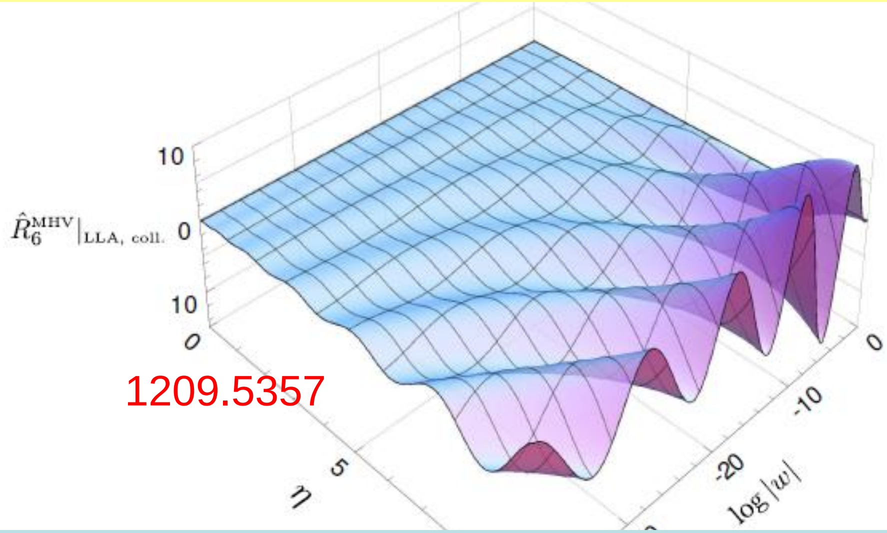
Single-valued Harmonic Polylogarithms and the Multi-Regge-Limit (Part I)



Lance Dixon (SLAC)

with C. Duhr and J. Pennington, arXiv:1207.0186

also: J. Pennington, arXiv:1209.5357

Amplitudes and Periods Workshop

Paris 3 Decembre 2012

Solving planar N=4 SYM scattering

- Exact exponentiation of 4 & 5 gluon amplitudes
- Dual (super)conformal invariance
- Amplitudes equivalent to Wilson loops
- Strong coupling → minimal area "soap bubbles"

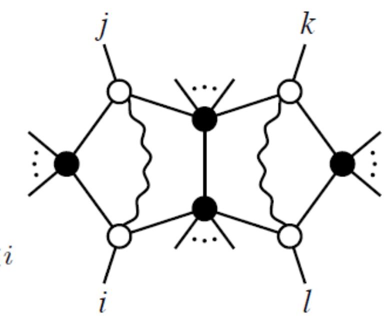
Can these structures be used to solve exactly in coupling for all planar N=4 SYM amplitudes? What is the first nontrivial case to solve?

All planar N=4 SYM integrands

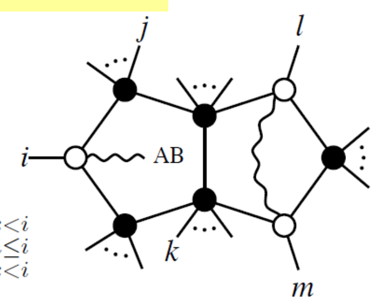
Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka, 1008.2958, 1012.6032

- All-loop BCFW recursion relation for integrand ☺
- Manifest Yangian invariance (huge group containing dual conformal symmetry).
- Multi-loop integrands written in terms of "momentum-twistors".
- Still have to do integrals over the loop momentum ☹

$$\mathcal{A}^{2-\mathrm{loop}}_{\mathrm{MHV}} = \frac{1}{2}\sum_{i<j<k<l<i}$$

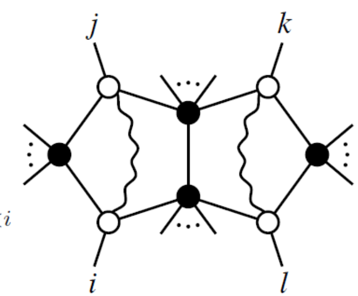


$$\mathcal{A}^{2-\mathrm{loop}}_{\mathrm{NMHV}} = \sum_{\substack{i<j<l<m\leq k<i\\ i<j<k<l<m\leq i\\ i\leq l<m\leq j<k<i}}$$



$$\times\,[i,j,j+1,k,k+1]$$

$$+\frac{1}{2}\sum_{i<j<k<l<i}$$



$$\times\left\{\begin{array}{c}\mathcal{A}^{\mathrm{tree}}_{\mathrm{NMHV}}(j,\ldots,k;\ l,\ldots,i)\\ +\mathcal{A}^{\mathrm{tree}}_{\mathrm{NMHV}}(i,\ldots,j)\\ +\mathcal{A}^{\mathrm{tree}}_{\mathrm{NMHV}}(k,\ldots,l)\end{array}\right\}$$

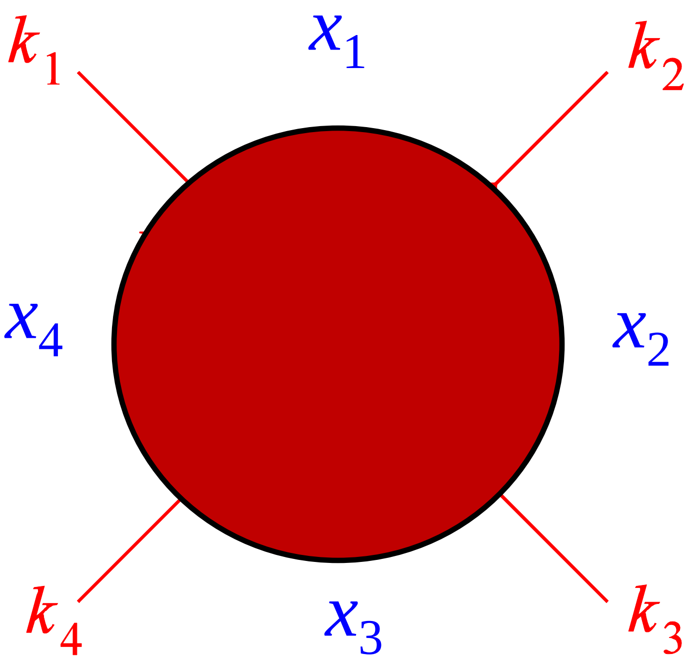
Do we need integrands?

In many cases, symmetries and other constraints on the multi-loop planar N=4 SYM amplitude are so powerful that we don't even need to know the integrand at all! ☺

Dual conformal invariance

Broadhurst (1993); Lipatov (1999); Drummond, Henn, Smirnov, Sokatchev, hep-th/0607160

Conformal symmetry acting in momentum space, on dual or sector variables x_i : $k_i = x_i - x_{i+1}$



invariance under inversion:

$$x_i^\mu \rightarrow \frac{x_i^\mu}{x_i^2}$$

$$x_{ij}^2 \rightarrow \frac{x_{ij}^2}{x_i^2 x_j^2}$$

Dual conformal constraints

- Symmetry fixes form of amplitude, up to functions of dual conformally invariant cross ratios:

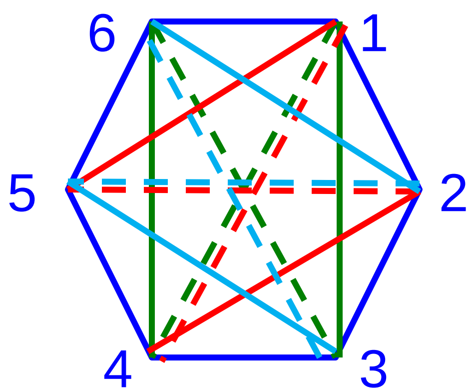
$$u_{ijkl} \equiv \frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2}$$

- Because $x_{i-1,i}^2 = k_i^2 = 0$ there are no such variables for n = 4,5
- Amplitude fixed to BDS ansatz: $\boxed{\mathcal{A}_{4,5}(\epsilon; s_{ij}) \ = \ \mathcal{A}_{4,5}^{\mathrm{BDS}}(\epsilon; s_{ij})}$

For $n = 6$, precisely 3 ratios:

$$u_1 = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2} = \frac{s_{12} s_{45}}{s_{123} s_{345}}$$

+ 2 cyclic perm's



$$\boxed{\mathcal{A}_6(\epsilon; s_{ij}) \ = \ \mathcal{A}_6^{\mathrm{BDS}}(\epsilon; s_{ij}) \exp[R_6(u_1, u_2, u_3)]}$$

MHV (--++++)

Formula for $R_6^{(2)}(u_1,u_2,u_3)$

• First worked out analytically from Wilson loop integrals
Del Duca, Duhr, Smirnov, 0911.5332, 1003.1702
17 pages of Goncharov polylogarithms.

• Simplified to just a few classical polylogarithms using symbology
Goncharov, Spradlin, Vergu, Volovich, 1006.5703

$$\boxed{\begin{aligned} R_6^{(2)}(u_1,u_2,u_3) &= \sum_{i=1}^{3}\left(L_4(x_i^+,x_i^-) - \frac{1}{2}\mathrm{Li}_4(1-1/u_i)\right) \\ &- \frac{1}{8}\left(\sum_{i=1}^{3}\mathrm{Li}_2(1-1/u_i)\right)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72} \end{aligned}}$$

$$L_4(x^+,x^-) = \frac{1}{8!!}\log(x^+x^-)^4 + \sum_{m=0}^{3}\frac{(-1)^m}{(2m)!!}\log(x^+x^-)^m(\ell_{4-m}(x^+)+\ell_{4-m}(x^-))$$

$$\ell_n(x) = \frac{1}{2}\left(\mathrm{Li}_n(x) - (-1)^n\,\mathrm{Li}_n(1/x)\right)$$

$$J = \sum_{i=1}^{3}(\ell_1(x_i^+) - \ell_1(x_i^-))$$

$$\boxed{x_i^\pm = u_i x^\pm, \qquad x^\pm = \frac{u_1+u_2+u_3-1\pm\sqrt{\Delta}}{2u_1u_2u_3} \qquad \Delta = (u_1+u_2+u_3-1)^2 - 4u_1u_2u_3}$$

Variables for $R_6^{(L)}(u_1,u_2,u_3)$

- A "pure" transcendental function of weight $2L$:

Differentiation gives a linear combination of weight $2L$-1 pure functions, divided by rational expressions r_i.

- Here,

$$r_i \in \{u_1, u_2, u_3, 1-u_1, 1-u_2, 1-u_3, y_1, y_2, y_3\}$$

with

$$y_i \equiv \frac{u_i - z_+}{u_i - z_-}$$

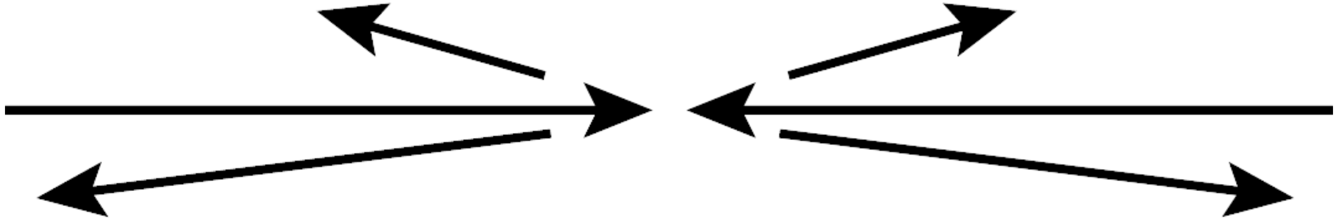
$$z_\pm = \frac{1}{2}\Big[-1 + u_1 + u_2 + u_3 \pm \sqrt{\Delta}\Big]$$

$$\Delta = (1 - u_1 - u_2 - u_3)^2 - 4u_1u_2u_3$$

y_i depend on u_i via square roots

The multi-Regge limit

- $R_6^{(2)}(u_1,u_2,u_3)$ is pretty simple. But to go to very high loop order, we take the limit of multi-Regge kinematics (MRK): large rapidity separations between the 4 final-state gluons:

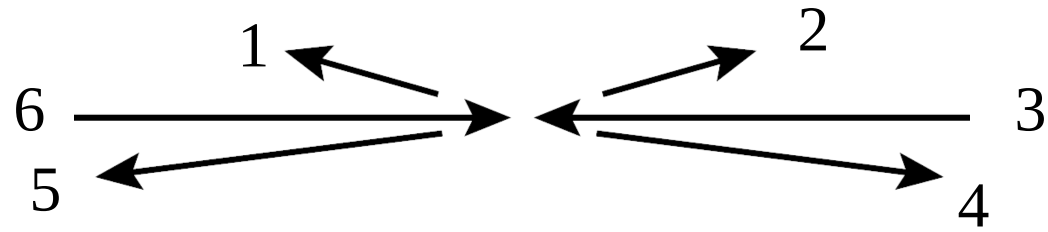


- Properties of planar N=4 SYM amplitude in this limit studied extensively already:

Bartels, Lipatov, Sabio Vera, 0802.2065, 0807.0894; Lipatov, 1008.1015;
Lipatov, Prygarin, 1008.1016, 1011.2673;
Bartels, Lipatov, Prygarin, 1012.3178, 1104.4709;
LD, Drummond, Henn, 1108.4461; Fadin, Lipatov, 1111.0782

Multi-Regge kinematics

$$u_1 = \frac{s_{12}^2 s_{45}^2}{s_{123}^2 s_{345}^2} \to 1$$



$$\frac{u_2}{1-u_1} \to x$$

$$\frac{u_3}{1-u_1} \to y$$

A very nice change of variables [LP, 1011.2673] is to (w, w^*) :

$$x = \frac{1}{(1+w)(1+w^*)}$$

$$y = \frac{ww^*}{(1+w)(1+w^*)}$$

$$y_1 \to 1$$

$$y_2 \to \frac{1+w^*}{1+w}$$

$$y_3 \to \frac{(1+w)w^*}{w(1+w^*)}$$

2 symmetries: conjugation $w \leftrightarrow w^*$
and inversion $w \leftrightarrow 1/w, \ w^* \leftrightarrow 1/w^*$

Physical 2→4 multi-Regge limit

- To get a nonzero result, for the physical region, one must first let $u_1 \to u_1\, e^{-2\pi i}$, and extract one or two discontinuities → factors of $-2\pi i$.
- Then let $u_1 \to 1$.

Bartels, Lipatov, Sabio Vera, 0802.2065, …

$$\boxed{R_6^{(L)} \to (2\pi i) \sum_{n=0}^{L-1} \ln^n(1-u_1)\, [g_n^{(L)}(w,w^*) + 2\pi i\, h_n^{(L)}(w,w^*)]}$$

imaginary part, from single discontinuity

real part, from double discontinuity

Simpler pure functions

$$R_6^{(L)} \to (2\pi i) \sum_{n=0}^{L-1} \ln^n(1-u_1) \, [g_n^{(L)}(w,w^*) + 2\pi i \, h_n^{(L)}(w,w^*)]$$

weight $2L$-n-1 $2L$-n-2

$$r_i \in \{w, 1+w, w^*, 1+w^*\}$$

- Single-valued in $(w,w^*) = (-z,-\bar{z})$ plane

This is precisely the class of functions defined by Brown:
F.C.S. Brown, C. R. Acad. Sci. Paris, Ser. I 338 (2004).

SVHPLs: $\mathcal{L}_{m_1,\ldots,m_w}(z,\bar{z}) \sim \sum_{i,j} c_{i,j} H_{\vec{m}_i}(z) H_{\vec{m}_j}(\bar{z})$

H = ordinary harmonic polylogarithms

Remiddi, Vermaseren, hep-ph/9905237

Harmonic Polylogarithms (HPLs)

Remiddi, Vermaseren, hep-ph/9905237

w = word formed from noncommuting letters x_0, x_1

$$\frac{\partial}{\partial z} H_{x_0 w}(z) = \frac{H_w(z)}{z} \qquad \frac{\partial}{\partial z} H_{x_1 w}(z) = \frac{H_w(z)}{1-z}$$

$$H_{x_0 w}(z) = \int_0^z dz' \, \frac{H_w(z')}{z'} \qquad H_{x_1 w} = \int_0^z dz' \, \frac{H_w(z')}{1-z'}$$

Special cases:

$$H_e(z) = 1 \qquad H_{x_0^n}(z) = \frac{1}{n!} \log^n z$$

Shuffle identity:

$$H_{w_1}(z) \, H_{w_2}(z) = \sum_{w \in w_1 \sqcup\!\sqcup \, w_2} H_w(z)$$

Shorthand example:

$$H_w(z) = H_{x_0 x_0 x_1 x_0 x_1}(z) = H_{0,0,1,0,1}(z) = H_{3,2}(z)$$

Brown construction of SVHPLs

$$\frac{\partial}{\partial z}\mathcal{L}_{x_0 w}(z,\bar{z}) = \frac{\mathcal{L}_w(z,\bar{z})}{z} \qquad \frac{\partial}{\partial z}\mathcal{L}_{x_1 w}(z,\bar{z}) = \frac{\mathcal{L}_w(z,\bar{z})}{1-z}$$

Special cases: $\mathcal{L}_e = 1 \qquad \mathcal{L}_{x_0^n} = \frac{1}{n!}\log^n |z|^2$

Shuffle identity: $\mathcal{L}_{w_1}\mathcal{L}_{w_2} = \sum_{w \in w_1 ш w_2} \mathcal{L}_w$

Main formula:

$$\boxed{\begin{gathered}\mathcal{L}(z,\bar{z}) = L_X(z)\tilde{L}_Y(\bar{z}) \equiv \sum_{w\in X^*}\mathcal{L}_w(z,\bar{z})w \\ L_X(z) = \sum_{w\in X^*} H_w(z)w \qquad \tilde{L}_Y(\bar{z}) = \sum_{w\in Y^*} H_{\phi(w)}(\bar{z})\tilde{w}\end{gathered}}$$

word reversal operator " ~ " $\qquad \phi$ renames y to x

The y alphabet

- Related to the x alphabet using the Drin'feld associator:

$$Z(x_0, x_1) = \sum_{w \in X^*} \zeta(w) w$$

and definition

$$\boxed{\tilde{Z}(y_0, y_1) y_1 \tilde{Z}(y_0, y_1)^{-1} = Z(x_0, x_1)^{-1} x_1 Z(x_0, x_1)}$$

$$\rightarrow \quad \begin{aligned} y_1 = \; & x_1 + \zeta_3(2x_0x_0x_1x_1 - 4x_0x_1x_0x_1 + 2x_0x_1x_1x_1 \\ & + 4x_1x_0x_1x_0 - 6x_1x_0x_1x_1 - 2x_1x_1x_0x_0 \\ & + 6x_1x_1x_0x_1 - 2x_1x_1x_1x_0) \\ & + \ldots \end{aligned}$$

Example: $$\begin{aligned} \mathcal{L}_{0,0,1,1}(z, \bar{z}) = \; & H_{0,0,1,1} + \overline{H}_{1,1,0,0} + H_{0,0,1} \overline{H}_1 \\ & + H_0 \overline{H}_{1,1,0} + H_{0,0} \overline{H}_{1,1} - 2\zeta_3 \overline{H}_1 \end{aligned}$$

y alphabet and $\bar{z}$ derivatives

$$\frac{\partial}{\partial z}\mathcal{L}_{x_0 w}(z,\bar{z}) = \frac{\mathcal{L}_w(z,\bar{z})}{z} \qquad \frac{\partial}{\partial z}\mathcal{L}_{x_1 w}(z,\bar{z}) = \frac{\mathcal{L}_w(z,\bar{z})}{1-z}$$

$$\Leftrightarrow \quad \frac{\partial}{\partial z}\mathcal{L}(z,\bar{z}) = \left(\frac{x_0}{z} + \frac{x_1}{1-z}\right)\mathcal{L}(z)$$

but

$$\frac{\partial}{\partial \bar{z}}\mathcal{L}(z,\bar{z}) = \mathcal{L}(z)\left(\frac{y_0}{\bar{z}} + \frac{y_1}{1-\bar{z}}\right)$$

Z_2 x Z_2 symmetry

- $z \leftrightarrow \bar{z}$

$$L_w(z) = \frac{1}{2}\Big(\mathcal{L}_w(z) - (-1)^{|w|}\,\mathcal{L}_w(\bar{z})\Big)$$

$$\cancel{\overline{L}_w(z) = \frac{1}{2}\Big(\mathcal{L}_w(z) + (-1)^{|w|}\mathcal{L}_w(\bar{z})\Big)}$$

reducible to products of lower weight

- $z \leftrightarrow 1/z$

$$\cancel{L_w(z) - (-1)^{|w|+d_w} L_w(\tfrac{1}{z})}$$

reducible to products of lower weight

$$\boxed{L_w^{\pm}(z) \equiv \frac{1}{2}\left[L_w(z) \pm L_w(\tfrac{1}{z})\right]}$$

Keep the irreducible one

MRK Master Formula

Fadin, Lipatov, 1111.0782

$$\boxed{\begin{aligned} e^{R+i\pi\delta}|_{\text{MRK}} \;=\; & \cos\pi\omega_{ab} + i\,\frac{a}{2}\sum_{n=-\infty}^{\infty}(-1)^n\left(\frac{w}{w^*}\right)^{\frac{n}{2}}\int_{-\infty}^{+\infty}\frac{d\nu}{\nu^2+\frac{n^2}{4}}\,|w|^{2i\nu}\,\Phi_{\text{Reg}}(\nu,n) \\ & \times\exp\left[-\omega(\nu,n)\left(\log(1-u_1)+i\pi+\frac{1}{2}\log\frac{|w|^2}{|1+w|^4}\right)\right] \end{aligned}}$$

BFKL eigenvalue

MHV impact factor

$$\omega(\nu,n) = -a(E_{\nu,n} + a\,E^{(1)}_{\nu,n} + a^2\,E^{(2)}_{\nu,n} + \cdots)$$

$$\Phi_{\text{Reg}}(\nu,n) = 1 + a\Phi^{(1)}_{\nu,n} + a^2\Phi^{(2)}_{\nu,n} + a^3\Phi^{(3)}_{\nu,n} + \cdots$$

LL NLL NNLL NNNLL

Formula may get corrections beyond NLL

Evaluating the master formula

- Every $g_n^{(L)}(w,w^*)$ and $h_n^{(L)}(w,w^*)$

is a linear combination of a finite basis of SVHPLs.

- Evaluate ν integral by residues

  → master formula leads to double sum.

- Truncating double sum ←→ truncating power series in $(w,w^*) = (-z,-\bar{z})$ around origin.
- Match the two series to determine the coefficients in the linear combination.
- LL and NLL ω and Φ known

Fadin, Lipatov 1111.0782

MHV LLA $g_{L\text{-}1}^{(L)}$ through 5 loops

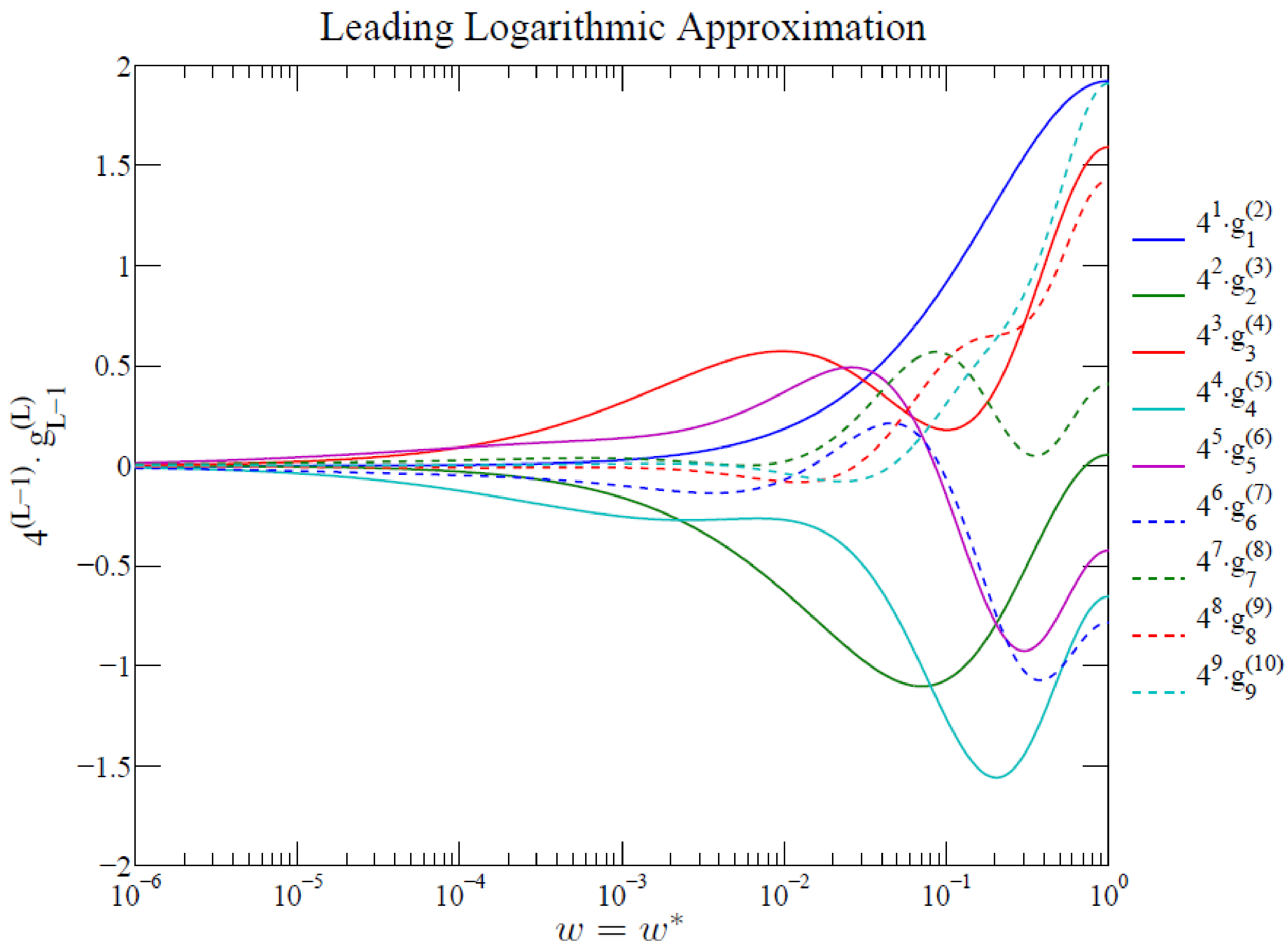
$$g_1^{(2)}(w,w^*) = \frac{1}{4}[L_1^+]^2 - \frac{1}{16}[L_0^-]^2 = \frac{1}{4}\ln|1+w|^2 \ln\frac{|1+w|^2}{|w|^2}$$

$$\begin{aligned} g_2^{(3)}(w,w^*) = -\frac{1}{8}L_3^+ + \frac{1}{12}[L_1^+]^3 \;=\; & -\frac{1}{8}\Big[\mathrm{Li}_3(-w) + \mathrm{Li}_3(-w^*) - \frac{1}{2}\ln|w|^2(\mathrm{Li}_2(-w)+\mathrm{Li}_2(-w^*)) \\ & + \ln|1+w|^2(\frac{2}{3}\ln^2|1+w|^2 - \ln|w|^2\ln|1+w|^2 + \frac{1}{4}\ln^2|w|^2)\Big] \end{aligned}$$

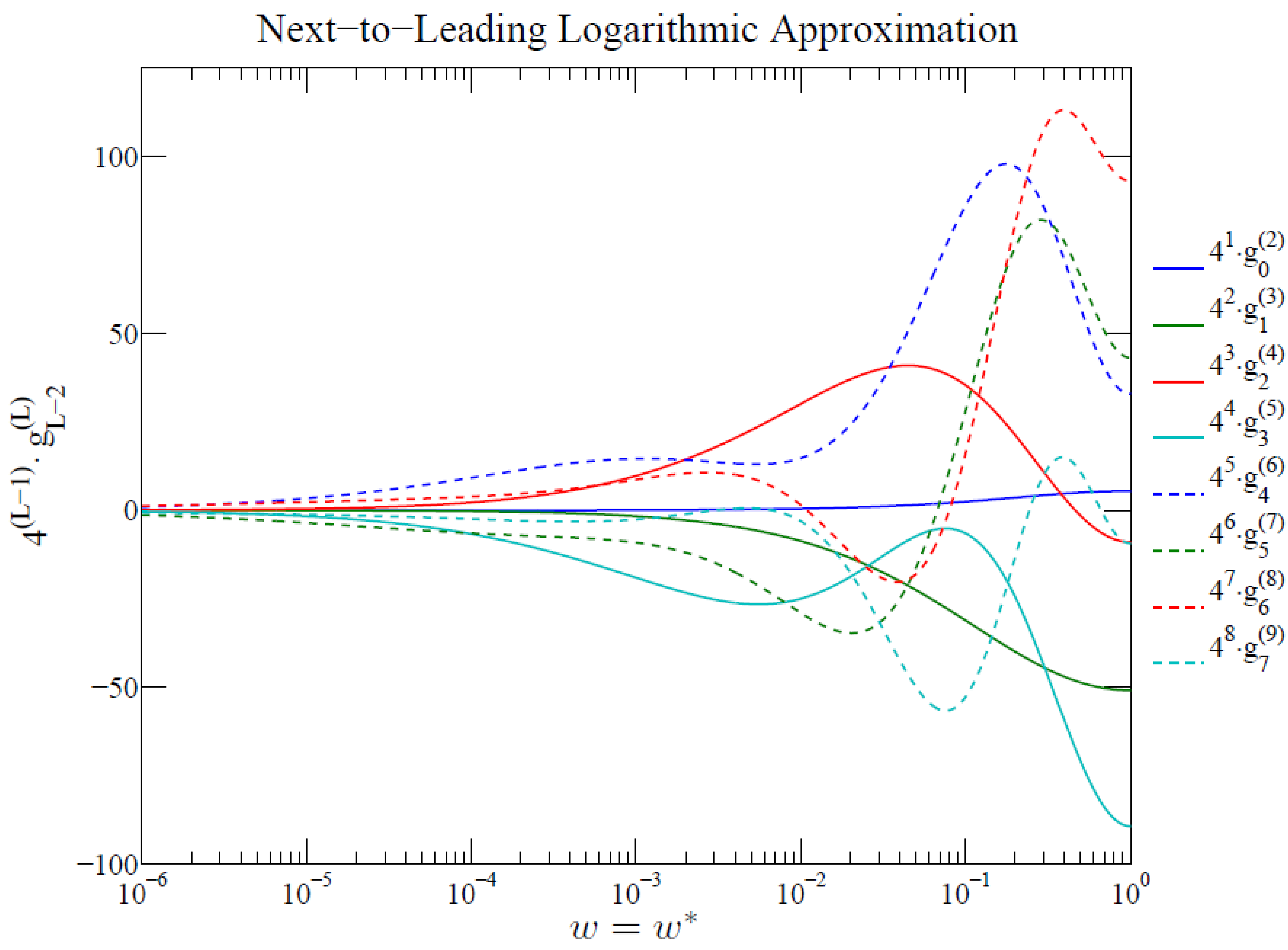
$$\begin{aligned} g_3^{(4)}(w,w^*) \;=\; & \frac{1}{48}[L_2^-]^2 + \frac{1}{48}[L_0^-]^2[L_1^+]^2 + \frac{7}{2304}[L_0^-]^4 + \frac{1}{48}[L_1^+]^4 - \frac{1}{16}L_0^- L_{2,1}^- \\ & - \frac{5}{48}L_1^+ L_3^+ - \frac{1}{8}L_1^+\zeta_3\,, \end{aligned}$$

$$\begin{aligned} g_4^{(5)}(w,w^*) \;=\; & \frac{1}{96}[L_0^-]^2[L_1^+]^3 + \frac{17}{9216}L_1^+[L_0^-]^4 - \frac{5}{384}L_3^+[L_0^-]^2 + \frac{1}{24}[L_0^-]^2\zeta_3 \\ & - \frac{1}{12}[L_1^+]^2\zeta_3 + \frac{1}{240}[L_1^+]^5 - \frac{1}{24}L_0^- L_{2,1}^- L_1^+ + \frac{43}{384}L_5^+ + \frac{1}{8}L_{3,1,1}^+ + \frac{1}{12}L_{2,2,1}^+ \\ & - \frac{1}{24}L_3^+[L_1^+]^2\,, \end{aligned}$$

MHV LLA $g_{L\text{-}1}{}^{(L)}$ through 10 loops



MHV NLLA $g_{L\text{-}2}^{(L)}$ through 9 loops



LLA to all orders

Pennington, 1209.5357

$$\eta = a \log(1 - u_1)$$

$$\rho(w) \equiv \mathcal{L}_w$$

$$R_6^{\rm MHV}\big|_{\rm LLA} = \frac{2\pi i}{\log(1-u_1)}\,\rho\Big(\mathcal{X}\,\mathcal{Z}^{\rm MHV} - \frac{1}{2}\,x_1\eta\Big)$$

$$\mathcal{X} = e^{\frac{1}{2}x_0\eta}\left[1 - x_1\left(\frac{e^{x_0\eta}-1}{x_0}\right)\right]^{-1},$$

$$\mathcal{Z}^{\rm MHV} = \frac{1}{2}\sum_{k=1}^{\infty}\left(x_1\sum_{n=0}^{k-1}(-1)^n x_0^{k-n-1}\sum_{m=0}^{n}\frac{2^{2m-k+1}}{(k-m-1)!}\,\mathfrak{Z}(n,m)\right)\eta^k$$

$$\exp\left[y\sum_{k=1}^{\infty}\zeta_{2k+1}x^{2k+1}\right] \equiv \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\mathfrak{Z}(n,m)\,x^n y^m$$

Matches series expansion through L = 14. All orders proof?

$\mathrm{N^kDLLA}$ limit to all orders

Collinear-Regge limit as $|w| \rightarrow 0$

Pennington, 1209.5357

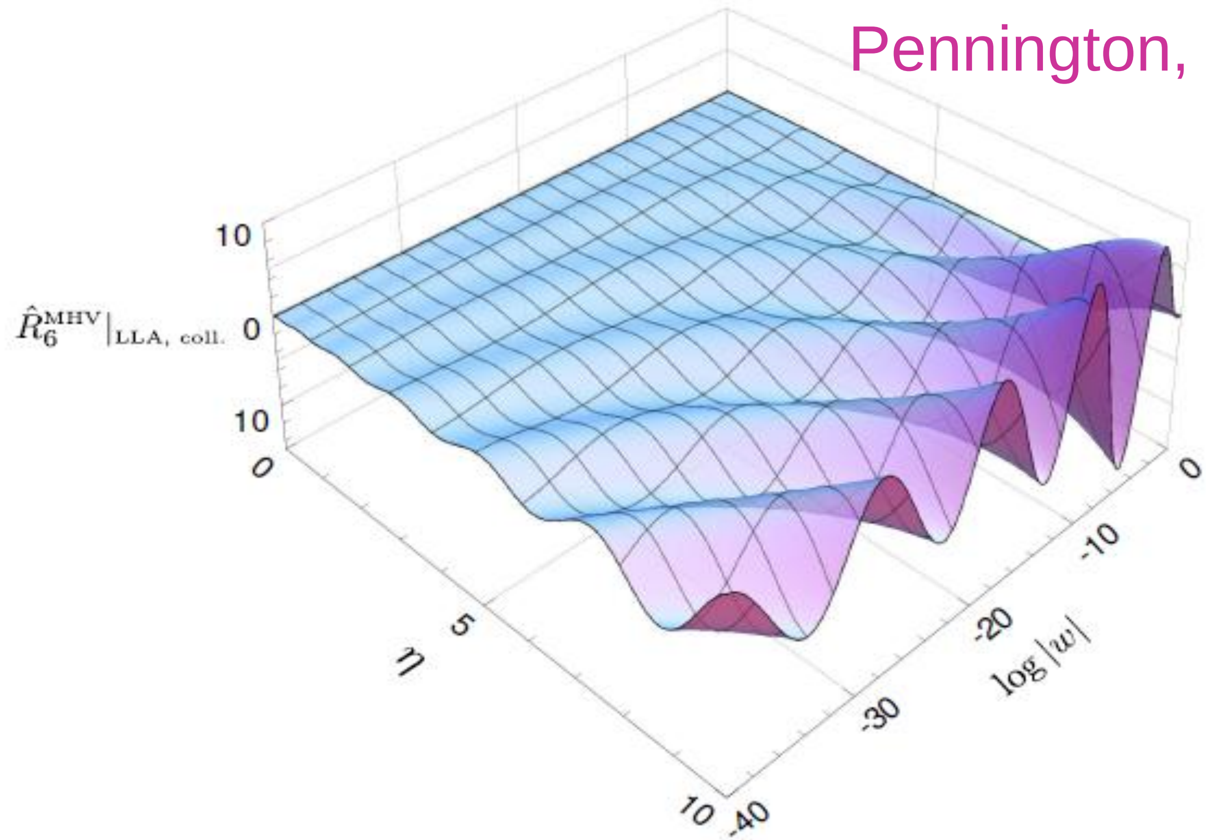
$$R_6^{\mathrm{MHV}}|_{\mathrm{LLA,\ coll.}} = \frac{2\pi i}{\log(1-u_1)}(w+w^*)\sum_{k=0}^{\infty}\eta^{k+1}\, r_k^{\mathrm{MHV}}(\eta\log|w|)$$

$$r_k^{\mathrm{MHV}}(x) = \frac{1}{2}\,\delta_{0,k} + \sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{j=k-m}^{2k-n-m}\frac{(-2)^{2m+j-k-1}}{(m+j-k)!}\,\mathfrak{Z}(n,m)\,x^{m-k+j/2}\,P_j^{(k-j-n,k-j-m)}(0)\,I_j(2\sqrt{x}\,)$$

$r_0(x)$ matches known DLLA result Bartels, Lipatov, Prygarin, 1104.4709

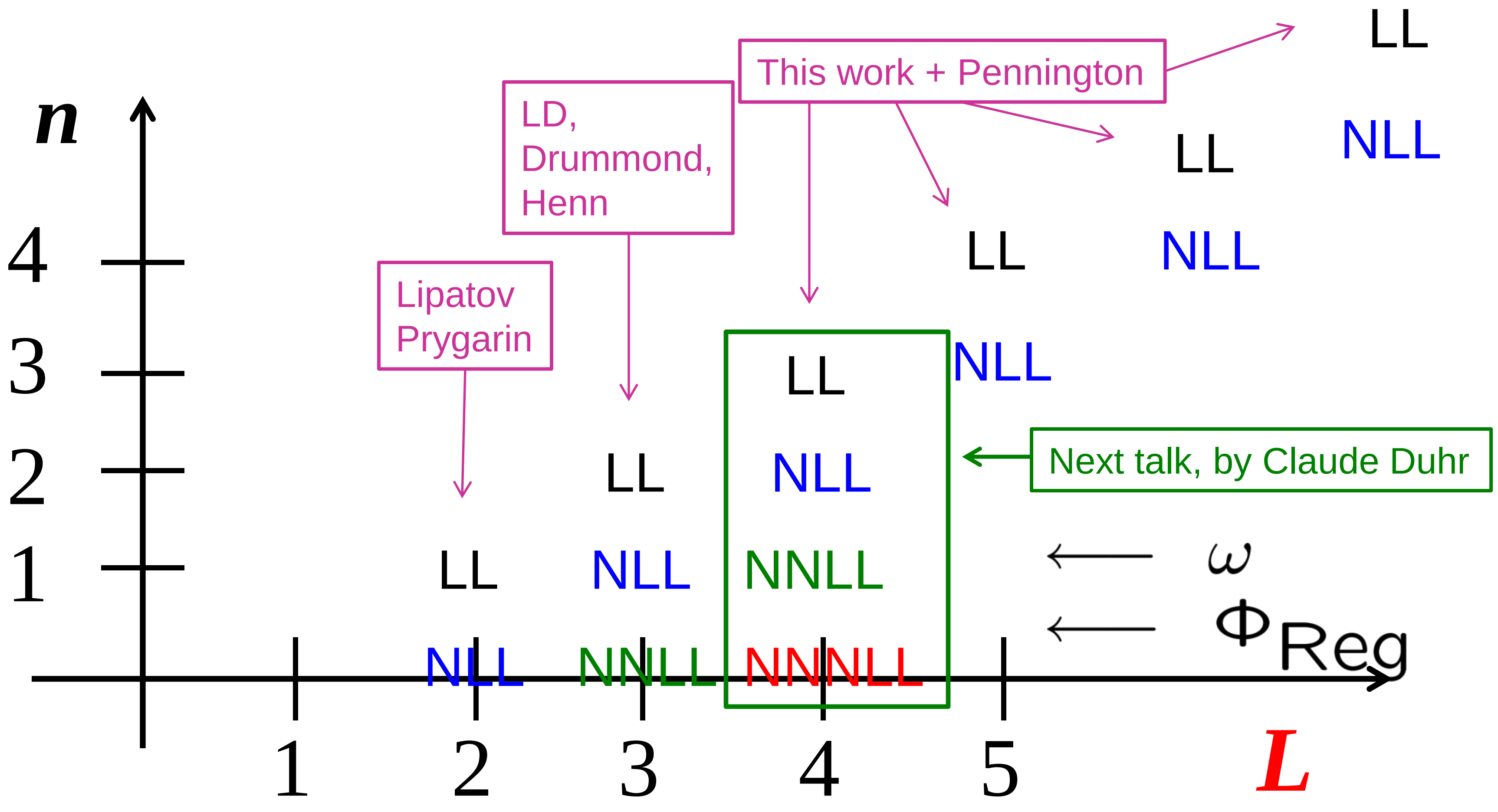
N^{k}DLLA limit to all orders

Pennington, 1209.5357



Should try to match to strong coupling results
Bartels, Kotanski, Schomerus, 1009.3938

Beyond NLL



(modulo some "beyond-the-symbol" constants starting at NNLL)

Same functions enter NMHV

Lipatov, Prygarin, Schnitzer, 1205.0186

Ratio of nonvanishing NMHV to MHV amplitudes in MRK is given by:

$$\mathcal{P}_{\mathrm{NMHV}}^{\mathrm{LLA}} = \frac{A_{\mathrm{NMHV}}^{\mathrm{LLA}}}{A_{\mathrm{MHV}}^{\mathrm{LLA}}} = \exp(R_{\mathrm{NMHV}}^{\mathrm{LLA}} - R_{\mathrm{MHV}}^{\mathrm{LLA}})$$

$$R_{\mathrm{NMHV}}^{\mathrm{LLA}} = 2\pi i \sum_{l=2}^{\infty} a^l \log^{l-1}(1-u_1) \left[\frac{1}{1+w^*} f^{(l)}(w,w^*) + \frac{w^*}{1+w^*} f^{(l)}\left(\frac{1}{w},\frac{1}{w^*}\right) \right]$$

$f^{(l)}(w,,w^*)$ a pure function

$$\int dw \, \frac{w^*}{w} \frac{\partial}{\partial w^*} g_{l-1}^{(l)}(w,w^*) = \frac{1}{1+w^*} f^{(l)}(w,w^*) + \frac{w^*}{1+w^*} f^{(l)}\left(\frac{1}{w},\frac{1}{w^*}\right)$$

Very simple action in Brown SVHPL basis

NMHV example

$$g_1^{(2)}(w,w^*) \;= \frac{1}{4}\,[L_1^+]^2 - \frac{1}{16}\,[L_0^-]^2 = \frac{1}{2}\mathcal{L}_{1,1} + \frac{1}{4}\mathcal{L}_{0,1} + \frac{1}{4}\mathcal{L}_{1,0}$$

First clip last index, with factor depending on whether it was `0’ or `1’ (up to ζ corrections):

$$\begin{aligned} w^*\frac{\partial}{\partial w^*}g_1^{(2)}(w,w^*) &= -\frac{1}{2}\left(\frac{w^*}{1+w^*}\right)\mathcal{L}_1 - \frac{1}{4}\left(\frac{w^*}{1+w^*}\right)\mathcal{L}_0 + \frac{1}{4}\mathcal{L}_1 \\ &= \frac{w^*}{1+w^*}\left[-\frac{1}{4}\,\mathcal{L}_1 - \frac{1}{4}\,\mathcal{L}_0\right] + \frac{1}{1+w^*}\left[\frac{1}{4}\,\mathcal{L}_1\right] \end{aligned}$$

Then prepend a `0’:

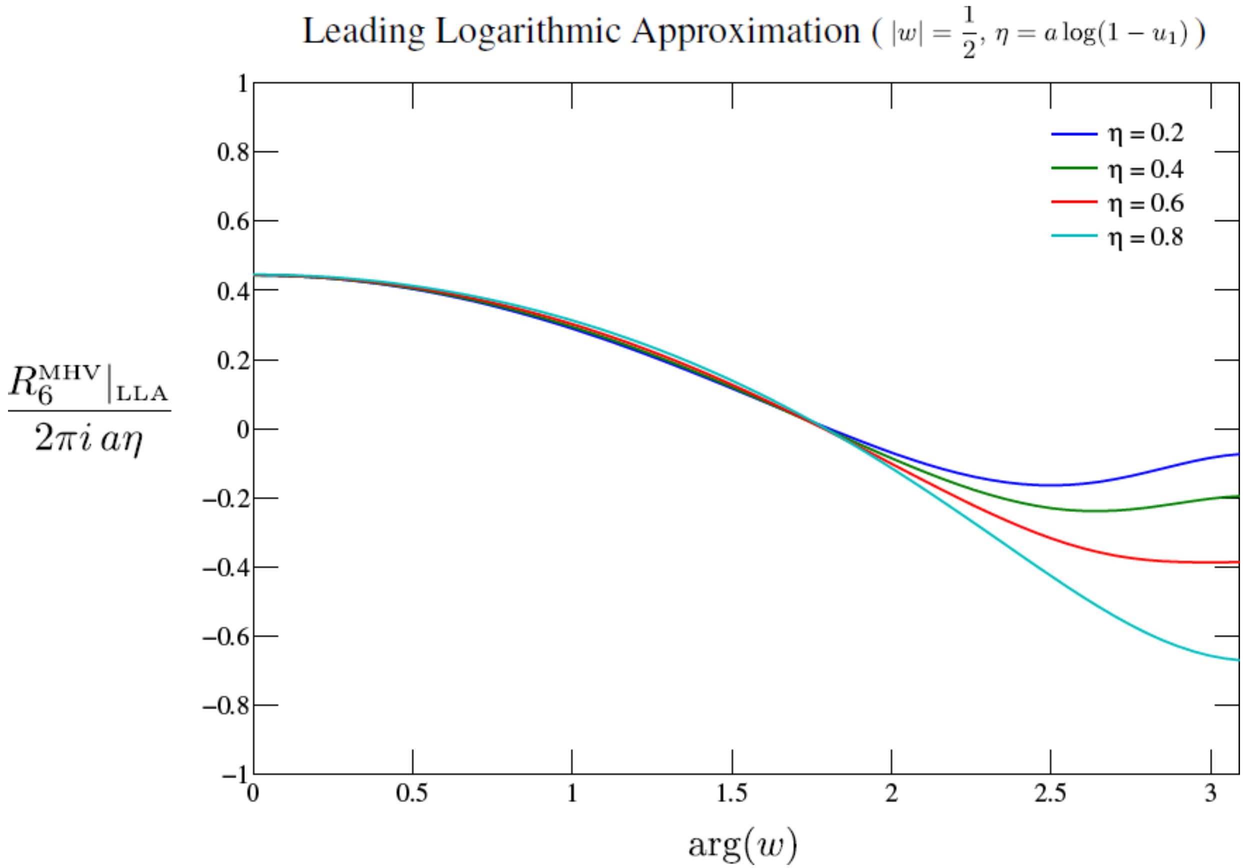
$$\begin{aligned} \int dw\,\frac{w^*}{w}\,\frac{\partial}{\partial w^*}g_1^{(2)} &= \frac{w^*}{1+w^*}\left[-\frac{1}{4}\,\mathcal{L}_{0,1} - \frac{1}{4}\,\mathcal{L}_{0,0}\right] + \frac{1}{1+w^*}\left[\frac{1}{4}\,\mathcal{L}_{0,1}\right] \\ &= \frac{1}{1+w^*}f^{(2)}(w,w^*) + \frac{w^*}{1+w^*}f^{(2)}\left(\frac{1}{w},\frac{1}{w^*}\right) \end{aligned}$$

Conclusions

- Planar N=4 SYM is a powerful laboratory for studying 4-d scattering amplitudes, thanks to dual (super)conformal invariance
- 6-gluon amplitude is first nontrivial case
- Multi-Regge limit offers a simpler setup to solve first, greatly facilitated by Brown's SVHPLs
- NMHV amplitudes in this limit also naturally described by same functions
- Multi-Regge limit of 6-gluon amplitude can be solved to all orders in LLA, especially in collinear corner ($|w| \rightarrow 0$).
- Bessel functions suggest integrability, localization.
- May be that full multi-Regge limit (i.e. N^kLLA terms) is next to be solved to all orders in the coupling?

Extra Slides

LLA Numerics for fixed $|w|$



Would be interesting to compare with numerical approach of Chachamis, Sabio Vera, 1112.4162, 1206.3140