and topology", Université Grenoble-Alpes, France, 2016
9 - Morse homology and problems of prescribed mean curvature, in "International Conference in Geometry", University of Macau, Macau, 2016
10 - How to perturb the Costa surface, in "Grupo de trabalho de geomtria diferencial", UFF, Brasil, 2015
11 - On translating solitons of the mean curvature flow that are of finite genus, in "Workshop on geometric flows", Universidad de Granada, Spain, 2015
12 - On singular perturbations of the Morse complex, in "30^o Colóquio Brasileiro de Matemática", IMPA, Brasil, 2015
13 - Perturbing the Costa surface, in "XLIV^o Escola de Verão", UnB, Brasil, 2015
14 - On complete embedded translating solitons of the mean curvature flow that are of finite genus, in "Workshop on Geometric Flows", Universidad de Granada, Spain, 2015
15 - On complete embedded translating solitons of the mean curvature flow that are of finite genus, in "V^o Workshop de Geometria Diferencial", UFA, Brasil, 2015
16 - A new Weierstrass type representation for constant extrinsic curvature surfaces in hyperbolic space, in "IV^o Workshop in Differential Geometry", UFA, Brasil, 2014
17 - On an Enneper-Weierstrass-type representation of constant Gaussian curvature surfaces in 3-dimensional hyperbolic space, in "Geometric Analysis at Roscoff", Université de Bretagne Occidentale, France, 2014
18 - On an Enneper-Weierstrass type representation of constant Gaussian curvature surfaces in 3-manifolds, in "XVIII^o Escola de Geometria Diferencial", UnB, Brasil, 2014
19 - On an Enneper-Weierstrass type representation of constant Gaussian curvature surfaces in 3-manifolds, in "New trends in differential geometry", Universitat Roma, Italy, 2014
20 - On an Enneper-Weierstrass-type representation of constant Gaussian curvature surfaces in 3-dimensional hyperbolic space, in "Teichmüller Theory and Surfaces in 3-Manifolds", Centro di Giorgi, Italy, 2014
21 - Free-boundary minimal surfaces in convex 3-manifolds, in "29^o Colóquio Brasileiro de Matemática", IMPA, Brasil, 2013
22 - Extremal hypersurfaces in convex 3-manifolds, in "III^o Workshop de Geometria Diferencial", UFA, Brasil, 2013
23 - Barrier techniques and the non-linear Plateau problem, in "Congreso de la Real Sociedad Matemática Española", Ávila, Spain, 2011
24 - Constant curvature immersed hypersurfaces and the Euler characteristic, in "International Conference on Surface Theory", Universidad de Sevilla, Spain, 2011
25 - The Plateau problem for general curvature functions, in "28^o Colóquio Brasileiro de Matemática", IMPA, Brasil, 2011
26 - The non-linear Plateau problem in Hadamard manifolds, in "Algebraic, geometric and analytic aspects of surface theory", IMPA, Brasil, 2010
27 - The Plateau problem in Hadamard manifolds, in "XVI^o Escola Brasileira de Geometria Diferencial",

USP, Brasil, 2010
28 - Non-linear Dirichlet problems in Hadamard manifolds, in "Jornada de Geometria", Universidad de Granada, Spain, 2009
29 - Constant curvature foliations of hyperbolic ends, in "Advanced Course in Geometric Flows and Hyperbolic Geometry", Centre de Recerca Matemàtica, Spain, 2008
30 - Constant curvature foliations of hyperbolic ends, in "Advanced Course in Geometric Flows and Hyperbolic Geometry", em "Dynamical Systems - Geometric structures and rigidity", Stefan Banach Centre, Poland, 2008
https://www.impan.pl/en/activities/banach-center/conferences?y=2008
31 - Positive Special Legendrian Submanifolds and Weingarten Problems, in "Summer School and Conference: Geometric Analysis and Nonlinear PDEs", Stefan Banach Centre, Poland, 2007
https://www.mimuw.edu.pl/~ga2007/

F - Academic output.

F.1 - Papers published or accepted for publication.

P1 - Smith G., Möbius structures, hyperbolic ends and k-surfaces in hyperbolic space, to appear in "In the Tradition of Thurston, Vol. II", (Ohshika K., Papadopoulos A. ed.), Springer Verlag, (2022)
P2 - Smith G., Stern A., Tran H., Zhou D., On the Morse index of higher-dimensional free boundary minimal catenoids, to appear in *Calc. Var. PDEs.*
P3 - Alvarez S., Prescription de courbure des feuilles des laminations: retour sur un théorème de Candel, to appear in *Ann. Inst. Fourier*
P4 - Smith G., On the Weyl problem in Minkowski space, *Int. Math. Res. Not.*, (2021)
P5 - Kilian M., Smith G., On the elliptic sinh-Gordon equation with Durham boundary conditions, *Nonlinearity*, **34**, no. 8, 5119–5135
P6 - Smith G., On an Enneper-Weierstrass-type representation of constant Gaussian curvature surfaces in 3-dimensional hyperbolic space, in "Minimal surfaces: Integrable systems and Visualisation" (Hoffmann T., Kilian M., Leschke K., Martin G. ed.), Springer Proceedings in Mathematics and Statistics, **349**, (2021)
P7 - Alvarez S., Smith G., Earthquakes and graftings of hyperbolic surface laminations, *Int. Math. Res. Not.*, (2020)
P8 - Smith G., A short proof of an assertion of Thurston concerning convex hulls, in "In the tradition of Thurston", (Alberge V., Ohshika K., Papadopoulos A. ed.), Springer Verlag, (2020)
P9 - Smith G., The Plateau problem for convex curvature functions, *Ann. Inst. Fourier*, **70**, no. 1, 1–66, (2020)
P10 - Fillastre F., Smith G., A note on invariant constant curvature immersions in Minkowski space, *Geom. Dedicata*, **206**, no. 1, 75–82, (2020)
P11 - Rosenberg H., Degree Theory of Immersed Hypersurfaces, *Mem. Amer. Math. Soc.*, **265**, no. 1290, (2020)
P12 - Smith G., Eternal forced mean curvature flows II - Existence, *Pacific J. Math.*, **299**, no. 1, 191–235, (2019)
P13 - Smith G., Zhou D., The Morse index of the critical catenoid, *Geom. Dedicata*, **201**, 13–19, (2019)
P14 - Fillastre F., Smith G., Group actions and scattering problems in Teichmueller theory, in The Handbook of Group Actions, Vol. III, Advanced Lectures in Mathematics, 40, International Press, Boston, (2018)
P15 - Smith G., Constant scalar curvature hypersurfaces in $(3 + 1)$-dimensional GHMC Minkowski spacetimes, *J. Geometry Phys.*, **128**, 99–117, (2018)
P16 - Máximo D., Nuñes I., Smith G., Free boundary minimal annuli in convex three-manifolds, *J. Diff. Geom.*, **106**, No. 1, (2017)
P17 - Smith G., Bifurcation of solutions to the Allen-Cahn equation, *J. London Math. Soc.*, **94**, no. 3, (2016), 667–687
P18 - Smith G., Global Singularity Theory for the Gauss Curvature Equation, *Ensaios Matemáticos*, **28**, (2015), 1–114
P19 - Smith G., Eternal forced mean curvature flows I - a compactness result, *Geom. Dedicata*, **176**, no. 1, (2014), 11–29
P20 - Smith G., Hyperbolic Plateau problems, *Geom. Dedicata*, **176**, no. 1, (2014), 31–44

P21 - Smith G., Compactness results for immersions of prescribed Gaussian curvature II - geometric aspects, *Geom. Dedicata*, **172**, no. 1, (2014), 303–350
P22 - Clarke A., Smith G., The Perron Method and the Non-Linear Plateau problem, *Geom. Dedicata*, **163**, no. 1, (2013), 159–165
P23 - Smith G., The non-linear Plateau problem in non-positively curved manifolds, *Trans. Amer. Math. Soc.*, **365**, (2013), 1109–1124
P24 - Smith G., Special Lagrangian curvature, *Math. Annalen*, **335**, no. 1, (2013), 57–95
P25 - Smith G., Compactness results for immersions of prescribed Gaussian curvature I - analytic aspects, *Adv. Math.*, **229**, (2012), 731–769
P26 - Smith G., Moduli of Flat Conformal Structures of Hyperbolic Type, *Geom. Dedicata*, **154**, no. 1, (2011), 47–80
P27 - Smith G., Equivariant Plateau problems, *Geom. Dedicata*, **140**, no. 1, (2009), 95–135
P28 - Smith G., An Arzela-Ascoli Theorem for Immersed Submanifolds, *Ann. Fac. Sci. Toulouse Math.*, **16**, no. 4, (2007), 817–866
P29 - Smith G., Problmes de Plateau equivariants, *Sémin. Théor. Spectr. Géom.*, **24**, (2007), 67–78
P30 - Smith G., Pointed k-surfaces, *Bull. Soc. Math. France*, **134**, no. 4, (2006), 509–557

F.2 - Translations.

T1 - *Tradução de alemão para inglês de* Hüber A., Zum potentialtheoretischen Aspekt der Alexandrowschen Flächentheorie, *Comm. Math. Helv.*, **34**, 99–126, (1960)

F.3 - Completed papers submitted for review.

S1 - Smith G, On the asymptotic Plateau problem in Cartan-Hadamard manifolds, arXiv:2107.14670
S2 - Magaño C., Smith G, On eternal mean curvature flows of tori in perturbations of the unit sphere, arXiv:2004.00054
S3 - Smith G., On the asymptotic geometry of finite-type k-surfaces in three-dimensional hyperbolic space, arXiv:1908.04834
S4 - Jiménez-Grande A., Smith G., On embedded minimal surfaces of Costa-Hoffman-Meeks type in hyperbolic space, arXiv:1805.12194
S5 - Smith G., On complete embedded translating solitons of the mean curvature flow that are of finite genus, arXiv:1501.04149

F.4 - Completed papers awaiting revision.

Although I am satisfied with the content of the following papers, I find that their presentation requires a complete revision.

R1 - Smith G., Eternal forced mean curvature flows III - Morse homology, arXiv:1601.03437
R2 - Smith G., Constant curvature hyperspheres and the Euler Characteristic, arXiv:1103.3235

F.5 - Others.

O1 - On groups of diffeomorphisms of Hölder type,
http://im.ufrj.br/~moriarty/fragments/diffeomorphism_groups_170719.pdf
O2 - On the Hausdorff property of the Cheeger-Gromov topology,
http://im.ufrj.br/~moriarty/fragments/cheeger_gromov_171118.pdf
O3 - On the Rauch and Topogonov comparison theorems,
http://im.ufrj.br/~moriarty/fragments/rauch_180306.pdf
O4 - The non-linear Dirichlet problem in Hadamard manifolds, arXiv:0908.3590
O5 - Finite area and volume of pointed k-surfaces, arXiv:0709.0393
O6 - A Brief Note on Foliations of Constant Gaussian Curvature, arXiv:0802.2202
O7 - A Brief Note on Special Lagrangian Submanifolds in Euclidean Space,
https://www.mis.mpg.de/publications/preprints/2007/prepr2007-80.html

G - Qualifications.

G.1 - Titles and diplomas.

1 - Habilitiation à diriger les recherches, Université Grenoble-Alpes, France, 08/02/2017
2 - PhD, supervised by François Labourie, Université Paris XI, France, 13/12/2004
3 - MA Mathematics, University of Cambridge, UK, 2002
4 - DEA (Masters II), Université Paris XI, France, 2001
5 - Certificate of Advanced Studies in Mathematics (Masters I), University of Cambridge, UK, 1999
6 - BA Mathematics, University of Cambridge, UK, 1998

G.2 - Scientific visits and postdoctoral fellowships.

1 - 6 month scientific visit, l'Institute des Hautes Études Scientifiques (IHES), Paris, France, 03/01/2022–30/06/2022
2 - Postdoc, IMPA, Rio de Janeiro, Brasil, 10/2012–02/2013
3 - Marie Curie Postdoctoral Fellow, Centre de Recerca Matemàtica, Barcelona, Spain, 10/2010–12/2012
4 - Postdoc, IMPA, Rio de Janeiro, Brasil, 03/2010-09/2010
5 - Visiting professer, Departament de Matemàtiques, Universitat Autònoma de Barcelona, Barcelona, Spain, 11/2009–02/2010
6 - Postdoc, Centre de Recerca Matemàtica, Barcelona, Spain, 10/2008–09/2009
7 - Postdoc, Max Planck Institute for Mathematics, Bonn, Germany, 10/2007–09/2008
8 - Postdoc, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany, 10/2005–09/2007

H - Bibliography.

[1] Andrews B., Contraction of convex hypersurfaces in Euclidean space, *Calc. Var. Partial Differential Equations*, **2**, (1994), no. 2, 151–171
[2] Barbot T., Béguin F., Zeghib A., Prescribing Gauss curvature of surfaces in 3-dimensional spacetimes, Application to the Minkowski problem in the Minkowski space, *Ann. Inst. Fourier*, **61**, no. 2, (2011), 511–591
[3] Bonsante F., Mondello G., Schlenker J. M., A cyclic extension of the earthquake flow I., *Geom. Topol.*, **17**, no. 1, (2013), 157–234
[4] Bonsante F., Mondello G., Schlenker J. M., A cyclic extension of the earthquake flow II., *Ann. Sci. Ec. Norm. Supér.*, **48**, no. 4, (2015), 811–859
[5] Caffarelli L., Nirenberg L., Spruck J., Nonlinear second-order elliptic equations. V. The Dirichlet problem for Weingarten hypersurfaces, *Comm. Pure Appl. Math.* **41** (1988), no. 1, 47–70
[6] Calegari D., Marques F. C., Neves A., Counting minimal surfaces in negatively curved 3-manifolds, to appear in *Duke Math. Journ.*
[7] Dávila J., Nguyen X. H., Del Pino M., Finite topology self-translating surfaces for the mean curvature flow in $\mathbb{R}^3$, *Adv. Math.*, **320**, 674–729
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[12] Fraser A., Extremal eigenvalue problems and free boundary minimal surfaces in the ball, in **Geometric Analysis**, Lecture Notes in Mathematics, 1–40, Springer/Fondatione C.I.M.E., (2020)
[13] Gromov M. L., Pseudo holomorphic curves in symplectic manifolds, *Inventiones Mathematicae*, **82** (1985), 82: 307–347
[14] Guan B., Spruck J., The existence of hypersurfaces of constant Gauss curvature with prescribed boundary, *J. Differential Geom.* **62** (2002), no. 2, 259–287
[15] Guan B., Spruck J., Locally convex hypersurfaces of constant curvature with boundary, *Comm. Pure Appl. Math.* **57** (2004), no. 10, 1311–1331

[16] Guan B., Spruck J., Hypersurfaces of constant curvature in hyperbolic space. II., *J. Eur. Math. Soc. (JEMS)* **12** (2010), no. 3, 797–817

[17] Harvey F. R., Lawson H. B., Pseudoconvexity for the special Lagrangian potential equation, *Calc. Var. PDEs.*, **60**, (2021)

[18] Hauswirth L., Pacard F., Minimal surfaces of finite genus with two limit ends, *Invent. Math.*, **169**, no. 3, (2007), 569–620

[19] Labourie F., Immersions isométriques elliptiques et courbes pseudoholomorphes, *J. Diff. Geom.*, **30**, (1989), 395–424

[20] Labourie F., Problème de Minkowski et surfaces à courbure constante dans les variétés hyperboliques, *Bull. Soc. Math. Fr.*, **119**, (1991), 307–325

[21] Labourie F., Métriques prescrites sur le bord des variétés hyperboliques de dimension 3, *Journal of Diff. Geo.* **35**, 609–626 (1992)

[22] Labourie F., Problèmes de Monge-Ampère, courbes pseudo-holomorphes et laminations, *G.A.F.A.*, **7**, (1997), 496–534

[23] Labourie F., Un lemme de Morse pour les surfaces convexes (French), *Invent. Math.* **141** (2000), no. 2, 239–297

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[35] Trudinger N. S., Wang X., On locally locally convex hypersurfaces with boundary, *J. Reine Angew. Math.* **551** (2002), 11–32

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