Particle life time in De Sitter Space

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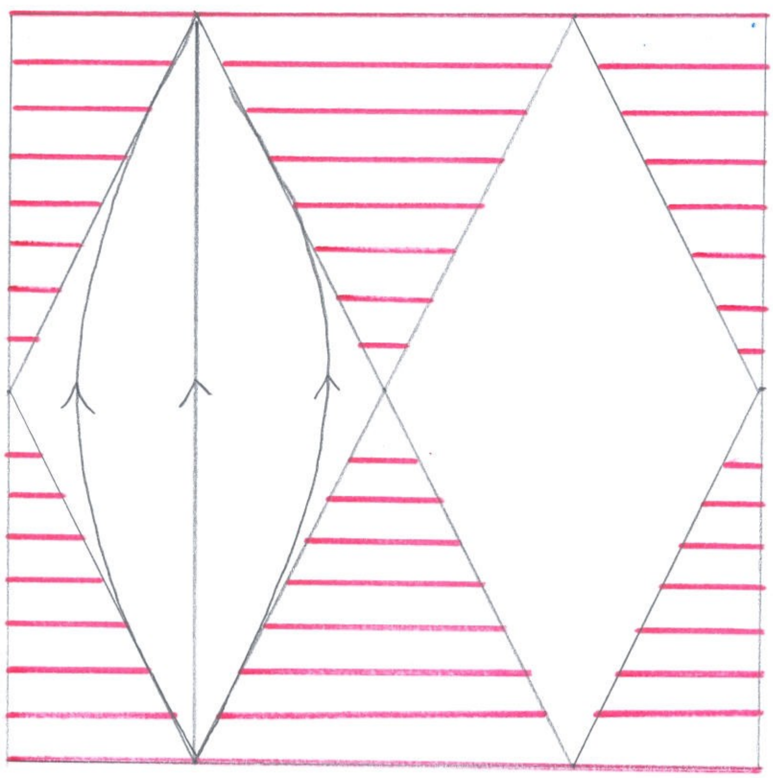
CMP and in preparation.

DE SITTER

Sphere $-T^2 + X^2 = R^2$

No global Killing vector

$$\begin{pmatrix} 0 \\ \cos\theta \\ \sin\theta \end{pmatrix} \longrightarrow \begin{pmatrix} \operatorname{sh} t \cos\theta \\ \operatorname{ch} t \cos\theta \\ \sin\theta \end{pmatrix}$$

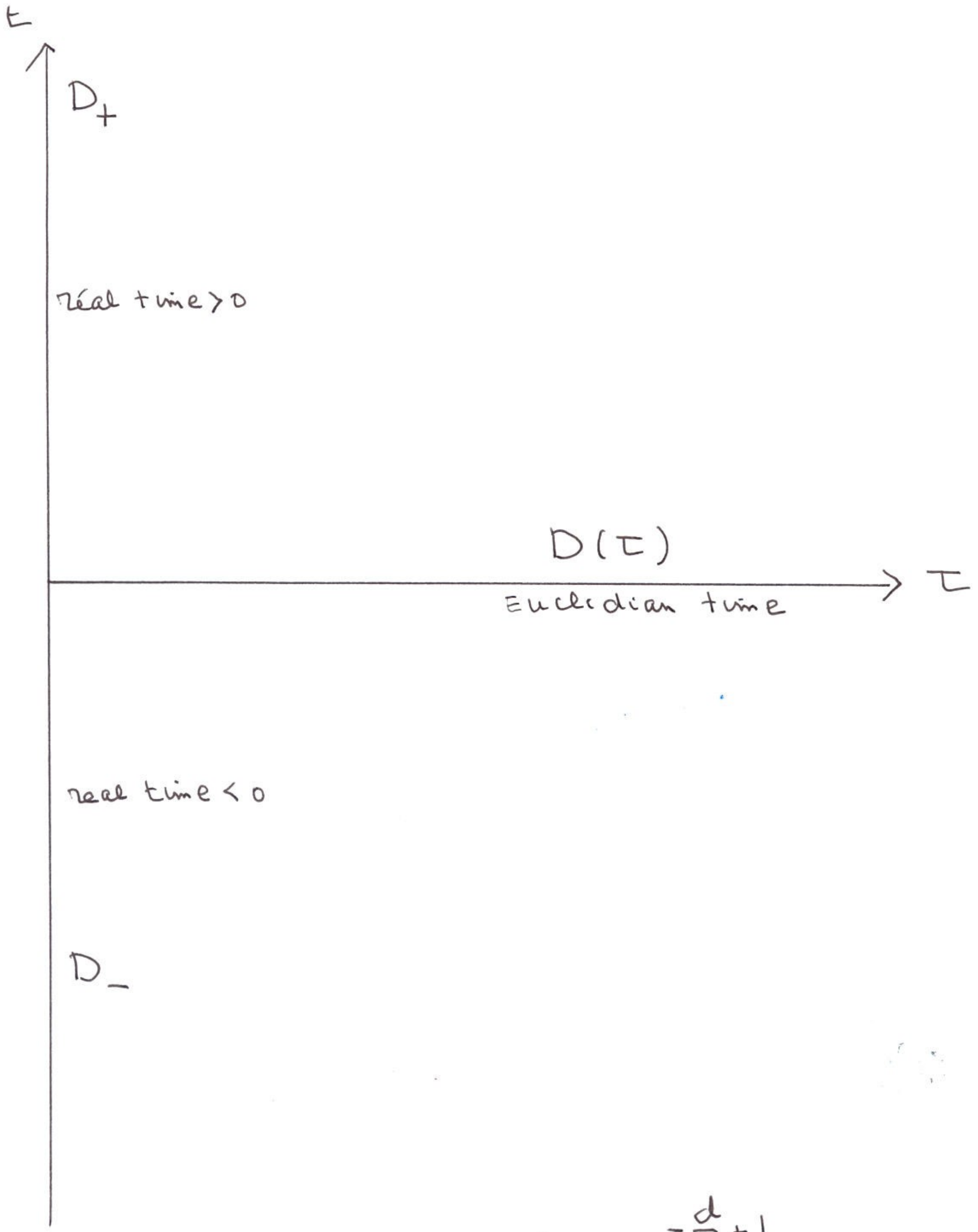


Whightman Schwinger functions

Minkowski space

$$\Box_x D(x, x') = 0$$

$$D\left(\sqrt{(x-x')^2}\right)$$

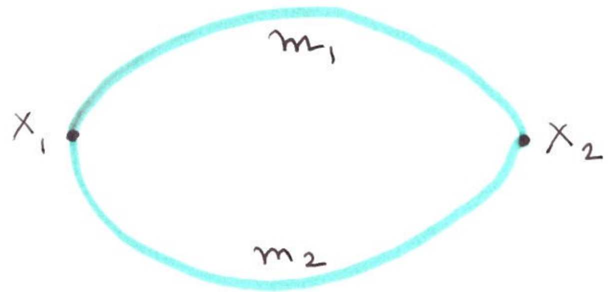


$$D_m(\tau) = (m\tau)^{-\frac{d}{2}+1} K_{\frac{d}{2}-1}(m\tau)$$

Källen - Lehmann

$$S^+ S = 1 \quad , \quad S = 1 + i\,T$$

$$i\,(T - T^+) = -\,T^+ T$$



$$D_{m_1}\, D_{m_2} = \int \rho(m)\, D_m \, dm$$

Fermi Golden rule :

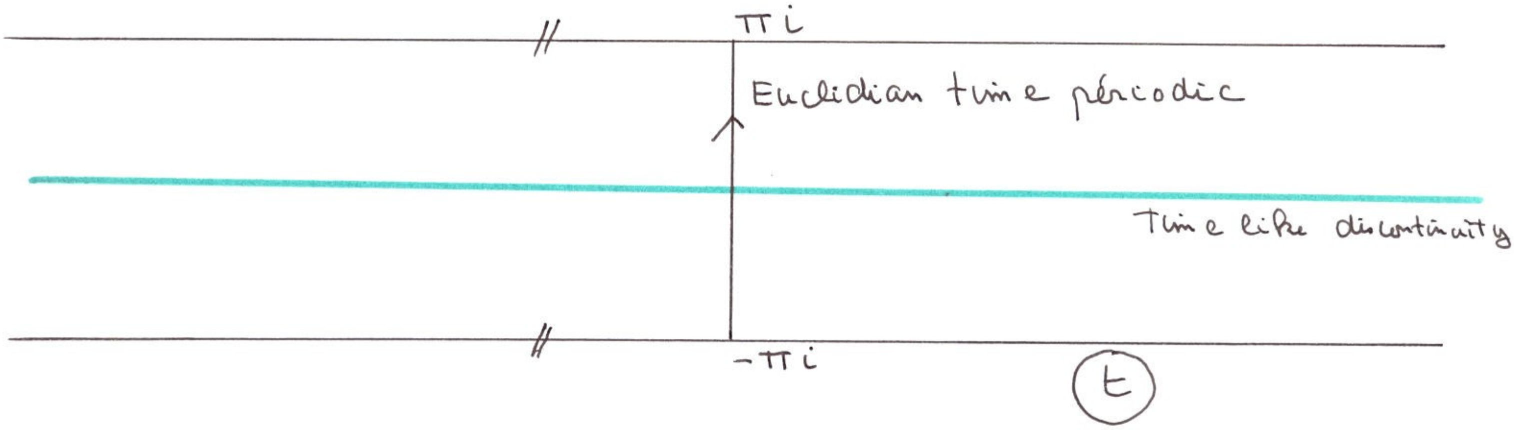
$$\rho = \int \delta_+(p^2 - m_1^2)\, \delta_+(q^2 - m_2^2)\, \delta^{(d)}(p + q - M)$$

$$= \left[\frac{\prod_{4t} (m \pm m_1 \pm m_2)}{m^2} \right]^{\frac{d-3}{2}} \Theta(m - m_1 - m_2)$$

De Sitter ?

$$D_n(t)$$

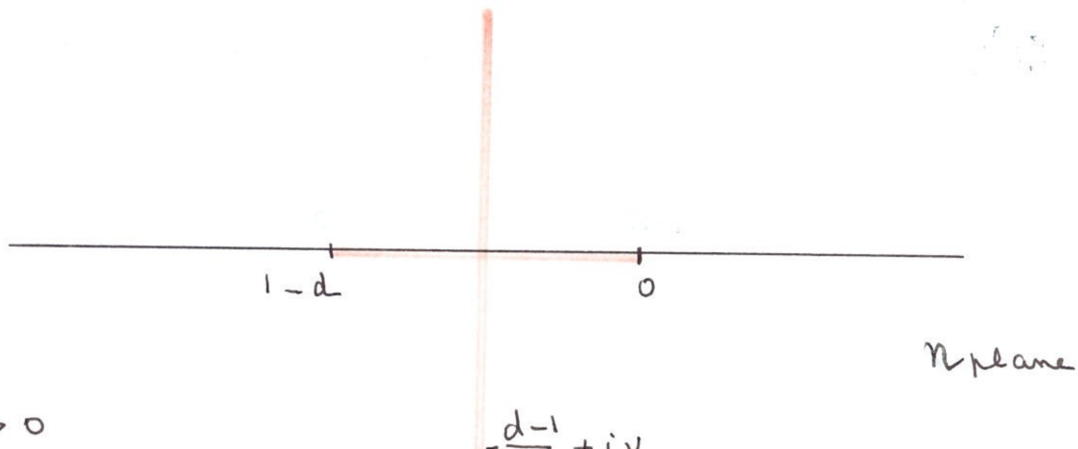
$$(x - x')^2 = \operatorname{ch} t - 1$$



$$D = P_n^{\left(\frac{d-1}{2}\right)}(-\operatorname{ch} t)$$

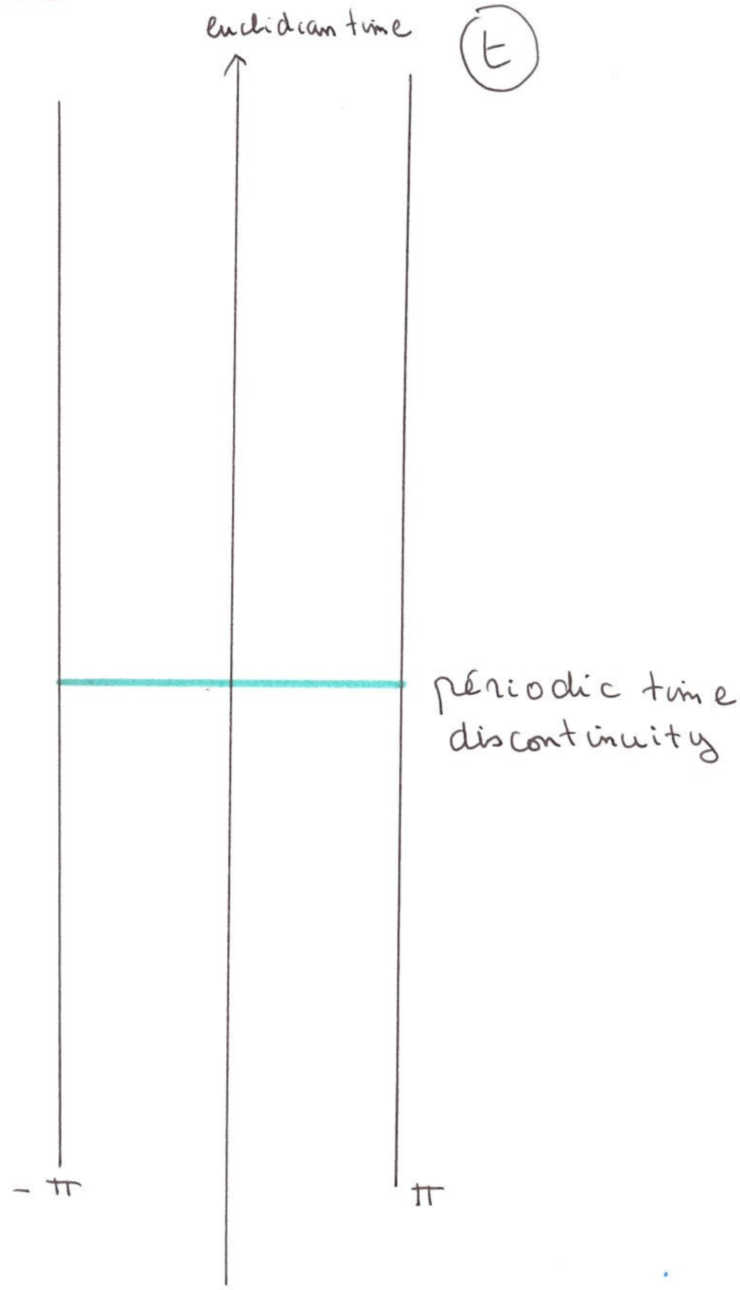
Periodicity in Euclidian time has <u>thermic interpretation</u> :

KMS condition.



$m^2 > 0$

AdS



$$(x - x')^2 = \cos t - 1$$

$$D_n = Q_n^{\left(\frac{d-1}{2}\right)}(\cos t)$$

Results

$$\lambda = \frac{d-1}{2}$$

$$\nu = n + \lambda, \quad \text{Imaginary}$$

$$W_\nu = \Gamma(\lambda+\nu)\,\Gamma(\lambda-\nu)\, F\left(\lambda+\nu, \lambda-\nu, \lambda+\tfrac{1}{2}, \tfrac{1-z}{2}\right)$$

$$\boxed{\begin{gathered} W_{\nu_1} W_{\nu_2} = \int_{i\mathbb{R}} d\nu\, \nu \sin(\pi\nu)\, \phi(\nu)\, \frac{W_\nu}{\Gamma(\lambda+\nu)\,\Gamma(\lambda-\nu)} \\ \phi(\nu) = \prod_{8 \text{ factors}} \Gamma(\lambda \pm \nu_1 \pm \nu_2 \pm \nu) \end{gathered}}$$

Formula giving the Källen Lehmann weight

Relation with Clebsh Gordan Coefficiants

Gegenbauer Polynomials are bi-invariant functions on the group $SO(d+1)$ $\left(SO(d)\backslash SO(d+1)/SO(d)\right)$ with respect to $SO(d)$

$$C_n^{\lambda} = \frac{\Gamma(n+2\lambda)}{\Gamma(n+1)\,\Gamma(2\lambda)}\, F_{\nu}$$

$$\nu = n+\lambda \quad \text{and} \quad n \in \mathbb{N}$$

$$\int_{-1}^{1} d\mu(x)\; C^{(\lambda)}_{n_1}(x)\; C^{(\lambda)}_{n_2}(x)\; C^{(\lambda)}_{n_3}(x) = C^{\lambda}_{n_1 n_2 n_3}$$

$C^{\lambda}_{n_1 n_2 n_3} \neq 0$ only if

$n_1+n_2+n_3$ is even and n_1, n_2, n_3 form a triangle. Szegö Orthogonal Polynomials.

$C^{\lambda}_{n_1 n_2 n_3}$ is a Clebsh gordan coefficiant

and K.L. is an extension of the result to the $SO(d,1)$ Unitary series

Intriguing Observation

(G.E. Volovick)

De Sitter violates the Lorentz invariance mass inequality:

$$M \longmapsto m_1 + m_2$$

$$M \geq m_1 + m_2$$

In the limit where the violation is small:

$$\Delta M \ll M$$

$$w \propto \exp\left(-\frac{\Delta M}{T}\right)$$

Thermal activation process with

$$T = \frac{\hbar H}{\pi} = 2\, T^{dS}$$

Idea for derivation

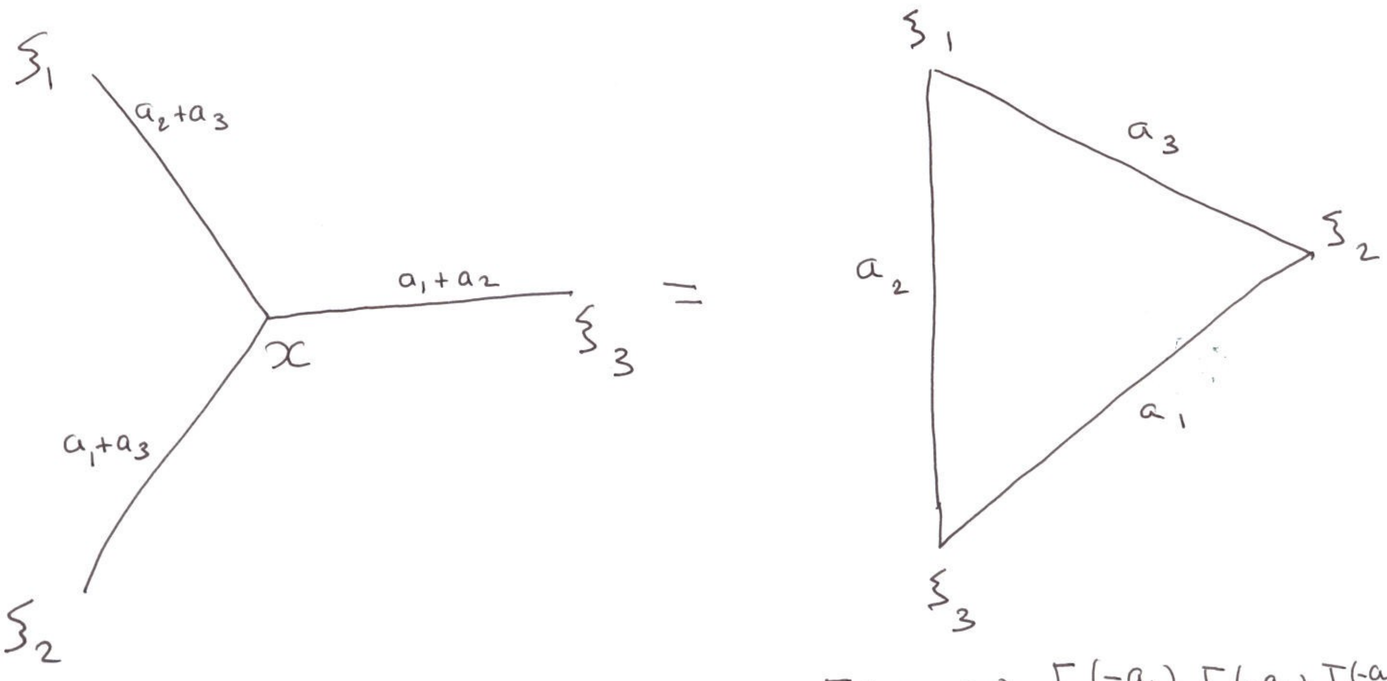
$$C_n^{(\lambda)}(x) = \int d\xi_1 \, (x \cdot \xi)^{-n}$$

ξ variable on the cône. $\xi^2 = 0$

$$\int_H C_{n_1} C_{n_2} C_{n_3} \, dx$$

$$= \int_{S^{d-1}} d\xi_1 \, d\xi_2 \, d\xi_3 \int_H dx \, (\xi_1 x)^{-n_1} (\xi_2 x)^{-n_2} (\xi_3 x)^{-n_3}$$

Integral over x

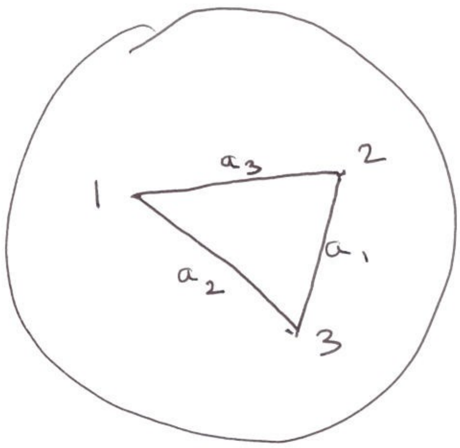


$$C(\lambda) \quad \Gamma(-a-\lambda) \frac{\Gamma(-a_1)\,\Gamma(-a_2)\,\Gamma(-a_3)}{\Gamma(-a_2-a_3)\,\Gamma(-a_1-a_3)\,\Gamma(-a_1 a_2)}$$

$$\int (\xi_1 \xi_2)^{a_3} (\xi_2 \xi_3)^{a_1} (\xi_1 \xi_3)^{a_2}$$

$$= \int \rho(r_1, r_2, r_3)\, r_{12}^{a_3}\, r_{23}^{a_1}\, r_{13}^{a_2}$$

Moments of the probability that 3 random points on the sphere S_{d-1} constitute a triangle with sides r_{12}, r_{23}, r_{13}.



$$= c(\lambda) \quad \Gamma(a + 2\lambda) \quad \frac{\Gamma(\lambda + a_1)\, \Gamma(\lambda + a_2)\, \Gamma(\lambda + a_3)}{\Gamma(2\lambda + a_2 + a_3)\, \Gamma(2\lambda + a_1 + a_3)\, \Gamma(2\lambda + a_1 + a_2)}$$

Conclusions

- De Sitter is very interesting to study for numerous reasons : Inflation, acceleration, ...

- No good notion of wave packets acceleration washes away all the structure, reason why it is interesting to study inflation.

- No killing vector makes Euclidian continuation difficult. How to use complex sphere to make predictions?

- Many connections with group theory Källen-lehmann weight = clebsch gordan coefficients, particles organize into representation of SO(d, 1) group.

- Is there any analogous of dS - guange correspondance?