Gravity coupled to the standard model, the spectral model

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Simplest example

$$\mathcal{A} = M_n(C^\infty(M))$$

algebra of $n \times n$ matrices of smooth functions on a manifold M.

$$1 \to \mathsf{Int}(\mathcal{A}) \to \mathsf{Aut}(\mathcal{A}) \to \mathsf{Out}(\mathcal{A}) \to 1$$

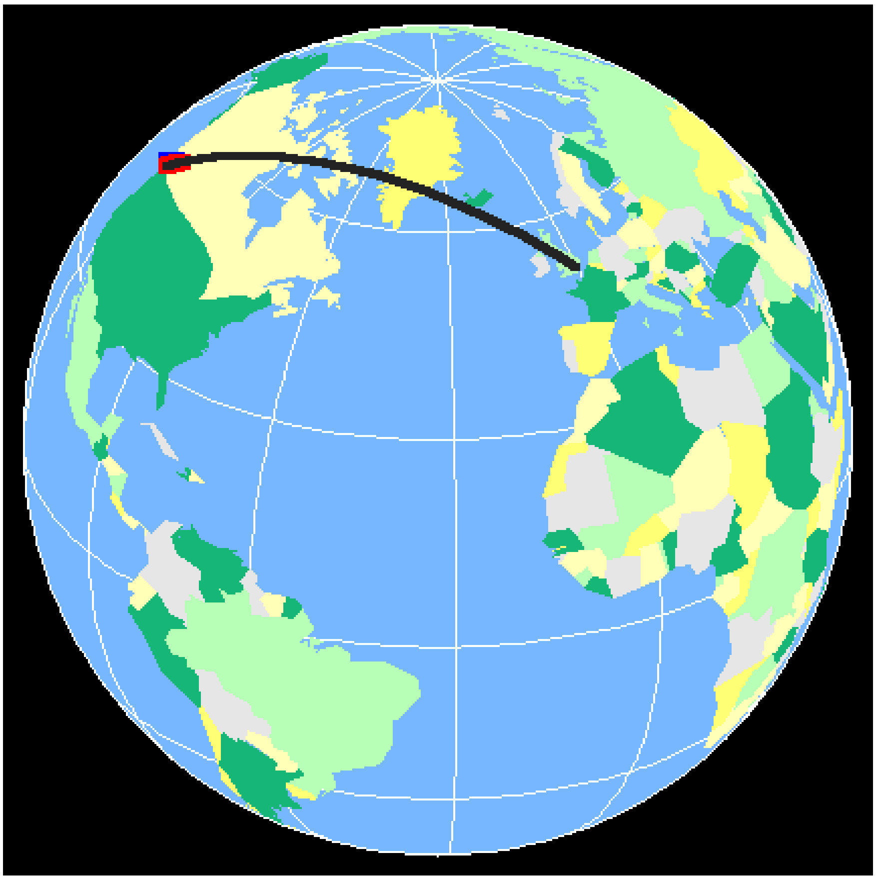
$$1 \to \mathsf{Map}(M, \mathsf{SU}(n)) \to \mathcal{G} \to \mathsf{Diff}(M) \to 1.$$

Gravity + Yang-Mills

We have shown that the study of pure gravity on this space yields Einstein gravity on M minimally coupled with Yang-Mills theory for the gauge group SU(n). The Yang-Mills gauge potential appears as the inner part of the metric, in the same way as the group of gauge transformations (for the gauge group SU(n)) appears as the group of inner diffeomorphisms.

What is a metric in spectral geometry

$$d(a, b) = \text{Inf} \int_\gamma \sqrt{g_{\mu\,\nu}\, dx^\mu\, dx^\nu}$$



Dirac's square root of the Laplacian

Spectral Geometry

$$d(a,b) = \text{Sup}\,\{|f(a)-f(b)|\,;\; f \in \mathcal{A}\,,\; \|[D,f]\| \leq 1\,\}$$

Manifold $\leftrightarrow$ Poincaré duality in KO-homology

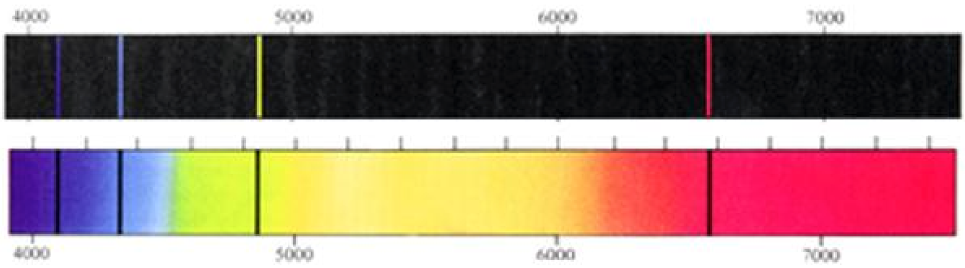
$$(\mathcal{A}, \mathcal{H}, D)\,, \quad ds = D^{-1}\,,\; J,\; \gamma$$

$$J^2 = \varepsilon\,,\; DJ = \varepsilon' JD, \quad J\,\gamma = \varepsilon''\gamma J, \quad D\gamma = -\gamma D$$

n	0	1	2	3	4	5	6	7
ε	1	1	-1	-1	-1	-1	1	1
ε'	1	-1	1	1	1	-1	1	1
ε''	1		-1		1		-1	

In physics

- $\mathcal{H}$: one particle Euclidean Fermions
- D : inverse propagator
- J : charge conjugation
- γ : chirality



Meter $\rightarrow$ Wave length (Krypton (1967) spectrum of 86Kr then Caesium (1984) hyperfine levels of C133)

Product : continuum $\times$ finite

Starting with an ordinary spin geometry M of dimension n and taking the product $M \times F$, one obtains a space whose metric dimension is still n but whose KO-dimension is the sum of n with the KO-dimension of F.

As it turns out the Standard Model with neutrino mixing favors the shift of dimension from the 4 of our familiar space-time picture to $10 = 4 + 6 = 2$ modulo 8.

Finite spaces F of given KO-dimension

We classified the irreducible $(\mathcal{A}, \mathcal{H}, J)$

$$[a, b^0] = 0\,,\ b^0 = J b^* J^{-1}$$

Two cases :

1. The center $Z(\mathcal{A}_{\mathbb{C}})$ is $\mathbb{C}$, then

$$\mathcal{A}_{\mathbb{C}} = M_k(\mathbb{C})$$

2. The center $Z(\mathcal{A}_{\mathbb{C}})$ is $\mathbb{C} \oplus \mathbb{C}$ then

$$\mathcal{A}_{\mathbb{C}} = M_k(\mathbb{C}) \oplus M_k(\mathbb{C})$$

KO-dimension 6 $\Rightarrow$ case 2.

A dress for the beggar

If one assumes that one is in the "symplectic–unitary" case and that the grading γ is given by a grading of the vector space over $\mathbb{H}$, one can show that the dimension of $\mathcal{H}$ is at least 2×16 while the simplest solution is given by the algebra $\mathcal{A} = M_2(\mathbb{H}) \oplus M_4(\mathbb{C})$.

The center $Z(\mathcal{A}_{\mathbb{C}})$ is $\mathbb{C} \oplus \mathbb{C}$, and the product of this finite geometry F by a manifold M appears, from the commutative standpoint, as two distinct copies of M.

Further breaking

We showed that requiring that these two copies of M stay a finite distance apart reduces the symmetries from the group SU(2)×SU(2)× SU(4) of inner automorphisms to the symmetries $U(1) \times \mathrm{SU}(2) \times \mathrm{SU}(3)$ of the Standard Model. This reduction of the gauge symmetry occurs because of the second kinematical condition $[[D,a],b] = 0$ which in the general case becomes :

$$[[D,a],b^0] = 0\,, \quad \forall\, a,b \in \mathcal{A}$$

Spectral Model

Let M be a Riemannian spin 4-manifold and F the finite noncommutative geometry of KO-dimension 6 described above. Let $M \times F$ be endowed with the product metric.

1. The unimodular subgroup of the unitary group acting by the adjoint representation Ad(u) in $\mathcal{H}$ is the group of gauge transformations of SM.
2. The unimodular inner fluctuations of the metric give the gauge bosons of SM.
3. The full standard model (with neutrino mixing and seesaw mechanism) minimally coupled to Einstein gravity is given in Euclidean form by the action functional

   $$S = \operatorname{Tr}(f(D_A/\Lambda)) + \frac{1}{2}\langle\, J\,\tilde{\xi}, D_A\,\tilde{\xi}\rangle\,, \quad \tilde{\xi} \in \mathcal{H}^+_{cl},$$

   where D_A is the Dirac operator with the unimodular inner fluctuations.

Standard Model	Spectral Action
Higgs Boson	Inner metric$^{(0,1)}$
Gauge bosons	Inner metric$^{(1,0)}$
Fermion masses u, ν	Dirac$^{(0,1)}$ in $\uparrow$
CKM matrix Masses down	Dirac$^{(0,1)}$ in $(\downarrow 3)$
Lepton mixing Masses leptons e	Dirac$^{(0,1)}$ in $(\downarrow 1)$
Majorana mass matrix	Dirac$^{(0,1)}$ on $E_R \oplus J_F E_R$
Gauge couplings	Fixed at unification
Higgs scattering parameter	Fixed at unification
Tadpole constant	$-\mu_0^2 \, \lvert\mathbf{H}\rvert^2$

The bosonic action takes the form

$$S = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^* R^*$$
$$+ \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$+ \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x$$

where

$$\frac{1}{\kappa_0^2} = \frac{96 f_2 \Lambda^2 - f_0 c}{12 \pi^2}$$
$$\mu_0^2 = 2 \frac{f_2 \Lambda^2}{f_0} - \frac{e}{a}$$
$$\alpha_0 = -\frac{3 f_0}{10 \pi^2}$$
$$\tau_0 = \frac{11 f_0}{60 \pi^2}$$
$$\gamma_0 = \frac{1}{\pi^2}(48 f_4 \Lambda^4 - f_2 \Lambda^2 c + \frac{f_0}{4} d)$$
$$\lambda_0 = \frac{\pi^2}{2 f_0} \frac{b}{a^2}$$
$$\xi_0 = \frac{1}{12}$$

Predictions

– Unification of couplings

$$\frac{g_3^2\, f_0}{2\pi^2} = \frac{1}{4}, \qquad g_3^2 = g_2^2 = \frac{5}{3}\, g_1^2\,.$$

– See-saw mechanism for neutrino masses with large $M_R \sim \Lambda$.

– The mass matrices satisfy the constraint at unification

$$\sum_\sigma (m_\nu^\sigma)^2 + (m_e^\sigma)^2 + 3\,(m_u^\sigma)^2 + 3\,(m_d^\sigma)^2 = \, 8\, M^2$$

$$Y_2 = \sum_\sigma (y_\nu^\sigma)^2 + (y_e^\sigma)^2 + 3\,(y_u^\sigma)^2 + 3\,(y_d^\sigma)^2$$

$$Y_2(S) = 4\, g^2.$$

This yields a value of the top mass which is 1.04 times the observed value when neglecting the yukawa couplings of the bottom quarks etc...and is hence compatible with experiment.

– The Higgs scattering parameter

$$\tilde{\lambda}(\Lambda) = g_3^2 \frac{b}{a^2}.$$

The numerical solution to the RG equations with the boundary value $\lambda_0 = 0.356$ at $\Lambda = 10^{17}$ GeV gives $\lambda(M_Z) \sim 0.241$ and a Higgs mass as in the predictions of the big desert.

– Newton constant ($f_2 \sim 5 f_0$)

– No proton decay