Causality constraints on modifications to gravity

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On work with:

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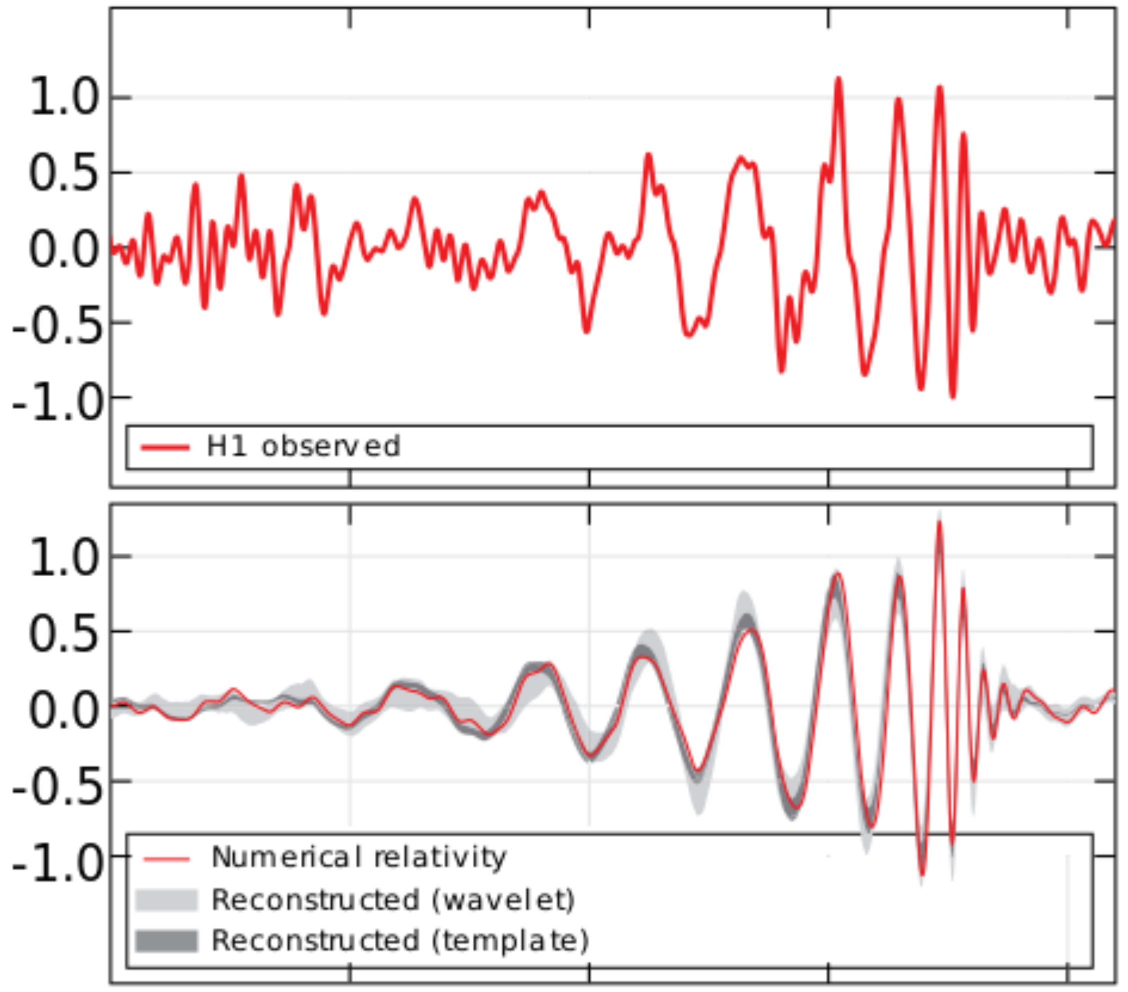
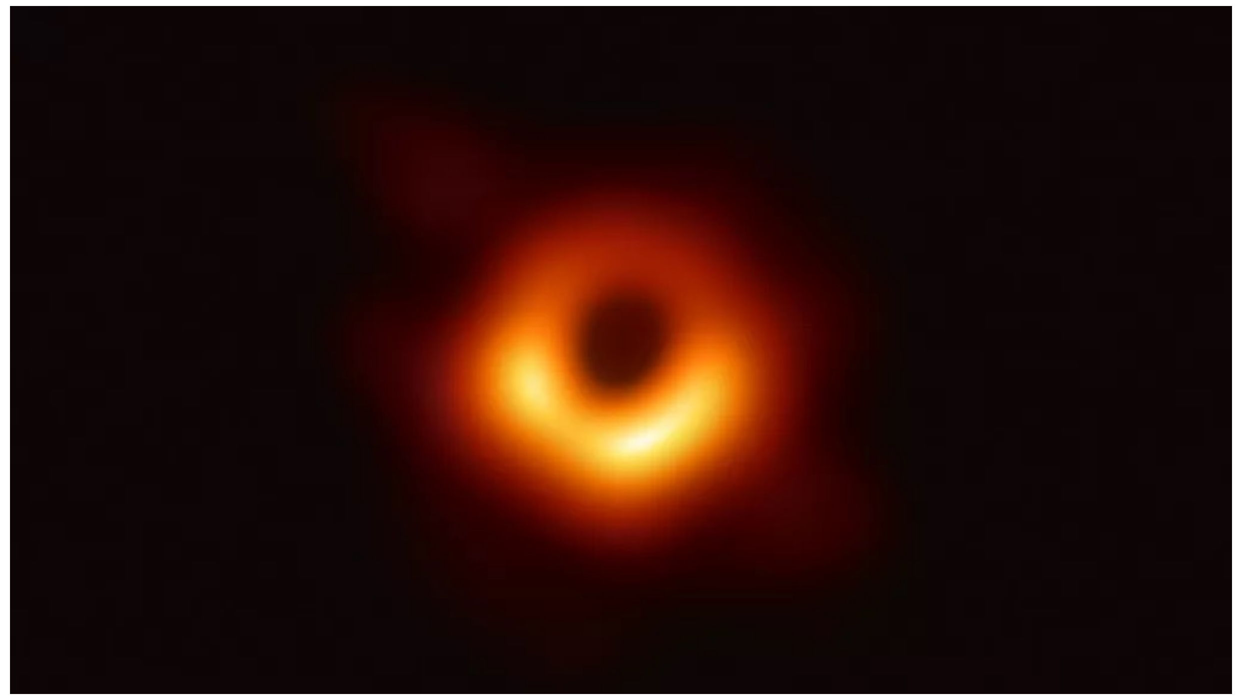
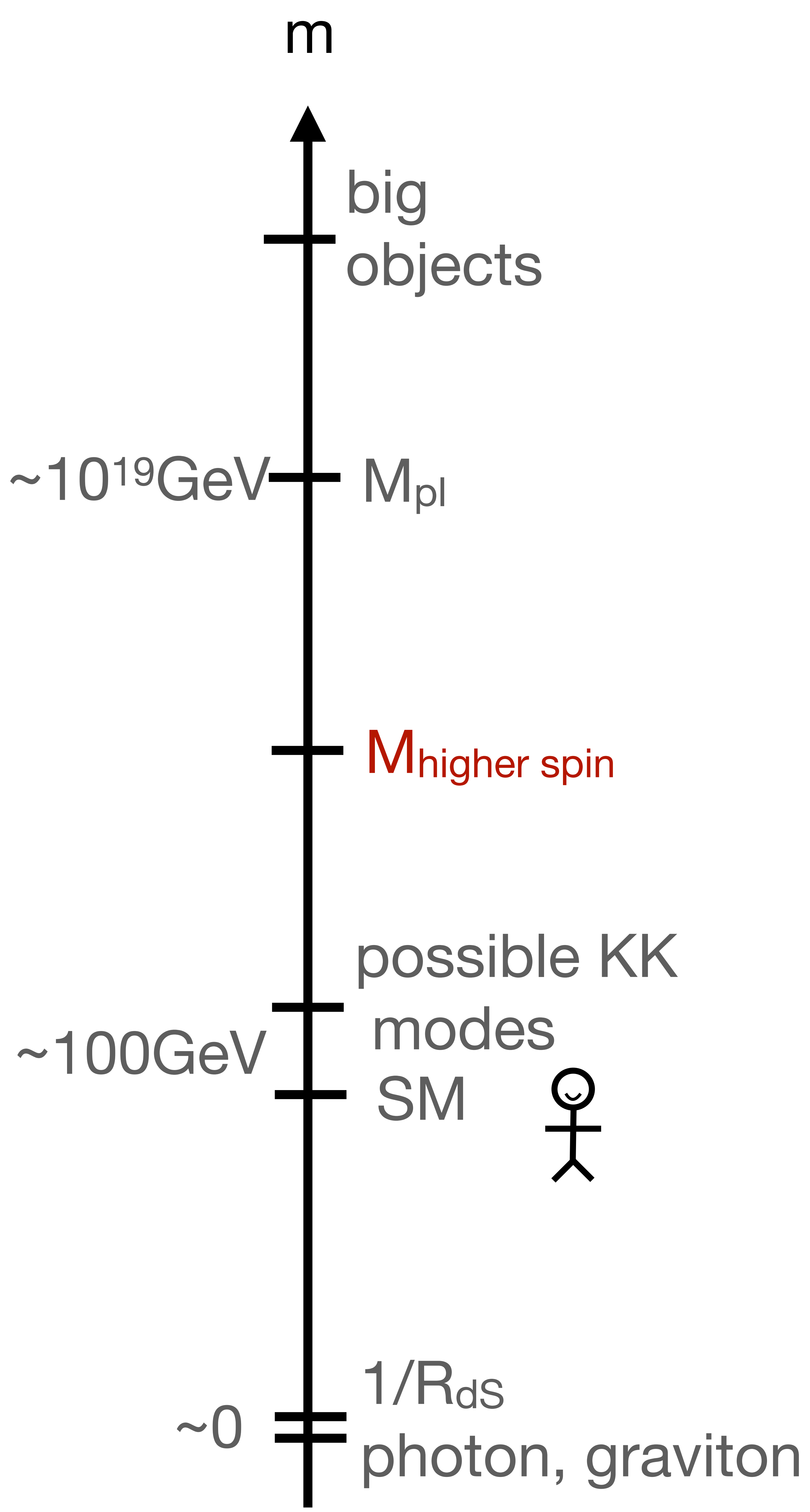
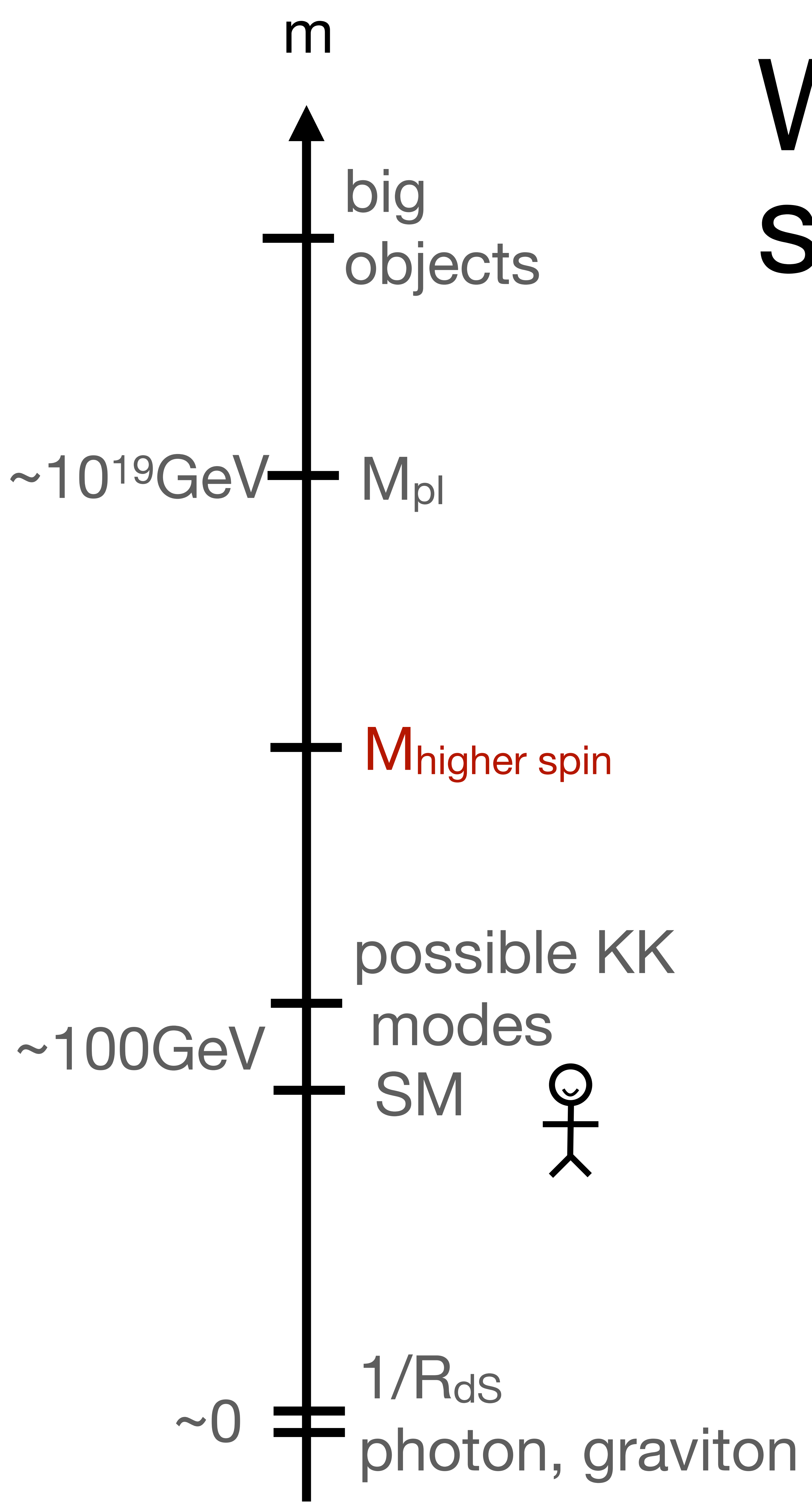
Einstein's General Relativity works well





How different could it be,
if world is causal+unitary ?


m
big objects
~10^19GeV
Mpl
Mhigher spin
possible KK modes
~100GeV
SM
1/RdS
~0
photon, graviton



We are interested in graviton-graviton scattering below $M_{\text{higher-spin}}$

$$S = \frac{1}{16\pi G}\left(R + g_3 \mathrm{Riem}^3 + g_4 \mathrm{Riem}^4 + \ldots\right) + \text{matter}$$

We'll look for effects suppressed by that scale:

$$g_3 \sim \frac{\#}{M^4_{\text{higher-spin}}} \gg \frac{1}{M^4_{\text{pl}}}, \quad g_4 \sim \frac{\#'}{M^6_{\text{higher-spin}}} \gg \frac{1}{M^6_{\text{pl}}}, \ldots$$

$\Rightarrow$ How large can #'s be ?

How small: [Guerrieri, Penedones& Vieira '21]

Outline

1. The question: What modifications can we bound?
   - Graviton scattering
   - causality+unitarity
   - CEMZ constraints

2. The method: Dispersive sum rules
   - axioms
   - scalar scattering
   - gravity

3. Results
   - what can hide from the SM?

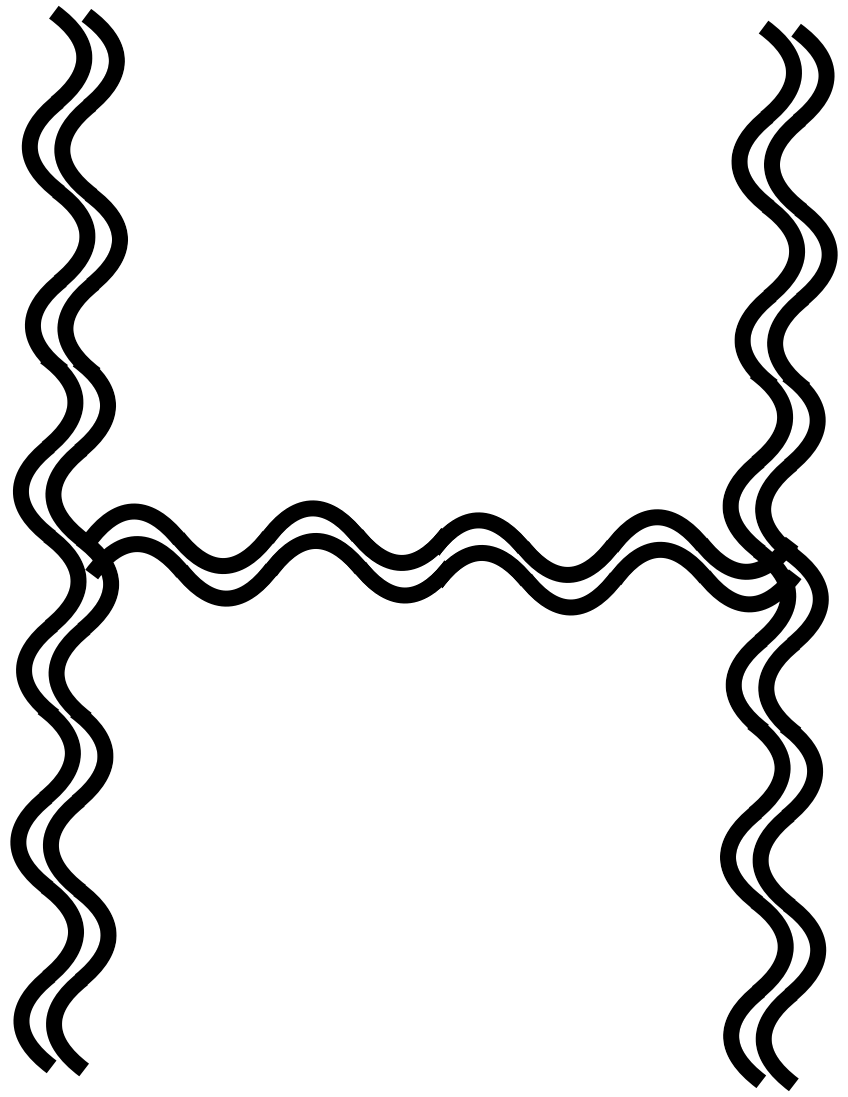
On:

SCH, Mazac,Rastelli& Simmons-Duffin '20	CFT
SCH& van Duong '20	Flat
SCH, Mazac,Rastelli& Simmons-Duffin '21	Flat
SCH, Mazac,Rastelli& Simmons-Duffin '21	CFT
SCH, Li, Parra Martinez& Simmons-Duffin '22	Flat

Low energy graviton scattering

$M_{\text{higher-spin}} << M_{pl}$: neglect loops.

3⁻⁻ 4⁺⁺



2⁻⁻ 1⁺⁺

scalar with ϕRiem²

$$M^{+--+} = [14]^4 \langle 23 \rangle^4 \times 8\pi G \left[\frac{1}{stu} + \frac{|g_3|^2 su}{4t} + \frac{|g_s|^2}{-t} + g_4 + g_5 t + \ldots \right]$$

Einstein!

Riem³ at vertices

contacts: Riem⁴ and derivatives

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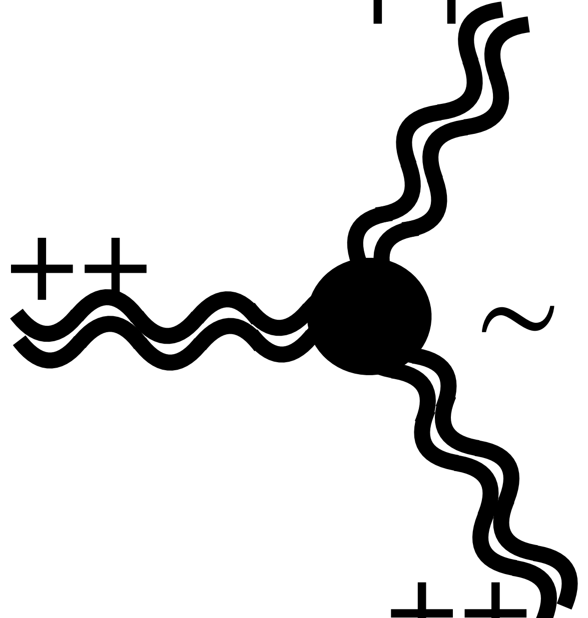


++

$$\sim \sqrt{8\pi G} g_3 [12]^2 [23]^2 [13]^2$$

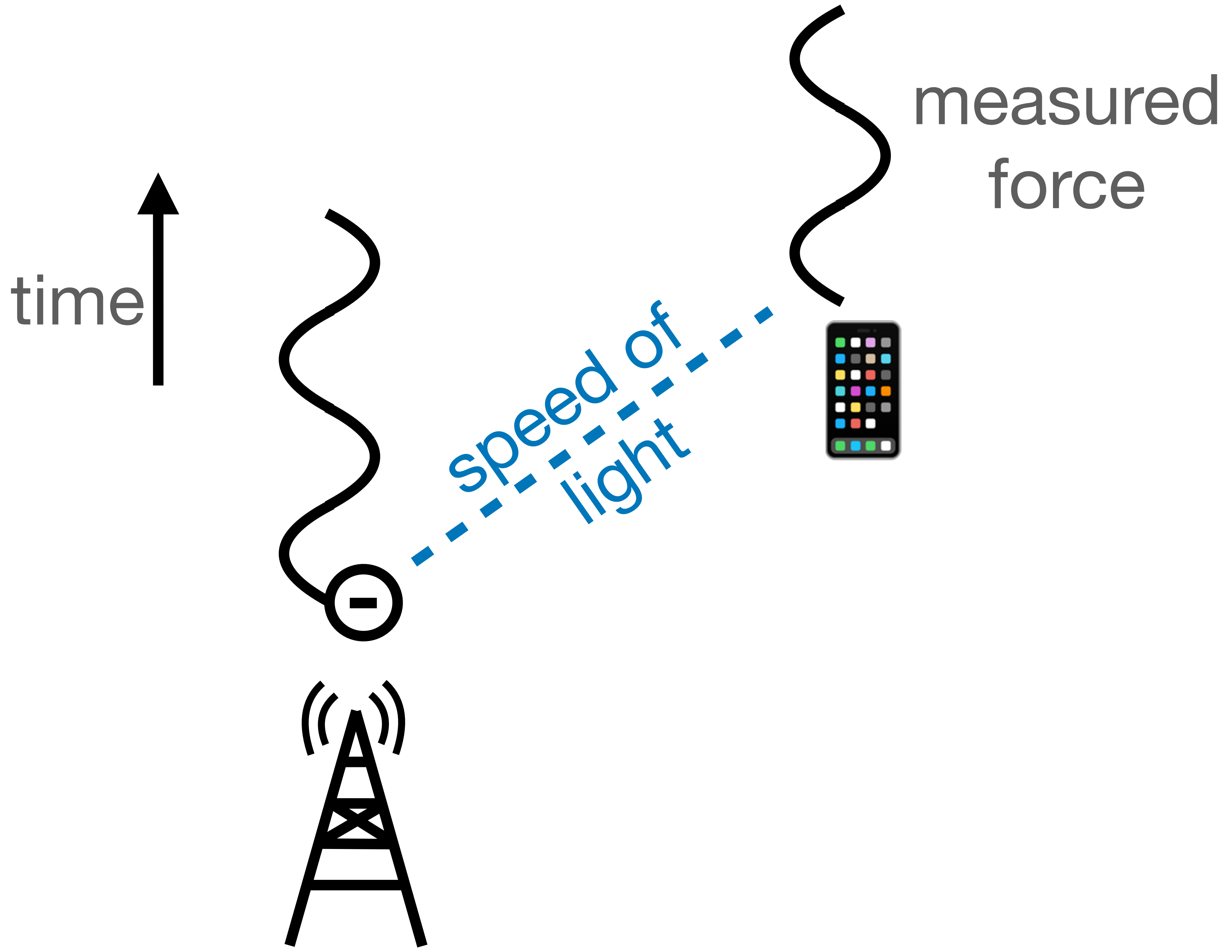
We don't bound:

- $f(R)$ ($\simeq$ Einstein + scalar field : no imprint in graviton scattering)
- Any term with Ricci tensor/scalar: removable by field redefinition (no imprint)
- Scalar potentials (don't grow with energy)
- Torsion etc: treat as extra matter fields / non-minimal couplings to matter

Causality

"signals can't travel faster than light"

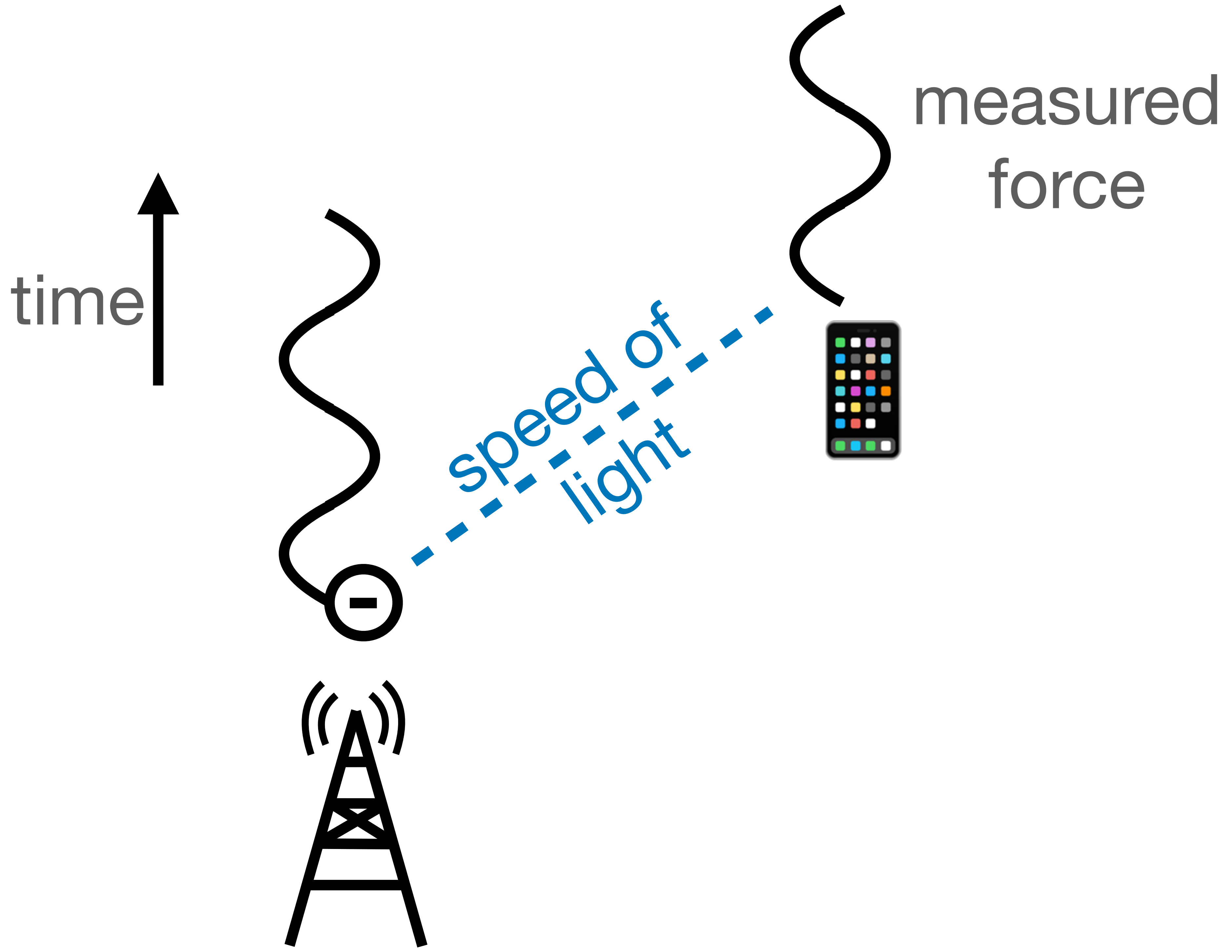
- Why waves, fields



Causality

"signals can't travel faster than light"

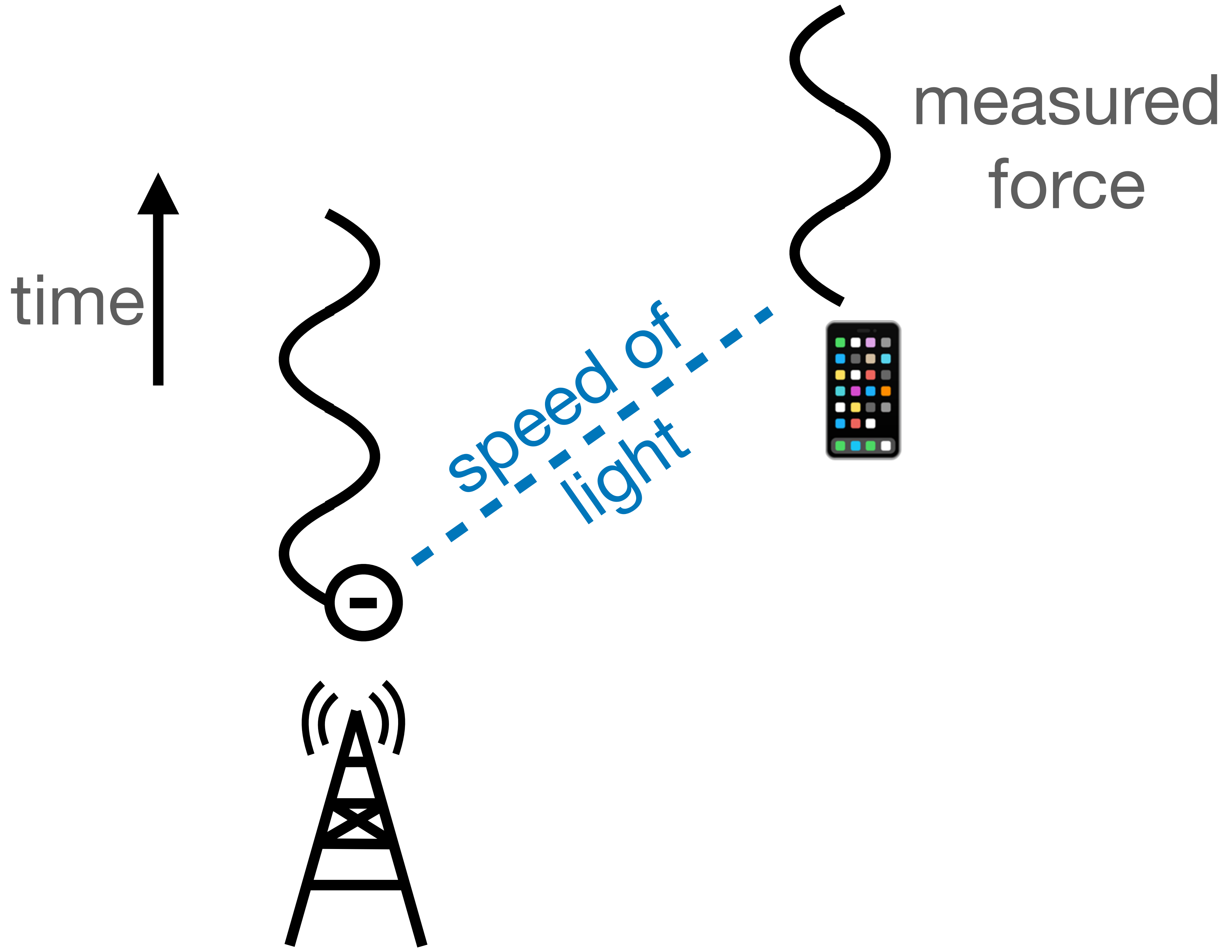
- Why waves, fields
- Why particles



Causality

"signals can't travel faster than light"

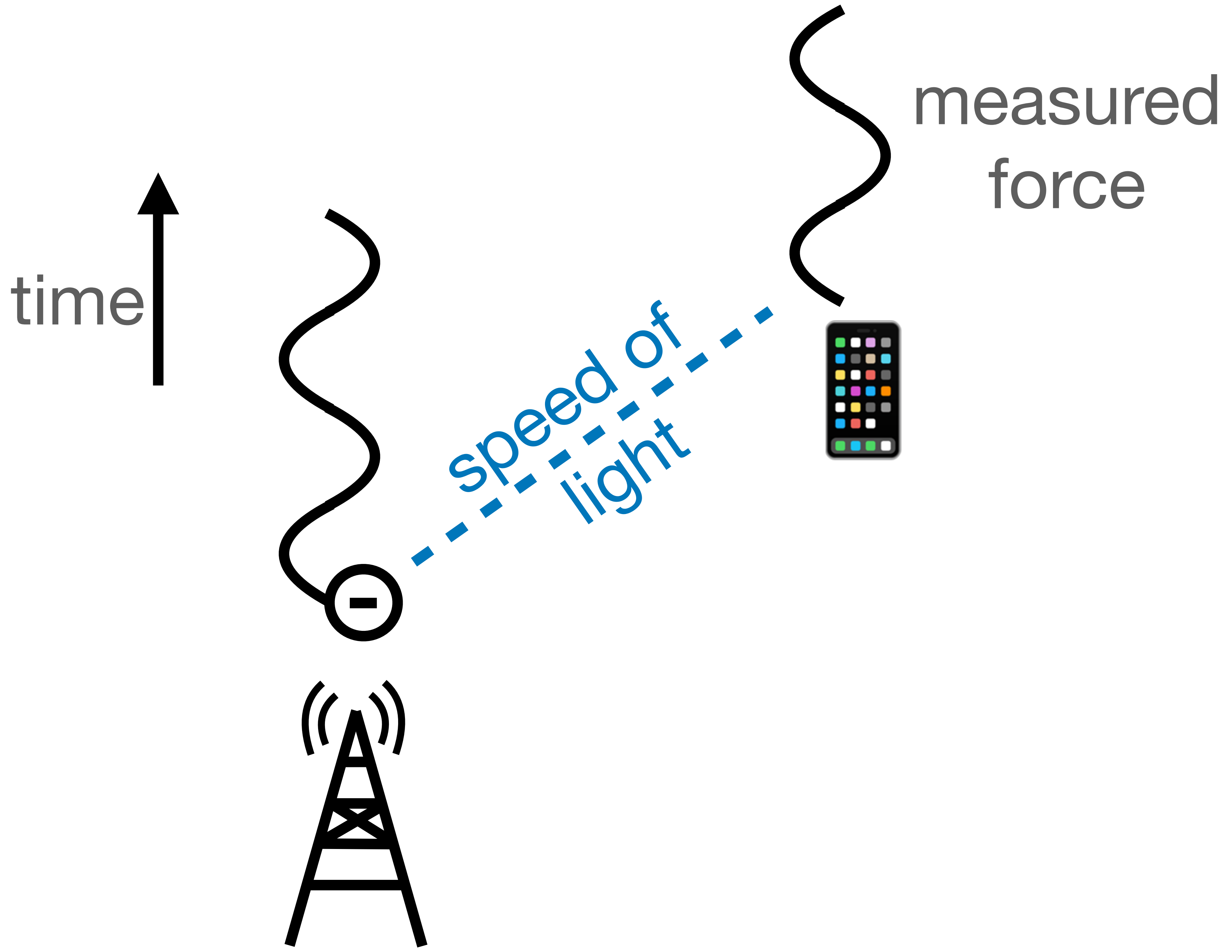
- Why waves, fields
- Why particles
- Why antiparticles



Causality

"signals can't travel faster than light"

- Why waves, fields
- Why particles
- Why antiparticles
- ...
- Why EFTs have to work
- Why gravity is attractive
- ...



- Experimentally tested to exquisite accuracy

  same limiting velocity for all species, antiparticle vs particle properties, ...

- Doesn't mean it's an exact property of Nature

  theoretically unknown how to define causality sharply for bulk observers, like us

- We assume 'relativistic causality' because we see no real alternative

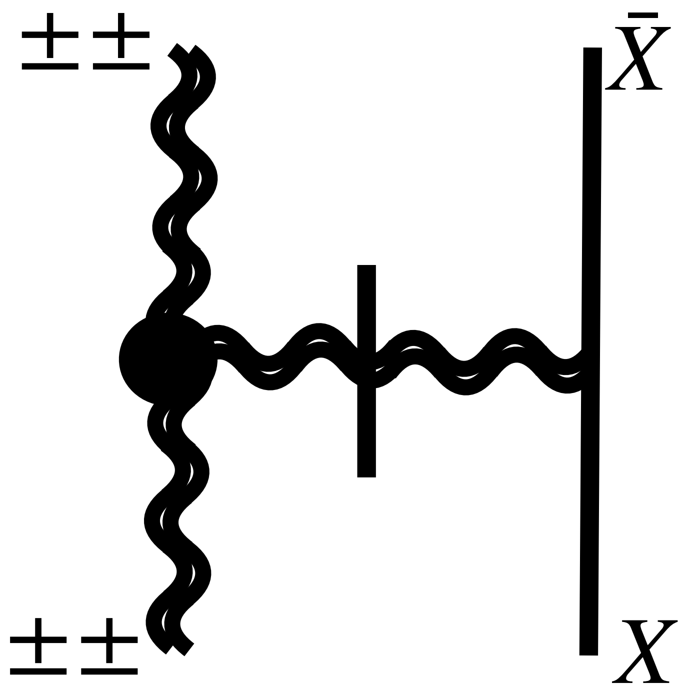
  simplest 'quantum gravity': $H = \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} - \sum_{i<j} V_{ij}, \quad V_{ij} = \frac{-Gm_im_j}{|\vec{x}_i - \vec{x}_j|} + ?$

Causality vs. gravity: some known results

0) at long distances, any Lorentz-invariant S-matrix of a massless spin-2 particle must agree with GR [Weinberg]

Causality vs. gravity: corrections?

1) Vertices: large impact-parameter scattering



$$M(s,b) \sim 8\pi G s \begin{pmatrix} \log(1/b\mu_{\rm IR}) & \frac{g_3}{b^4} \\ \frac{g_3}{b^4} & \log(1/b\mu_{\rm IR}) \end{pmatrix}$$

eigenvalue = time delay $\Rightarrow$ Causality requires low cutoff $\Rightarrow$ $|g_3| < \dfrac{\log}{b^4_{\rm min}} \sim \dfrac{\log}{M^4_{\rm higher-spin}}$

[Camanho,Edelstein,Maldacena& Zhiboedov '14]

2) Contacts: forward limit (!) of Kramers-Kronig dispersion relations

$$g_4 = \int_{M^2_{\rm heavy}}^{\infty} \frac{ds}{s}\, {\rm Im}\, f(s,t=0) \geq 0$$

[Bellazzini,Cheung& Remmen '15]
[Adams,Arkani-Hamed,Dubovsky,Nicolis&Rattazzi '06]

Our approach will merge these two.

Common complaints

and why we won't worry today

- We live in 4d. S-matrix doesn't exist. (IR divergences)

  ⇒divergent scattering phase has simple physical origin (Newton potential).
  if you can remove it, we'll just get stronger bounds!

- We live in an expanding universe. S-matrix doesn't exist.

  ⇒ Curvature / thermal effects? $H \ll M_{\text{higher-spin}}$ ....

  [de Rham, Grall, Melville, Jazayeri, Payer, Stefanyszyn; Dvali,…]

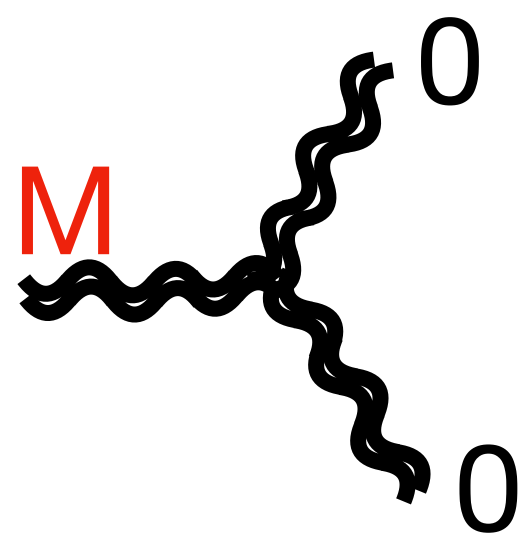
  ⇒ energy limit $E < M_{\text{pl}}^2/H \sim 10^{80}\text{GeV}$? I can't conceptualize its effects.

- Others?

Why spin≥4 states drive corrections to GR ?

Light spin 2’s are natural in Kaluza-Klein reductions.

But they don’t easily decay to massless gravitons.



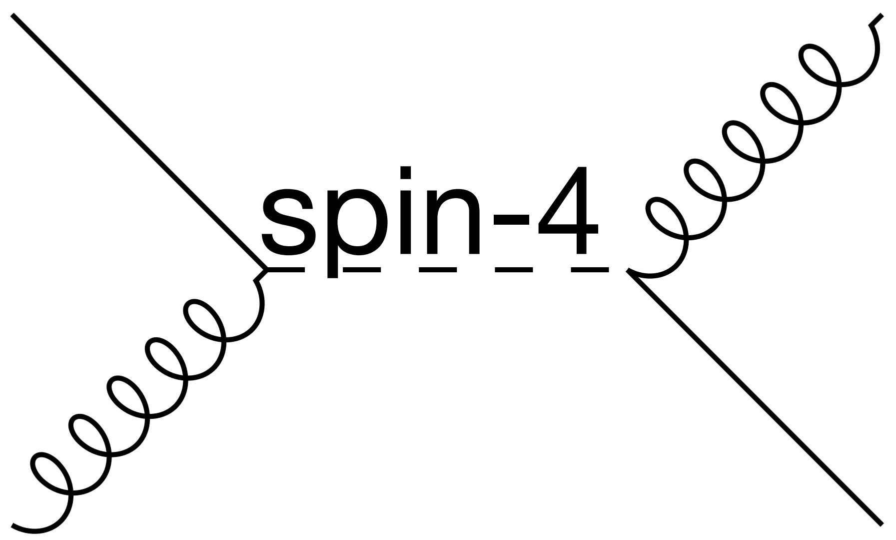
$\propto \int_M \sqrt{g}(1)(1)(\psi) \;\; = 0$ in GR by orthogonality of modes.

$\Rightarrow$ even if $M_{\rm KK} \ll M_{\rm higher-spin}$, anticipate couplings suppressed by $1/M^{\#}_{\rm higher-spin}$

$M_{\text{higher-spin}}$: What do we know?

1. LHC: string-like resonances M> ~7TeV

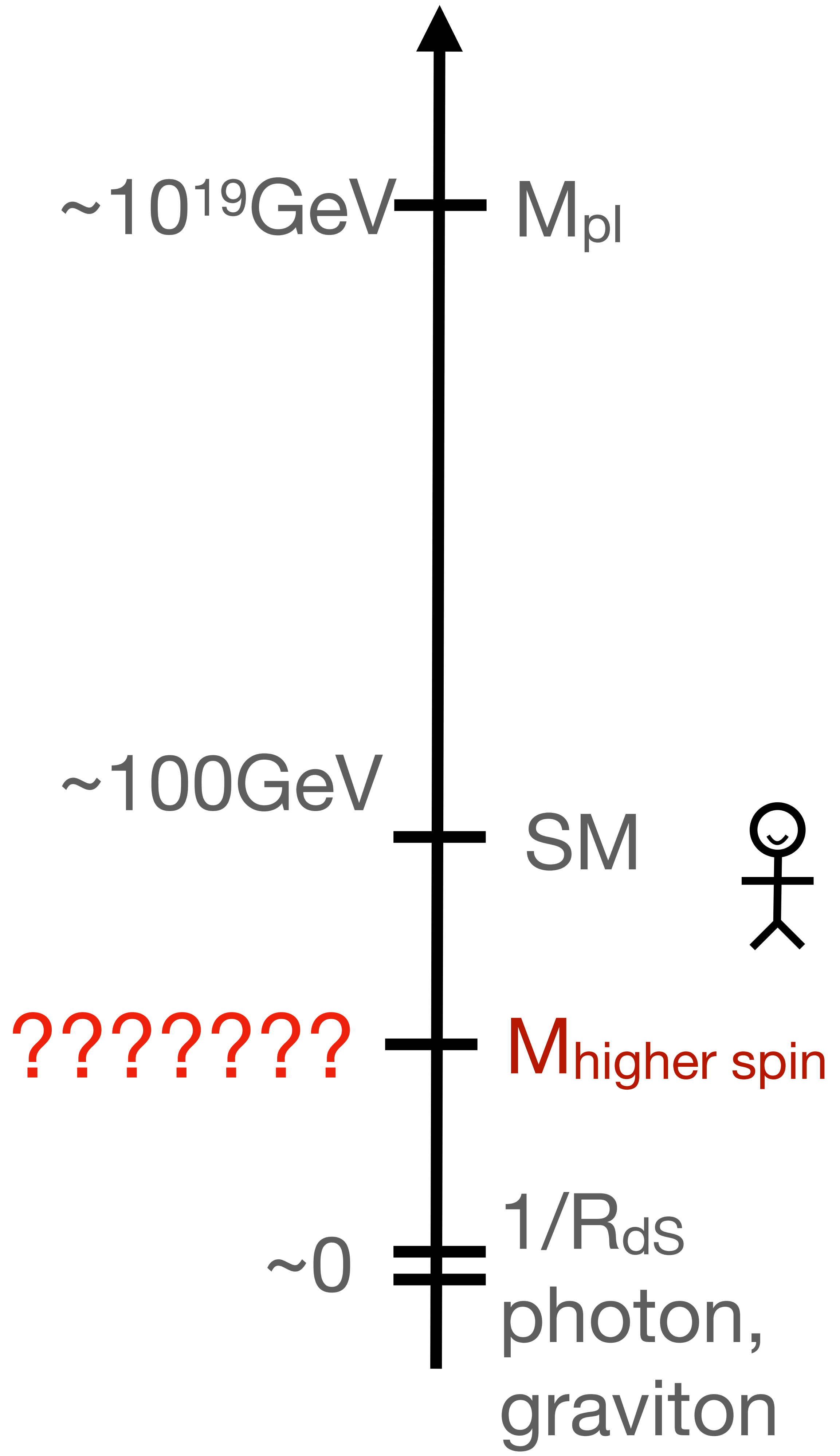
   only because of non-grav couplings to SM!



2. LIGO, EHT, neutron stars, solar system...
   test GR below $M \sim (1\text{km})^{-1} \sim 10^{-10}$eV

3. Cavendish-type experiments probe $M \sim (10^{-5}\text{m})^{-1} \sim 10^{2}$eV

   note: static 'fifth forces' ≠ 'gravity' in our language



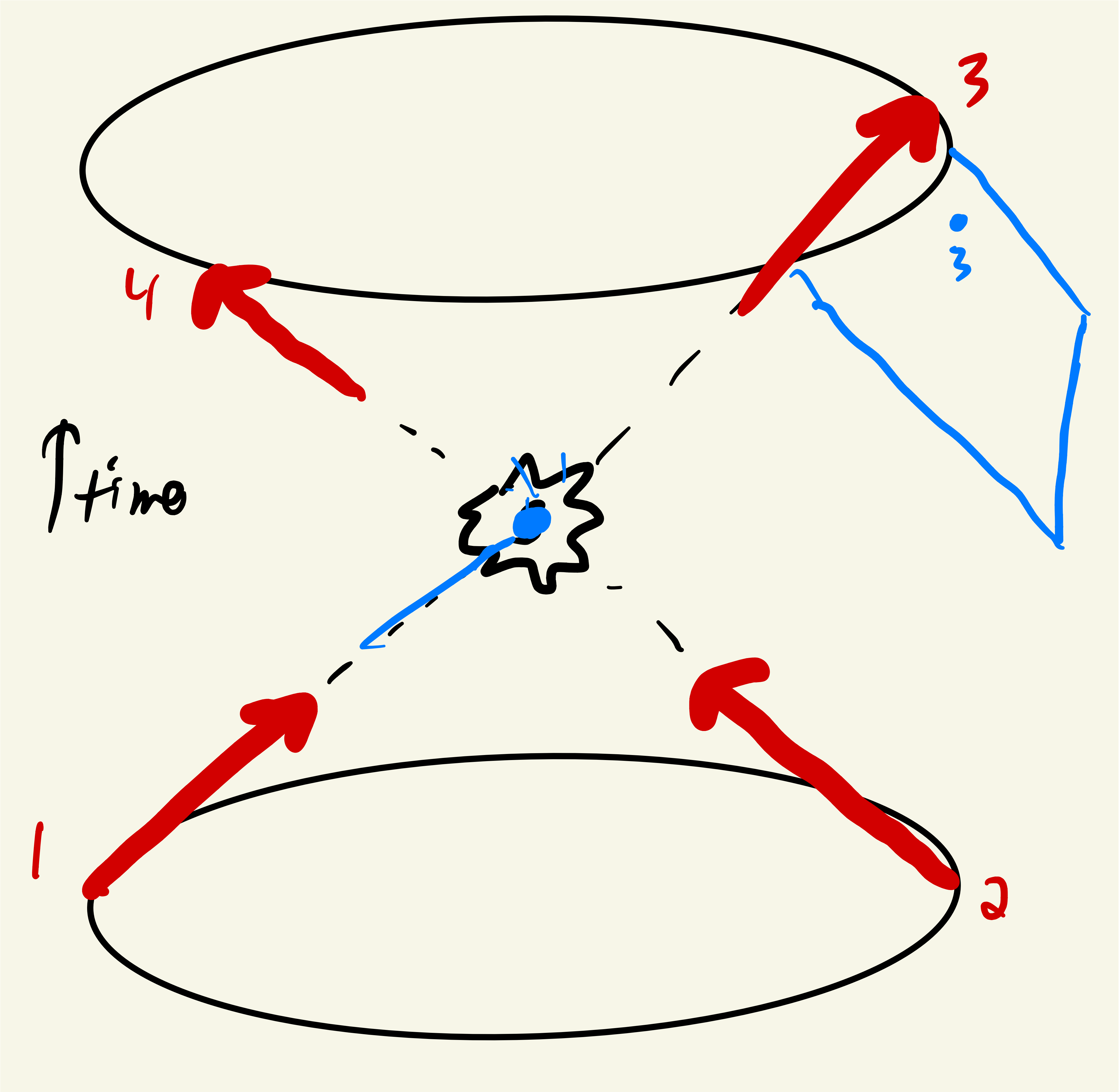
What is interplay between collider and astrophysical constraints ?

Do we really know that $M_{\text{higher spin}} >> 10^{-10}\text{eV}$?

REALLY?

Method

Causality for 2->2 scattering

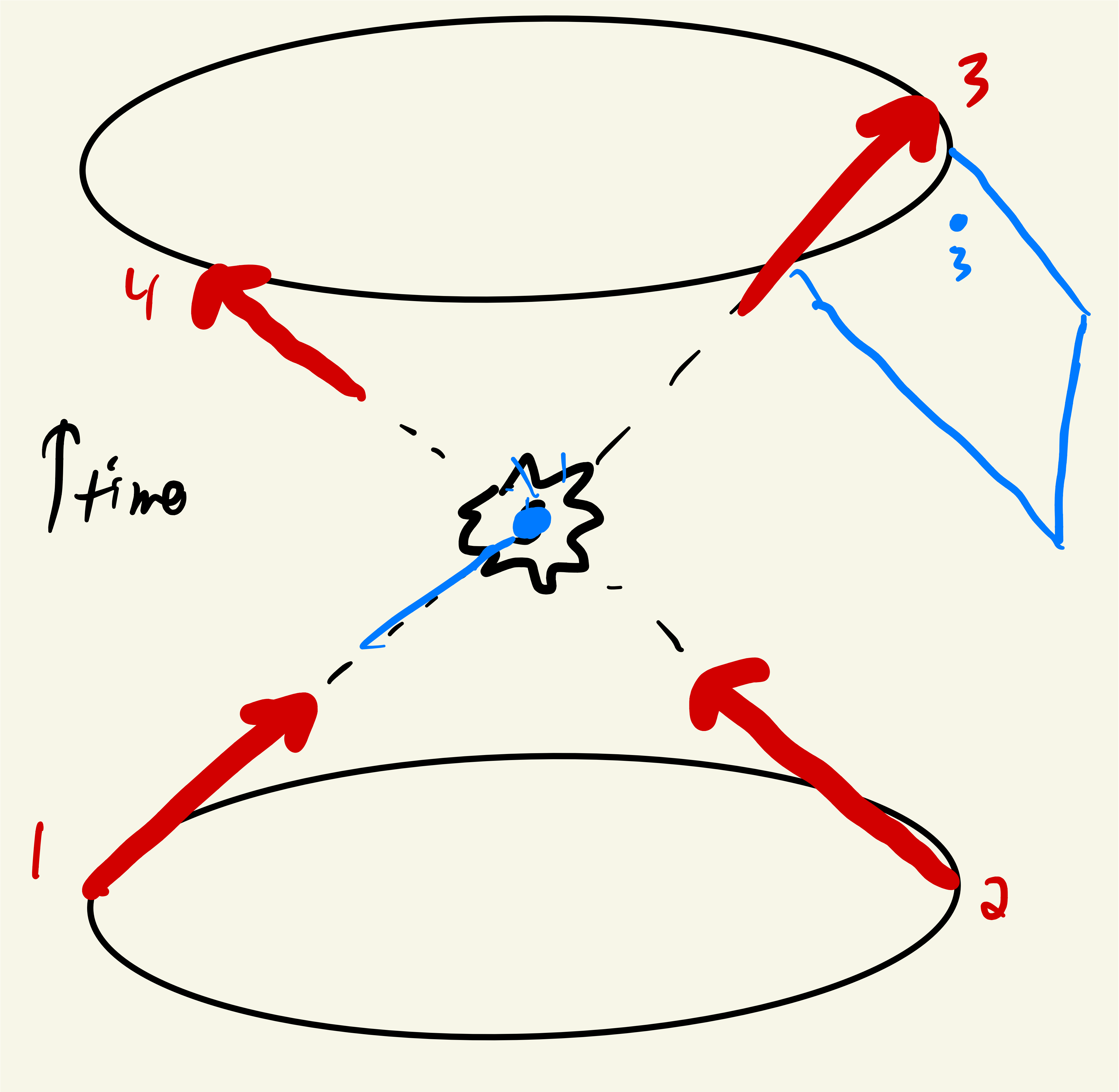


i) Fixed angle scattering can show time advances

[Giddings+Porto '09]

$\Rightarrow$ causality controls Regge limit $\begin{array}{l} s \to \infty \\ t \text{ or } b \text{ fixed} \end{array}$

Causality for 2->2 scattering



i) Fixed angle scattering can show time advances

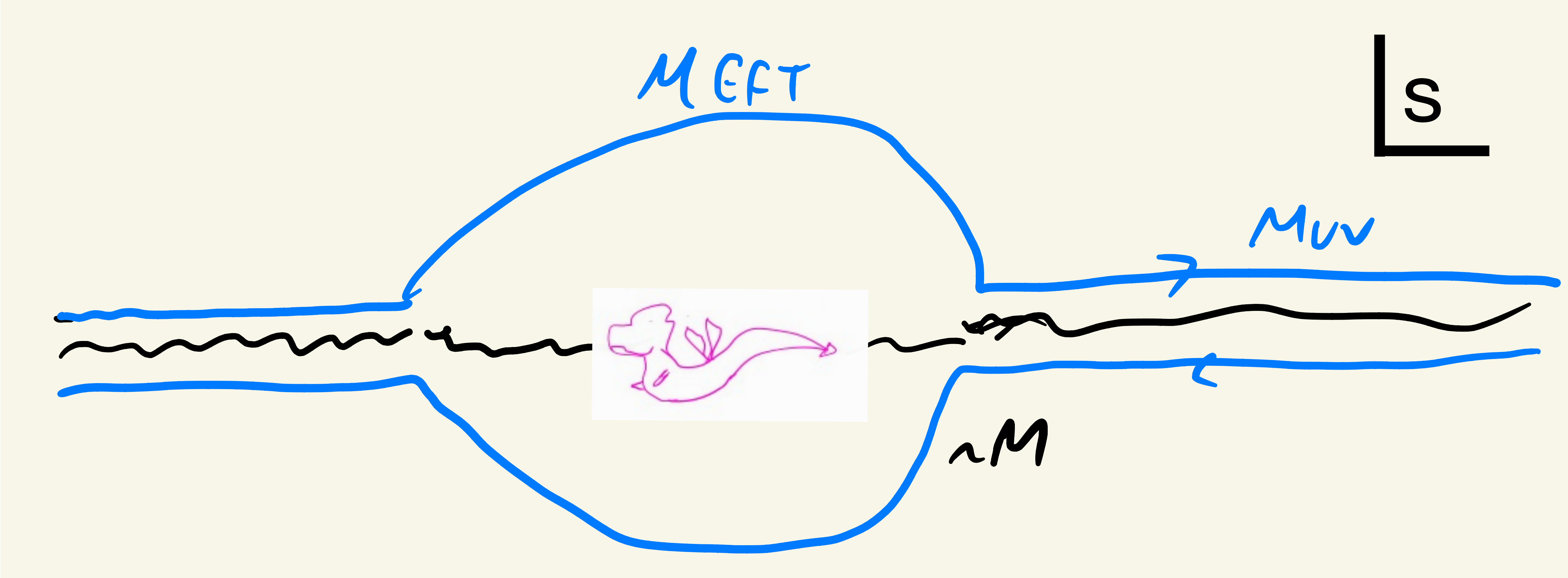
[Giddings+Porto '09]

$\Rightarrow$ causality controls Regge limit $\begin{array}{l} s \to \infty \\ t \text{ or } b \text{ fixed} \end{array}$

ii) Strongest statement involves crossing:

particle $1 \to 3 \simeq$ antiparticle $3 \to 1$

Assume: $M_{low}(s,t)$ has a causal+unitary (relativistic) UV completion



Minimal axioms:

i) Analyticity of M(s,t) in $\{t \in (-M^2, 0)\} \times \left\{ \begin{array}{l} \text{real } s > M^2 \cup \text{real } u > M^2 \\ \cup \text{ upper-half-plane connecting them} \end{array} \right\}$

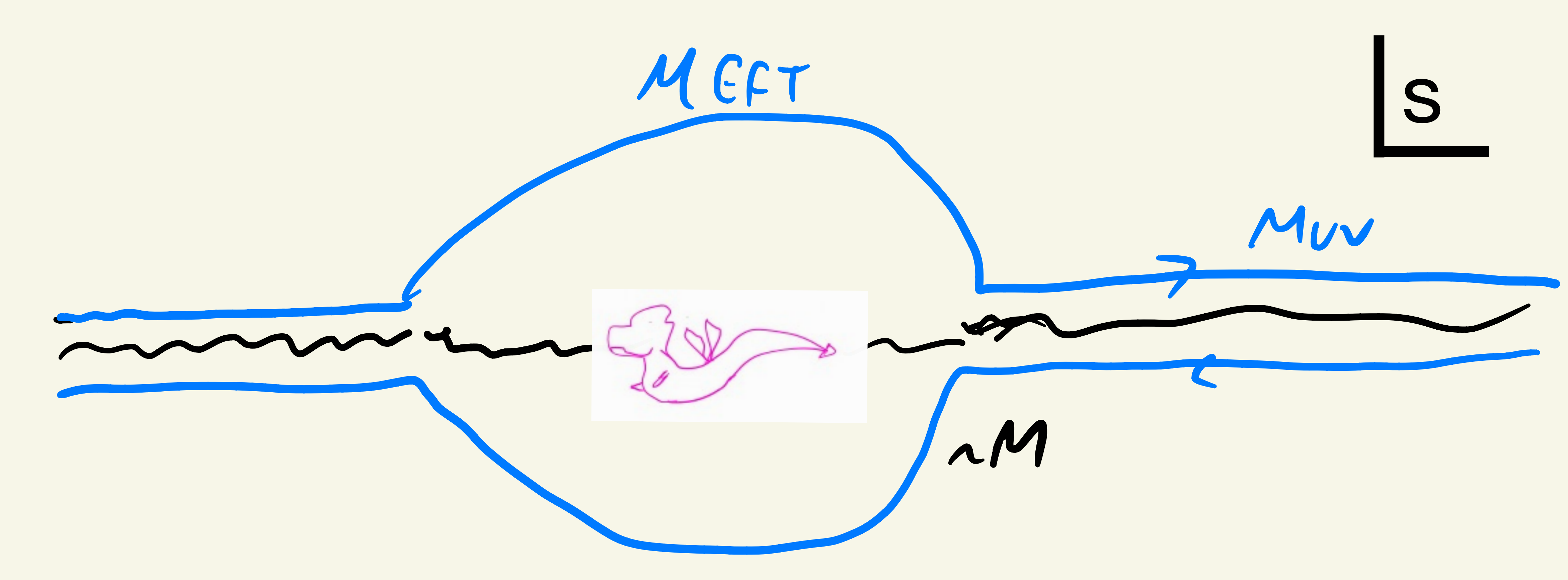
ii) Boundedness $|M_\psi(s)/s| \leq \text{const as } |s| \to \infty$

for smeared amplitude: $M_\psi(s) = \int_0^M \psi(p) M(s, -p^2)$

ψ: compact support in p, fast decay in b

holds for AdS gravity / large-N large-gap CFTs: [SCH,Mazac,Rastelli& Simmons-Duffin '21]

Axioms ensure Kramers-Kronig dispersive sum rules



Relate IR and UV:

$$0 = \oint_{|s|=\infty} ds \frac{\mathcal{M}_\psi(s)}{s^{k+1}} \Rightarrow \oint_{\text{EFT}} (\cdots) = \int_{\text{heavy}} \frac{ds}{s^{k+1}} \text{Im} \mathcal{M}_\psi(s) \qquad (k > 1)$$

$$\text{Im} \mathcal{M}_\psi(s) = \sum_J |c_J|^2 P_J(1 - 2p^2/s)_\psi$$

(Im M = sum of Legendre's with positive coefficients)

(low-energy couplings) = (sums of high-energy unknowns)

positive

Warm-up: non-gravitational real scalar

- weakly coupled EFT below M
- anything above M, just causal and unitary

$$
\begin{aligned}
\mathcal{L}_{\text{low}} = &-\frac{1}{2}(\partial_\mu\phi)^2 - \frac{g}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 \\
&+ \frac{g_2}{2}\left[(\partial_\mu\phi)^2\right]^2 + \frac{g_3}{3}(\partial_\mu\partial_\nu\phi)^2(\partial_\sigma\phi)^2 + 4g_4\left[(\partial_\mu\partial_\nu\phi)^2\right]^2 + \cdots
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{\text{low}}(s,t) = &-g^2\left[\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right] - \lambda \\
&+ g_2(s^2+t^2+u^2) + g_3(stu) + g_4(s^2+t^2+u^2)^2 + g_5(s^2+t^2+u^2)(stu) + \ldots
\end{aligned}
$$

Goal: bound higher-derivative terms

First few sum rules: (k=2, 4, …)

$$B_2: \quad 2g_2 - g_3 t + 8 g_4 t^2 + \ldots \qquad = \left\langle \frac{\left(2m^2+t\right)\mathcal{P}_J\left(1+\frac{2t}{m^2}\right)}{m^2\left(m^2+t\right)^2} \right\rangle_{m \geq M}$$

$$B_4: \quad 4g_4 + \ldots \qquad = \left\langle \frac{\left(2m^2+t\right)\mathcal{P}_J\left(1+\frac{2t}{m^2}\right)}{m^4\left(m^2+t\right)^3} \right\rangle_{m \geq M}$$

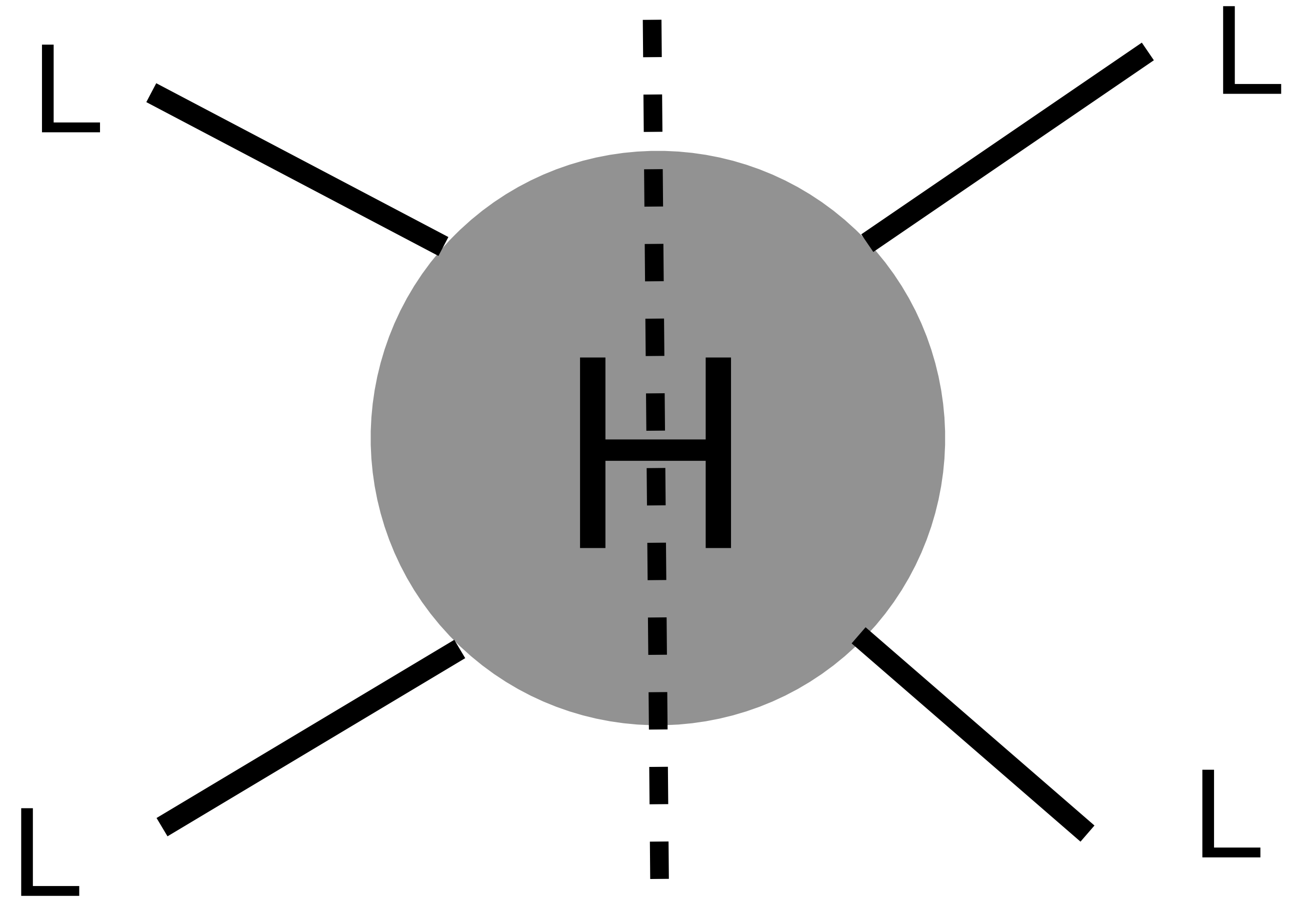
Instead of smearing, expand around t=0 (simpler, but requires Froissart)

$$g_2 = \left\langle \frac{1}{m^4} \right\rangle, \qquad g_3 = \left\langle \frac{3 - \frac{4}{d-2}\mathcal{J}^2}{m^6} \right\rangle \qquad g_4 = \left\langle \frac{1}{2m^8} \right\rangle$$

clearly: $g_2 \geq 0$ $\qquad g_3 \leq \dfrac{3g_2}{M^2}$ $\qquad 0 \leq g_4 \leq \dfrac{g_2}{2M^4}$

how about lower bound on g_3?

null constraints from IR crossing: $$0 = \left\langle \frac{\mathcal{J}^2(2\mathcal{J}^2 - 5d + 4)}{m^8} \right\rangle$$



this constrains UV spectral density! (light-light-heavy couplings)

[Tolley, Wang& Zhou ’20]
[SCH& van Duong ’20]

$$\left\langle \frac{1}{m^4} \overbrace{\frac{J^2}{m^2}}^{\sim b^2} \right\rangle_{m\geq M} \leq \frac{\#}{m^2} \left\langle \frac{1}{m^4} \right\rangle_{m\geq M}$$

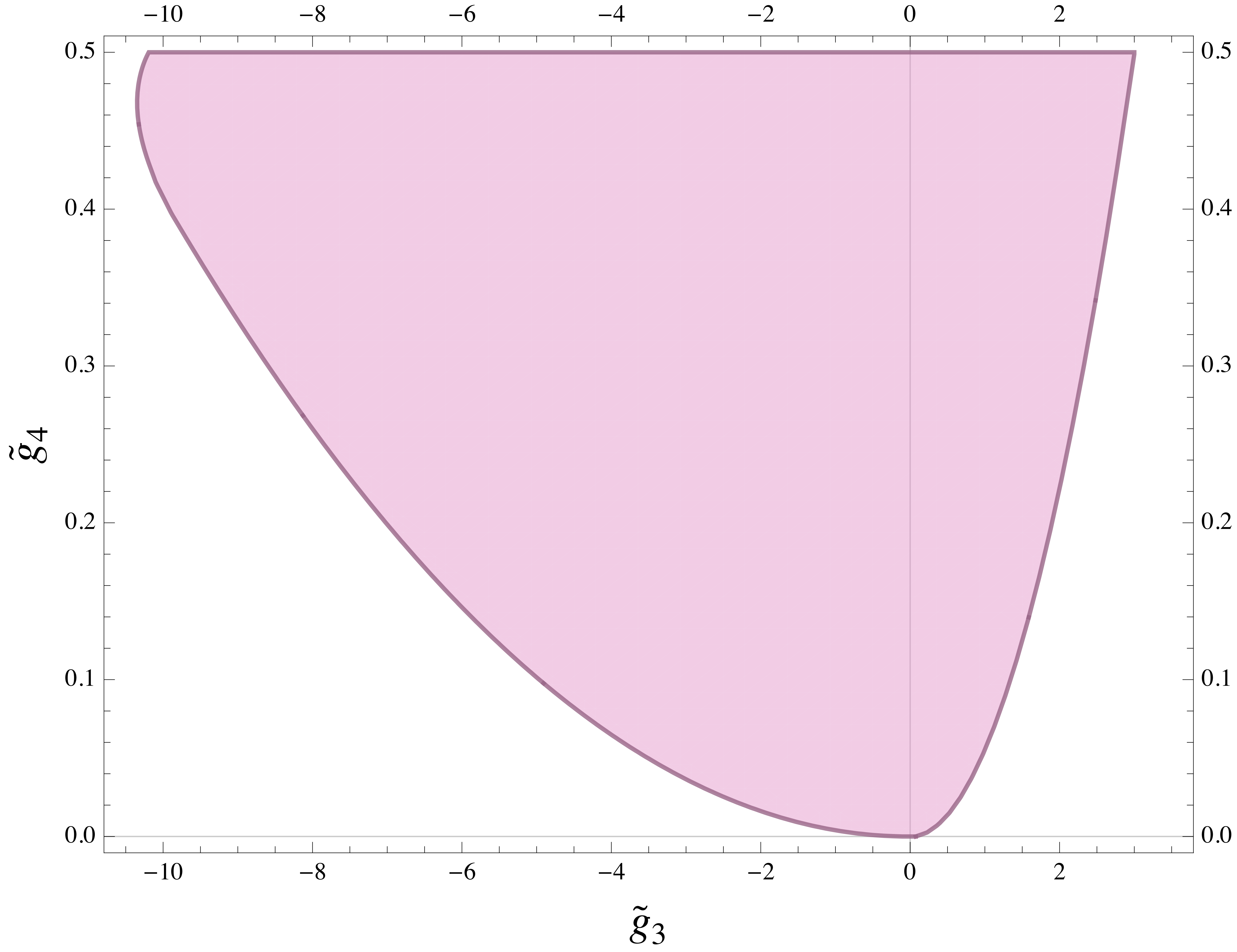
$\Rightarrow$ As far as sum rules are concerned,

heavy states with large spin (large b) can’t couple strongly

(ie. large black holes, long strings, etc, can never dominate sum rules)

'dimensional analysis' is a theorem

$$\tilde{g}_k \equiv g_k M^{\#} / g_2$$



EFT coefficient	Lower bound	Upper bound
$\tilde{g}_3$	-10.346	3
$\tilde{g}_4$	0	0.5
$\tilde{g}_5$	-4.096	2.5
$\tilde{g}_6$	0	0.25
$\tilde{g}'_6$	-12.83	3
$\tilde{g}_7$	-1.548	1.75
$\tilde{g}_8$	0	0.125
$\tilde{g}'_8$	-10.03	4
$\tilde{g}_9$	-0.524	1.125
$\tilde{g}'_9$	-13.60	3
$\tilde{g}_{10}$	0	0.0625
$\tilde{g}'_{10}$	-6.32	3.75

grow like geometric series

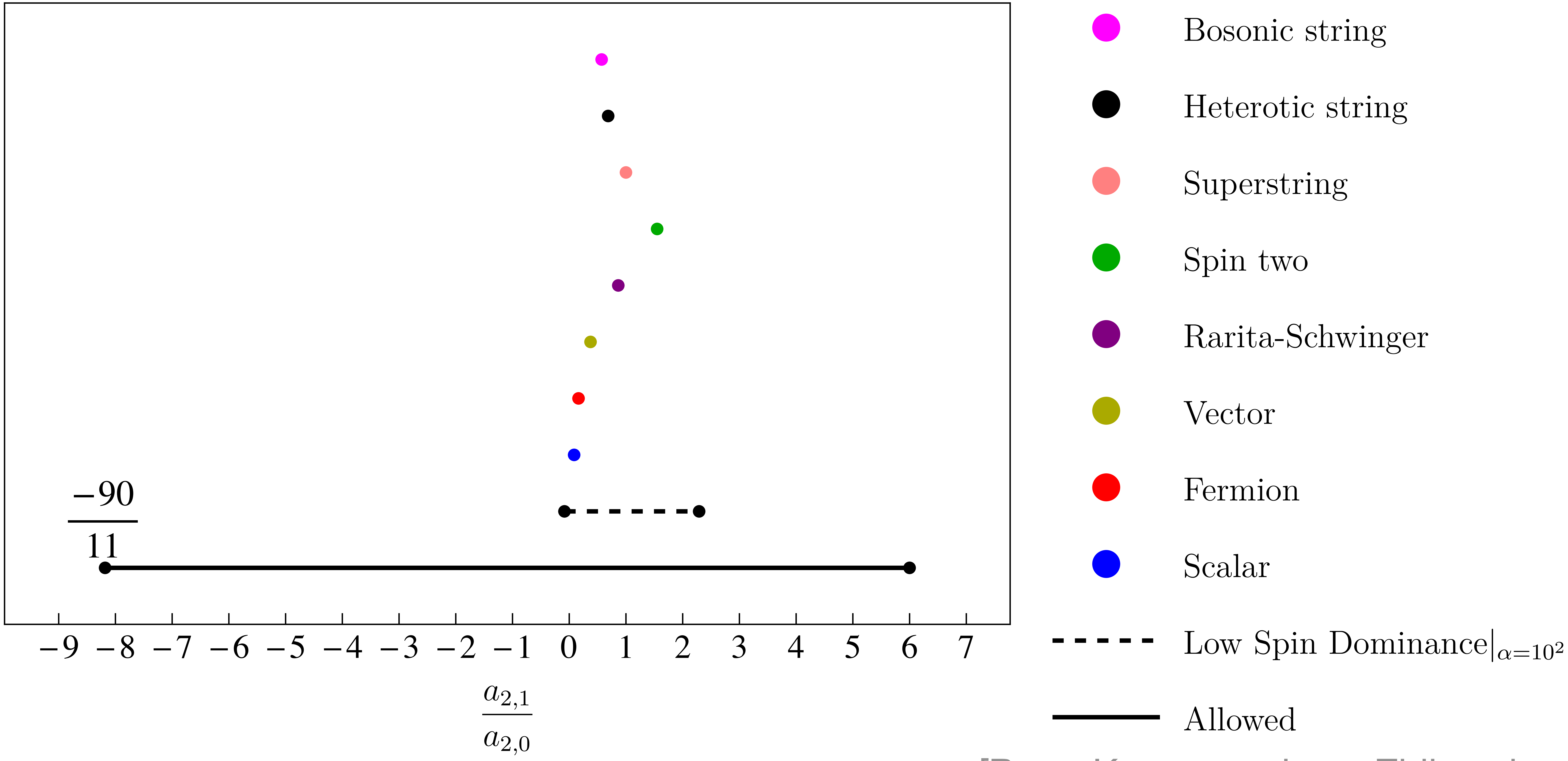
shape approximated analytically by EFThedron

[Tolley, Wang& Zhou '20]
[SCH& van Duong '20]
[Arkani-Hamed, Huang& Huang '20]
[Chiang, Huang, Li, Rodina& Weng '21]
[mixed correlators numerics: Du, Zhang& Zhou '21]

Gravitons: homogeneous bounds vs known theories

(with spin>=4 near-forward sum rules)

$$f\Big|_{D^4R^4} = a_{2,0}\, t^2 - a_{2,1} su$$



[Bern, Kosmopoulous, Zhiboedov '21]

[photons: Henriksson, McPeak, Russo, Vichi'21]

Graviton pole: scalars with graviton exchange

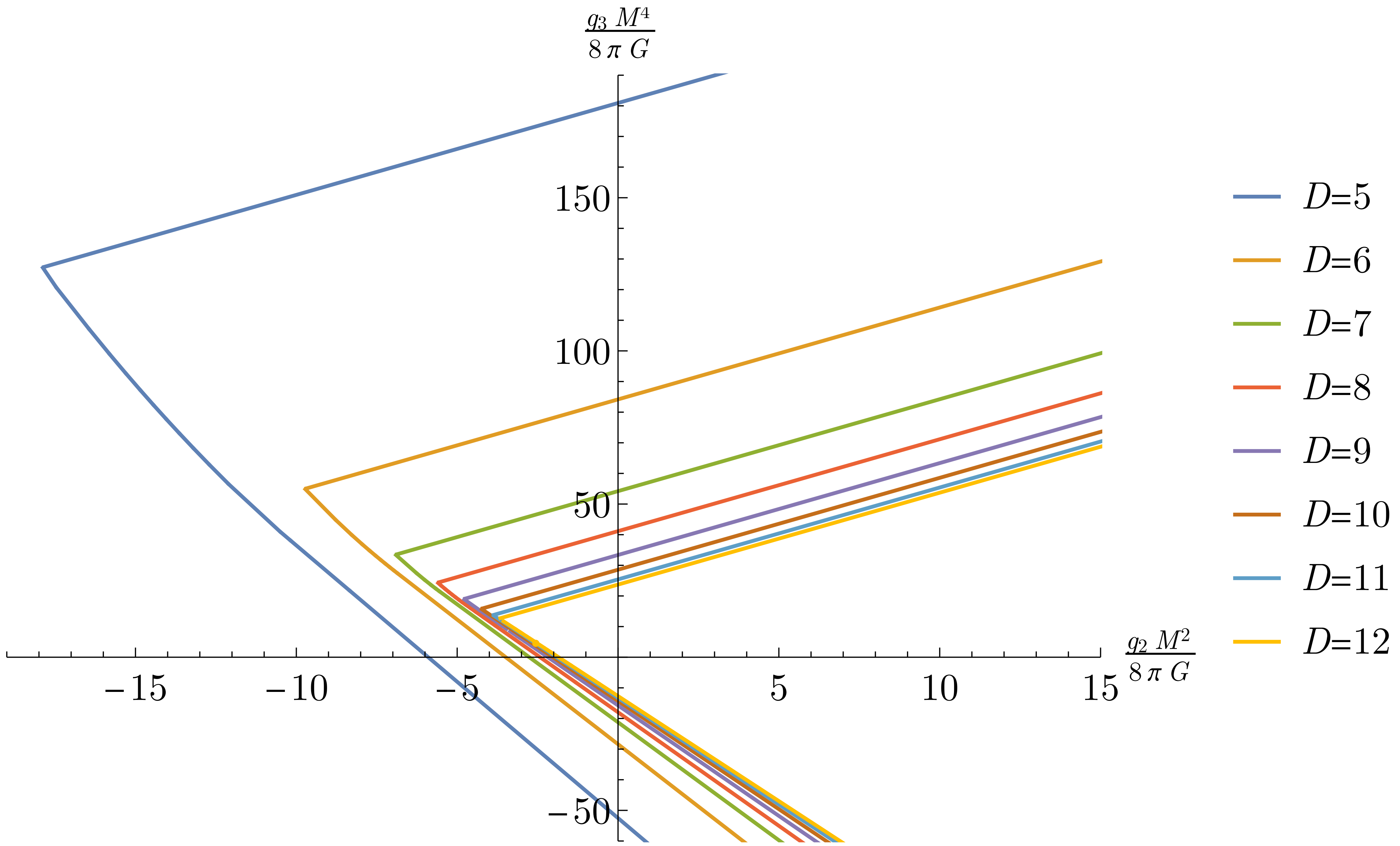
[SCH, Mazac, Rastelli& Simmons-Duffin ’21]

$$B_2(-p^2)\Big|_{\text{low}} = \frac{8\pi G}{p^2} + 2g_2 + g_3 p^2 + 8g^4 p^4 + \ldots$$

Can't use forward limits. Solution:

-Integrate over $p \in [0,M]$

-Use crossing to eliminate all but finitely many contacts



$(\partial\phi)^4$ term can be slightly negative:

$$g_2 \geq -\frac{\#G}{M^2}$$

Gravity: new results

Dispersive sum rules for gravitons

- Spin helps:

$$M^{+--+} = \underbrace{[14]^4\langle 23\rangle^4}_{\propto t^4} \times 8\pi G \left[\underbrace{\frac{1}{stu} + \frac{|g_3|^2 su}{4t} + \frac{|g_s|^2}{-t} + g_4 + g_5 t + \dots}_{f(s,t)} \right]$$

- The prefactor grants antisubtracted sum rules:

$$B_2(u) : 0 = \oint_{s=\infty} (s-t) ds [f(s,t) + f(t,s)]\,, \qquad B_3(u) : 0 = \oint_{s=\infty} ds [f(s,t) - f(t,s)]$$

- Superconvergence is awesome

- Superconvergent sum rules automatically kill all contacts!

$$B_2(-p^2): \quad \frac{8\pi G}{p^2} + |g_s|^2 p^2 + |g_3|^2 p^6 = \int_{M^2 > M^2_{\text{higher-spin}}} \frac{dm^2}{m^8}(2m^2 - p^2) \sum_J \left[|c_{++}(m)|^2 d^J_{++}(1 + \frac{2p^2}{m^2}) + |c_{+-}(m)|^2 d^J_{+-}(1 + \frac{2p^2}{m^2}) \right]$$

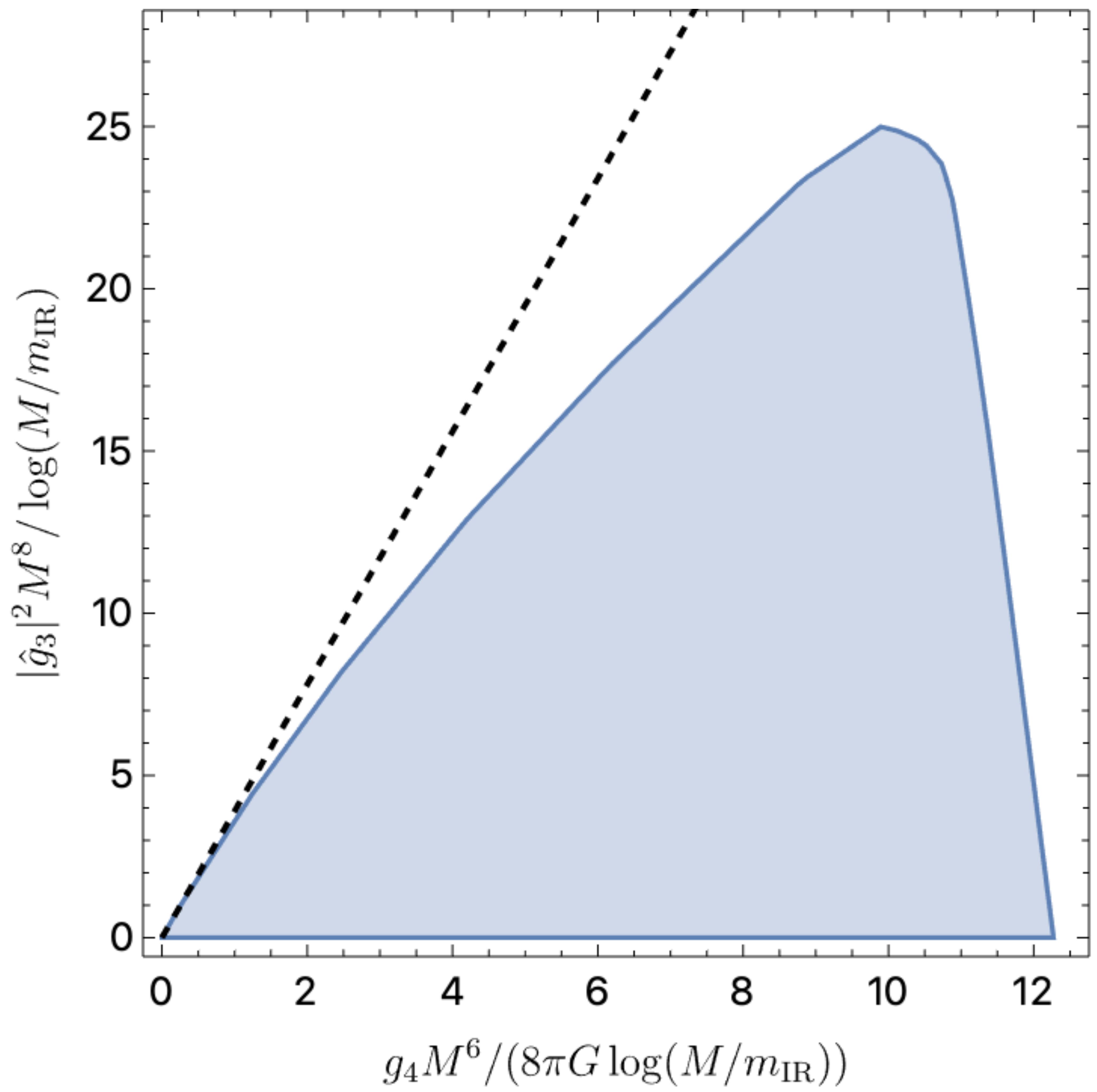
exact! (neglecting loops $\sim 1/M_{\text{pl}}^4$)

- to follow CEMZ, could study mixed problem with other helicity amplitudes.
  (using crossing to remove towers of ++++ contacts and reach p~M)

- Much simpler: find a single MAGIC combination that writes G = positive sum.

  It will automatically dominate every other coupling!

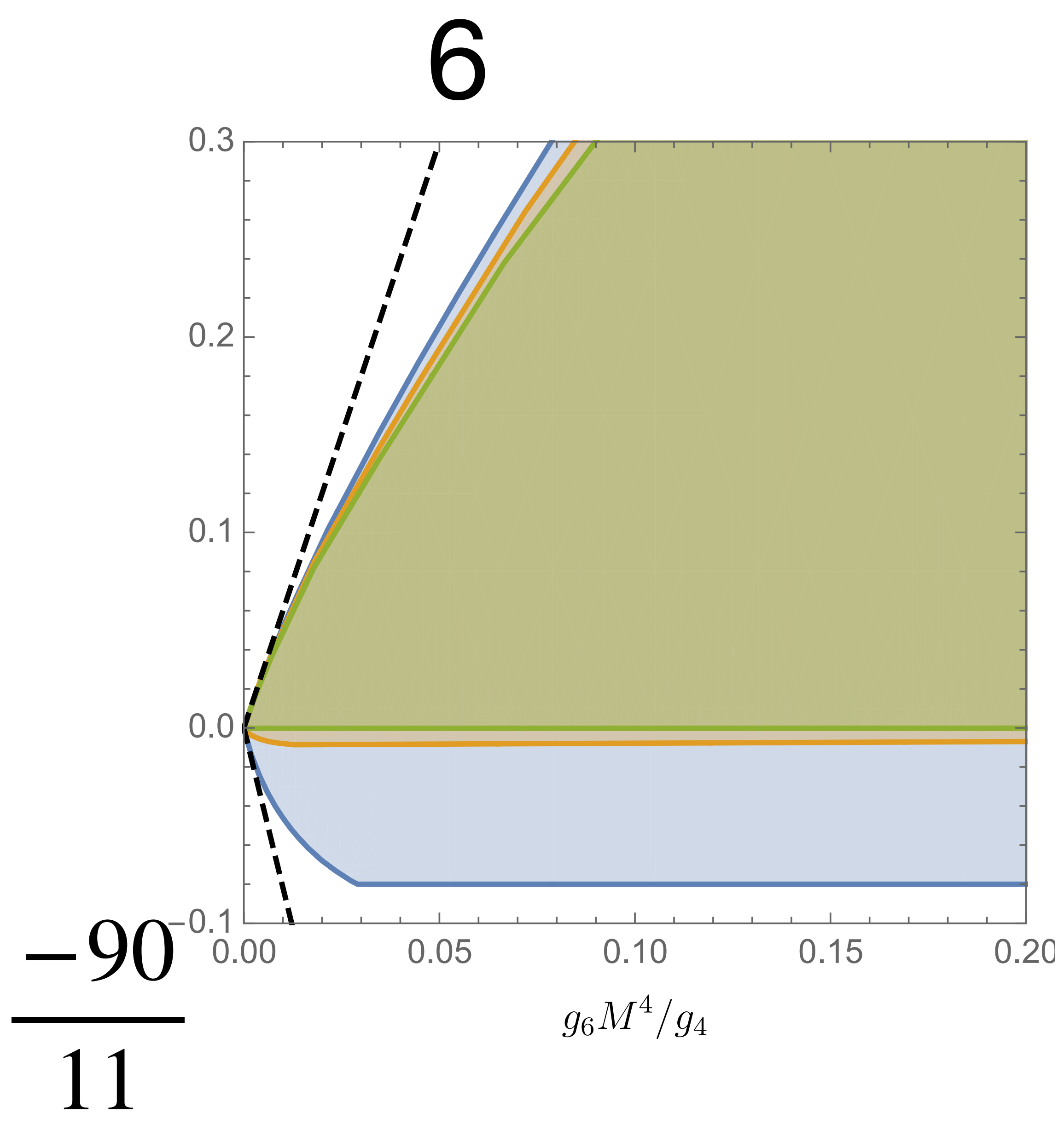
Riem^3 and Riem^4 can't exceed GR

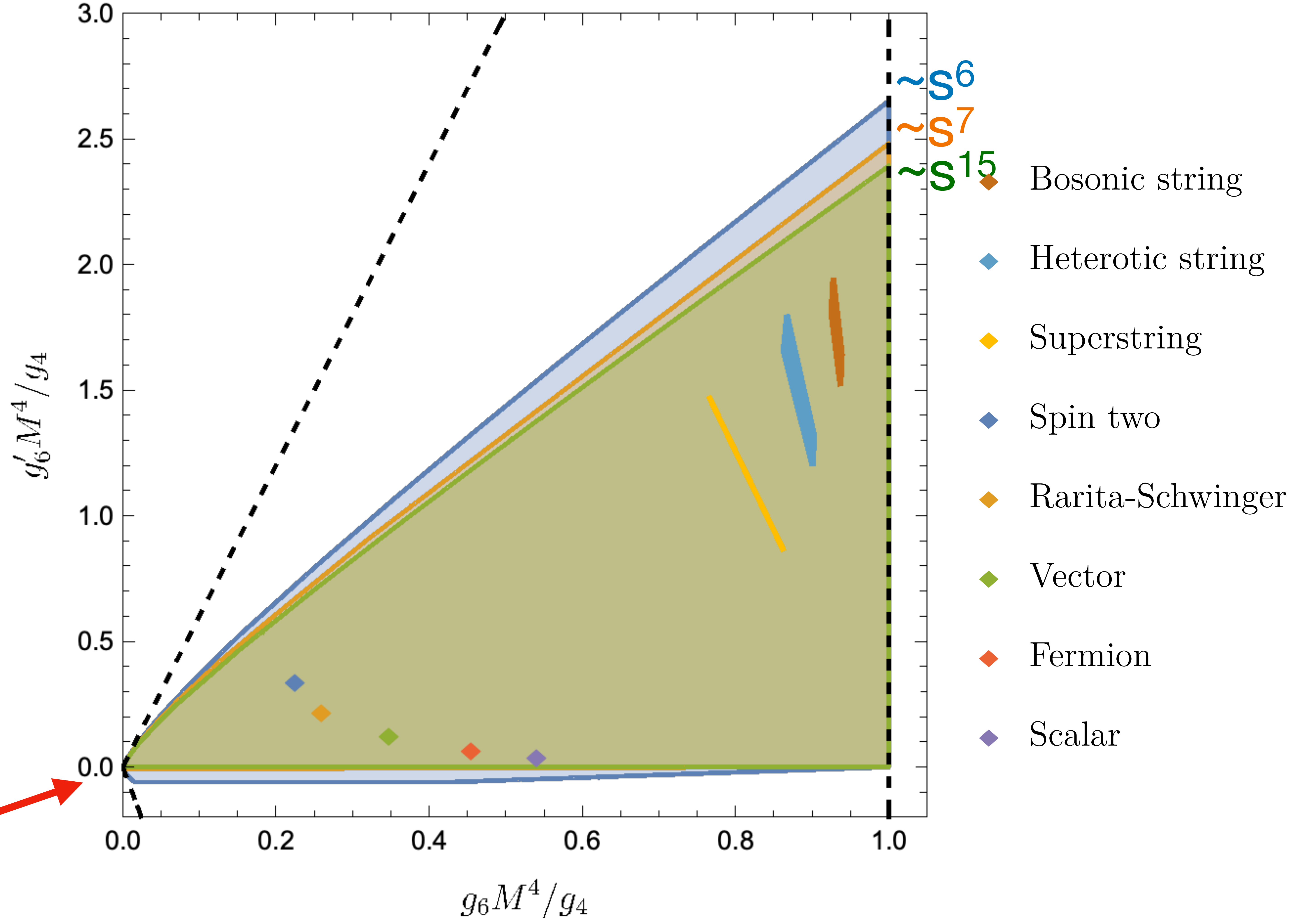


$$S = \frac{1}{16\pi G}\int\left(R + \frac{\tilde{g}_3\ \mathrm{Riem}^3}{M^4_{\text{higher-spin}}} + \frac{\tilde{g}_4\ \mathrm{Riem}^4}{M^6_{\text{higher-spin}}} + \dots\right)$$

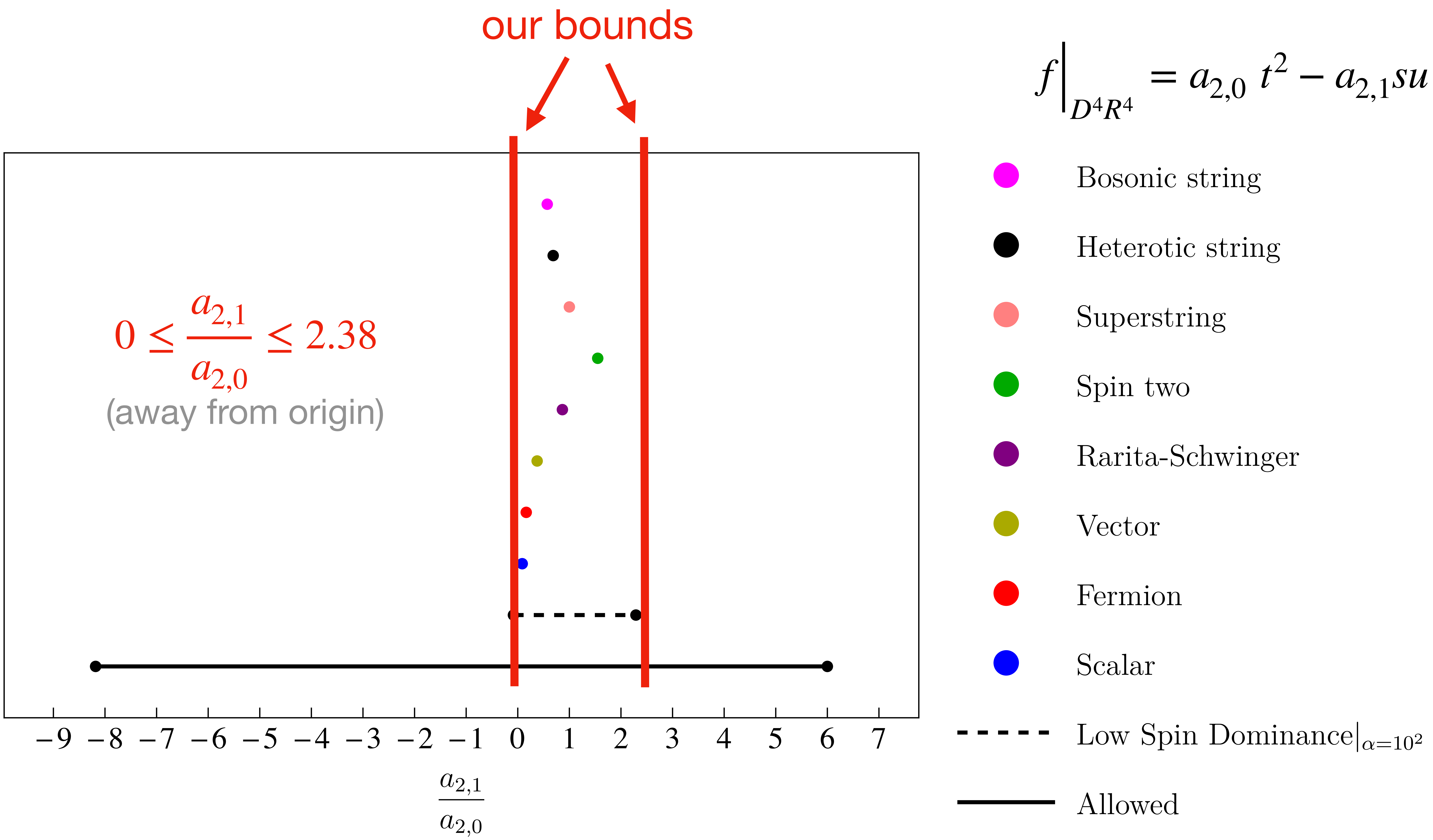
rescaled $\tilde{g}$'s can't exceed O(1) without violating causality

[SCH, Li, Parra Martinez, Simmons-Duffin] more on contact interactions using (more) spin>=4 null constraints: (two D^4R^4)/R^4





extremal slopes are only realized in region that disappears asymptotically!



our bounds
$f\big|_{D^4R^4} = a_{2,0}\, t^2 - a_{2,1} su$
$0 \leq \frac{a_{2,1}}{a_{2,0}} \leq 2.38$
(away from origin)
Bosonic string
Heterotic string
Superstring
Spin two
Rarita-Schwinger
Vector
Fermion
Scalar
Low Spin Dominance$|_{\alpha=10^2}$
Allowed
−9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7
$\frac{a_{2,1}}{a_{2,0}}$

A tale of 3 effective field theorists:

$$\sim h\partial^2 h + c'' \frac{\partial^6 h^3}{M^5}$$

$$L \supset m_{\rm pl}^2 R + c\frac{\text{Riem}^3}{M^2}$$

"c<O(1) since couplings at cutoff should be O(1)"

$$L \supset m_{\rm pl}^2 \left(R + c' \frac{\text{Riem}^3}{M^4} \right)$$

"c'<O(1): corrections can never dominate GR below the cutoff"

$$L \supset m_{\rm pl}^2 R + c'' m_{\rm pl}^3 \frac{\text{Riem}^3}{M^5}$$

"c"<O(1) so gravitons stay weakly coupled below M"

When $M << m_{pl}$, what is the correct scaling of higher-derivative corrections with M & m_{pl} ?

A tale of 3 effective field theorists:

$$L \supset m_{\rm pl}^2 R + c\frac{\mathrm{Riem}^3}{M^2}$$

"c<O(1) since couplings at cutoff should be O(1)"

too restrictive
(untrue in string theory...)

$$L \supset m_{\rm pl}^2\left(R + c'\frac{\mathrm{Riem}^3}{M^4}\right)$$

"c'<O(1): corrections can never dominate GR below the cutoff"

= what we find!

$$\sim h\partial^2 h + c''\frac{\partial^6 h^3}{M^5}$$

$$L \supset m_{\rm pl}^2 R + c'' m_{\rm pl}^3\frac{\mathrm{Riem}^3}{M^5}$$

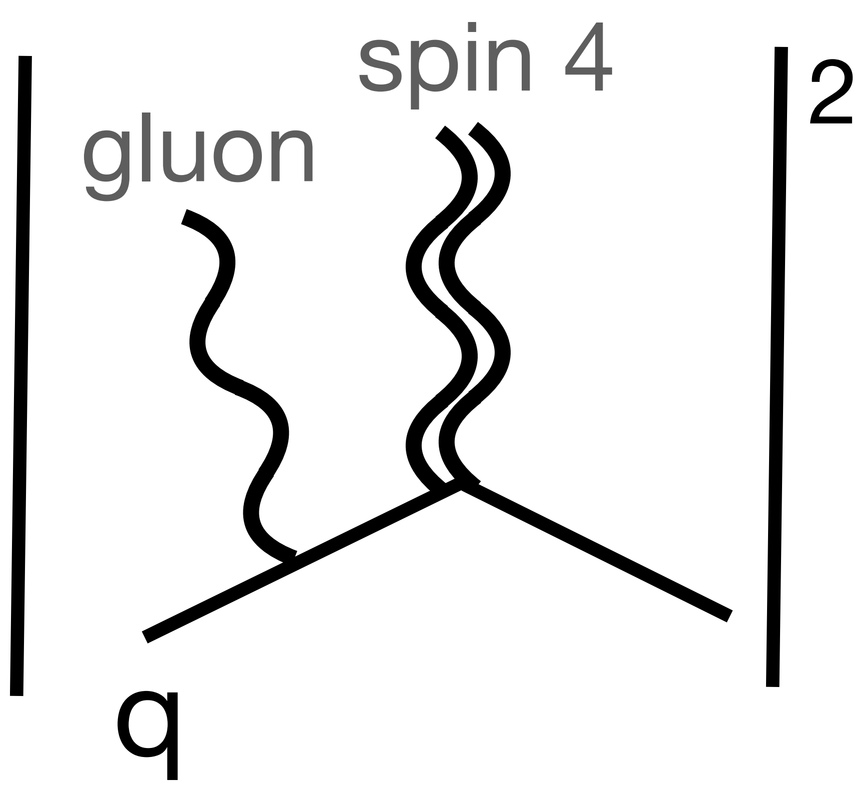
"c"<O(1) so gravitons stay weakly coupled below M"

too permissive
(ruled out by our causality bounds!)

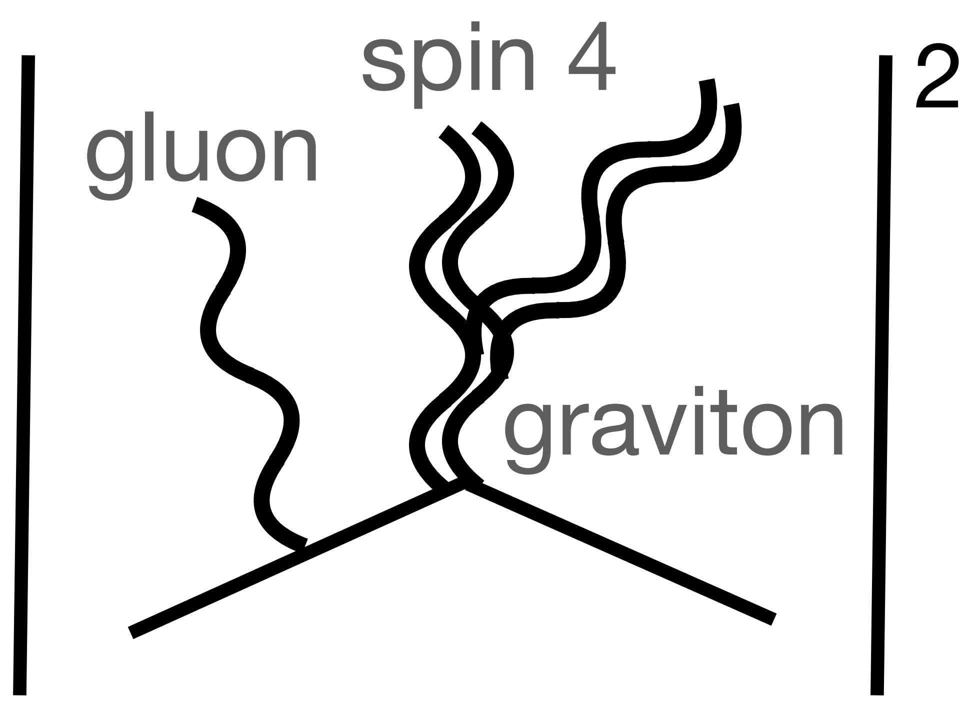
When $M \ll m_{pl}$, what is the correct scaling of higher-derivative corrections with M & m_{pl} ?

lesson: a gravitational EFT can never significantly differ from GR within its regime of validity.

What do we know about $M_{\text{higher-spin}}$?



$$\sim \frac{\alpha_s E^6}{M^4 M_{\text{pl}}^2}$$



$$\sim \frac{\alpha_s E^{16}}{M^{12} M_{\text{pl}}^4}$$

$$\text{or} \sim \frac{\alpha_s E^8}{M^4 M_{\text{pl}}^4}$$

- Very conservatively: hard to imagine not seeing 'missing energy' at LHC from a gravitationally-coupled spin-4 particle with M<MeV.
- Corresponds to a length scale: $M^{-1}_{\text{higher-spin}} < 10^{-13}\text{m}$...
- Phenomenological constraints should be analyzed carefully.

Conclusion

Suppose light-bending by Sun disagreed by more than ~10^{-40} from GR.

0. new light scalars [constrained by other experiments]

~~1. Other conventional new physics / new particles~~ might be impossible

2. Experimental error

3. Causality doesn't work like we thought

Open questions

- Bound couplings to matter, light, SM?
- Expect loops only $O(N/M_{\mathrm{pl}}^4)$. Check?
- Remove Log[IR]'s (dressing, …)?
- Higher spacetime dimensions? Where is weakly coupled string theory?
- What if $M \sim M_{\mathrm{pl}}$: how close to classical GR can 4d quantum gravity be?
- …

Assumptions

- causality (relativistic): can't send info faster than light

  served us well in past century.
  without it, particle physics methods lose predictive power.

  any $V(r)$ would do.

  e.g. quantum gravity would be trivial: $$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} - \sum_{i<j} \frac{G m_i m_j}{|\vec{x}_i - \vec{x}_j|}$$

- unitarity (probabilities can't be negative)

  interesting interplay: wrong-sign kinetic terms can be quantized so
  positive-frequency modes have positive norm, but propagate backward in time.

  (problem: negative-E propagating forward makes vacuum unstable.)

[Ostrogradsky; Cline, Jeon& Moore '03; Woodard '15]
[Lee-Wick '69, 'killed' by Cutkosky et al '69]

- The first few sum rules for +--+ :

$$B_2(-p^2): \quad \frac{8\pi G}{p^2} + |g_s|^2 p^2 + |g_3|^2 p^6 = \int_{\text{heavy}} (\ldots)$$

$$B_3(-p^2): \quad -|g_s|^2 - p^2 |g_3|^2 = \int_{\text{heavy}} (\ldots)$$

$$B_4(-p^2): \quad g_4 + |g_3|^2 p^2 + \ldots = \int_{\text{heavy}} (\ldots)$$

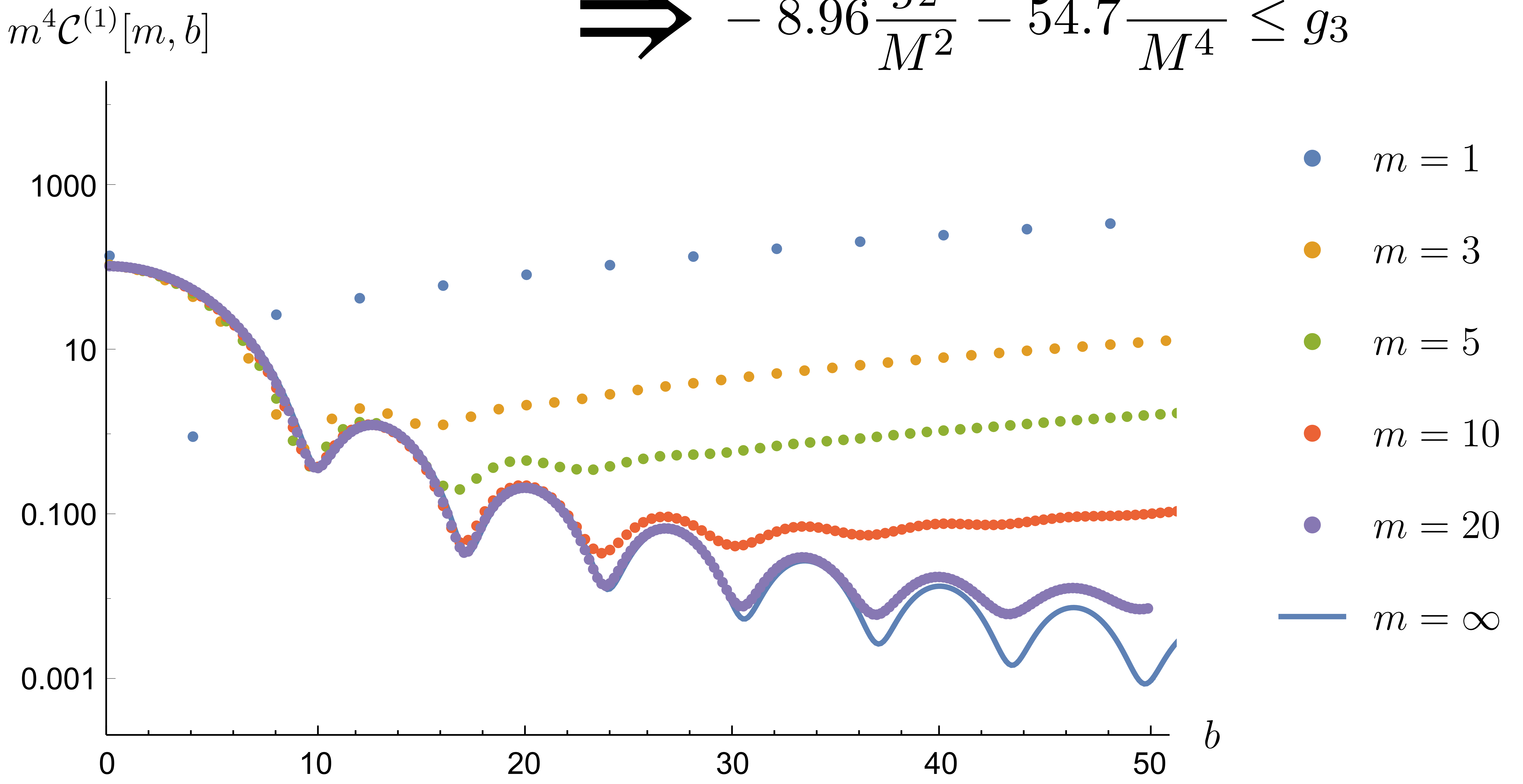
infinite sum

- For k=2,3 we integrate against ψ's, for $k \geq 4$ we expand in forward limit.

Example functional (d=5)

$$\mathcal{C}^{(1)} \equiv \int_0^1 dp\ \mathcal{C}^{\text{improved}}_{2,-M^2p^2} \times p^2(1-p)^2\left[2280 - 665p + 2964p^5 - 8280p^6\right]$$

$$\Rightarrow\quad -8.96\frac{g_2}{M^2} - 54.7\frac{8\pi G}{M^4} \leq g_3$$



$J_{\max}$	dimension	interactions
0	dim. 3	$\cancel{\phi^{(a}\phi^b\phi^{c)}}$
	dim. 4	$\phi^{(a}\phi^b\phi^c\phi^{d)}$
$\frac{1}{2}$	dim. 4	$\psi^{(i}\psi^{j)}\phi$
	dim. 5	$\psi^{(i}\psi^{j)}\phi^{(a}\phi^{b)}$
1	dim. 5	$\cancel{F\psi^{[i}\psi^{j]}}$
	dim. 6	$F^{[a}F^bF^{c]}$, $\phi^{[a}D\phi^{b]}\phi^{[c}D\phi^{d]}$, $\psi^i\bar{\psi}_j\phi^{[a}D\phi^{b]}$, $\psi^{(i}\psi^{j)}\psi^{(k}\psi^{l)}$, $\psi^{(i}\psi^{j)}\bar{\psi}_{(k}\bar{\psi}_{l)}$, $F\psi^{[i}\psi^{j]}\phi$, $F^{(a}F^{b)}\phi^{(c}\phi^{d)}$
	dim. 7	$\cancel{F\phi^{[a}D\phi^bD\phi^{c]}}$, $\cancel{F^{[a}F^bF^{c]}\phi}$, $D\psi^{[i}\psi^j\psi^{k]}\bar{\psi}$
higher-points	dim. 6	ϕ^6, $\psi^2\phi^3$
	…	

Table 1. Interactions which have spin ≤ 1 in all channels and are thus not probed by dispersion relations; ϕ are scalars, ψ Weyl fermions, and F field strengths. Adding any further derivative or graviton coupling pushes these above the $J_{\max} = 1$ threshold. Struck-out interactions $\cancel{\phi\phi\phi}$ are incompatible with SM gauge invariance.

[unpublished; see Durieux, Kitahara, Machado, Shadmit Weiss '20]

$J_{\max}$	dimension	interactions
$\frac{3}{2}$	dim. 7	$\psi\psi\phi\phi D^2$, $F\psi\bar{\psi}\phi D$, $FF\psi\psi$, $FF\bar{\psi}\bar{\psi}$, $\cancel{RF\psi\psi}$
	dim. 8	$\psi\bar{\psi}\phi\phi D^3$, $F\psi\psi\phi D^2$, $F\bar{\psi}\bar{\psi}\phi D^2$, $FF\psi\bar{\psi}D$
2	dim. 8	$\phi\phi\phi\phi D^4$, $\psi\psi\psi\psi D^2$, $F\bar{F}\psi\bar{\psi}D$, $FFFF$, $FF\bar{F}\bar{F}$
	dim. 9	$\psi\psi\phi\phi D^4$, $\psi\psi\psi\bar{\psi}D^3$, $F\phi\phi\phi D^2$, $F\psi\bar{\psi}\phi D^3$, $F\bar{F}\psi\psi D^2$, $FF\psi\psi D^2$, $FFF\phi D^2$, $FF\bar{F}\phi D^2$
	dim. 10	$\phi\phi\phi\phi D^6$, $\psi\psi\psi\psi D^4$, $\psi\psi\bar{\psi}\bar{\psi}D^4$, $FF\phi\phi D^4$, $F\bar{F}\phi\phi D^4$, $FFF\bar{F}D^2$, F^4D^2
2 w/ gravity	dim. ≤ 6	S_{GB}, S_{R^3}, $S'^{(D\geq 7)}_{R^3}$, RFF, $RR\phi\phi$,
	dim. 7	$RFF\phi$, $RRF\phi$, $RRR\phi$,
	dim. 8	$RF\phi\phi D^2$, $R\psi\psi\phi D^2$, $RFFF$, $RRFF$
	dim. 9	$\cancel{R\phi^3D^2}$, $\cancel{RFF\phi D^2}$, $\cancel{RRFFD^2}$

Table 2. Four-point interactions with maximum spin $\frac{3}{2}$ or 2, which are all detectable by some dispersion relation. The positioning of the derivatives D is schematic, but only some index contractions have the quoted spin.

All graviton couplings are boundable.

[see also: Chowdhury, Gadde, Gopalka, Halder, Janagal& Minwalla '20]

For CFTs, our axiom $|M_\psi(s)/s| \leq \mathrm{const}$ is the familiar 'Cauchy-Schwartz' bound ($j_* \leq 1$) on the Regge limit of correlators.

Note this is weaker than 'Froissart-like' bound $|M(s,t)/s^2| \to 0$

The latter may also hold in gravity, but is inessential for our story.

[see Chandorkar, Chowdhury, Kundu& Minwalla '21]

We showed that any positive S-matrix sum rule derived from these axioms uplifts to a positive CFT sum rule up to $1/\Delta_{\text{gap}}$ corrections.

(no need to assume sequences of CFTs)

[SCH,Mazac,Rastelli& Simmons-Duffin '21]

The axioms are compatible with quantum gravity.