Holographic Entanglement and Interaction

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"Intrication holographique et interaction" à l'IHES le 30 janvier 2014

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[S. Ryu and T. Takayanagi, Phys.Rev.Lett. 96 (2006) 181602]

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[J. Maldacena and L. Susskind, arXiv:1306.0533]

3. Accelerating quark and anti-quark

[K. Jensen and A. Karch, Phys.Rev.Lett. 111 (2013) 211602]

4. Gluon scattering

[SS and S.-J. Sin, to appear]

Quantum entanglement and holography

Quantum entanglement

Let's consider two systems: A and B

Hilbert space: $\mathcal{H}_A \otimes \mathcal{H}_B$

$\{|i\rangle_A\}$ $\{|i\rangle_B\}$ basis

A general state in total system is denoted by

$$|\psi\rangle_{\rm tot} = \sum_{i,j} c_{ij} |i\rangle_A \otimes |j\rangle_B$$

If $c_{ij} = c_i^A c_j^B$, it is a non-entangled state.

If $c_{ij} \neq c_i^A c_j^B$, it is an entangled state.

What is an order parameter of entanglement?

⇨ Entanglement Entropy

Density matrix: for $|\Psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$

$$\rho_{\rm tot} = |\Psi\rangle\langle\Psi|$$

Reduced density matrix

$$\rho_A := {\rm tr}_B \rho_{\rm tot} = \sum_j {}_B\langle j|(|\Psi\rangle\langle\Psi|)|j\rangle_B$$

The entanglement entropy is defined by Von Neumann entropy as

$$S_A := -{\rm tr}(\rho_A \log \rho_A)$$

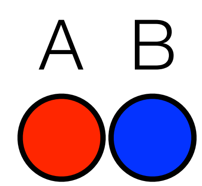
EPR pair

<u>Einstein-Podolsky-Rosen pair</u>

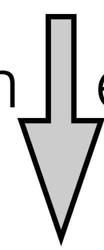
[Einstein-Podolsky-Rosen, Phys.Rev. 47 (1935) 777]

entangled two particles

e.g. a spin-0 particle decays to two spin-1/2 particles.



Separate them from each other at long distance



Observe A $\longrightarrow$ The state of B is determined.

A and B are still entangled.

e.g. Consider the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B \right)$$

Since the reduced density matrix is

$$\rho_A = \frac{1}{2} \left(|\downarrow\rangle_A \langle\downarrow|_A + |\uparrow\rangle_A \langle\uparrow|_A \right)$$

we obtain the entanglement entropy

$$S_A = -\mathrm{tr}(\rho_A \log \rho_A) = \log 2$$

Note that the pure state

$$\rho_P = \frac{1}{2} \left(|\downarrow\rangle_A + |\uparrow\rangle_A \right) \left(\langle\downarrow|_A + \langle\uparrow|_A \right)$$

has zero entanglement entropy

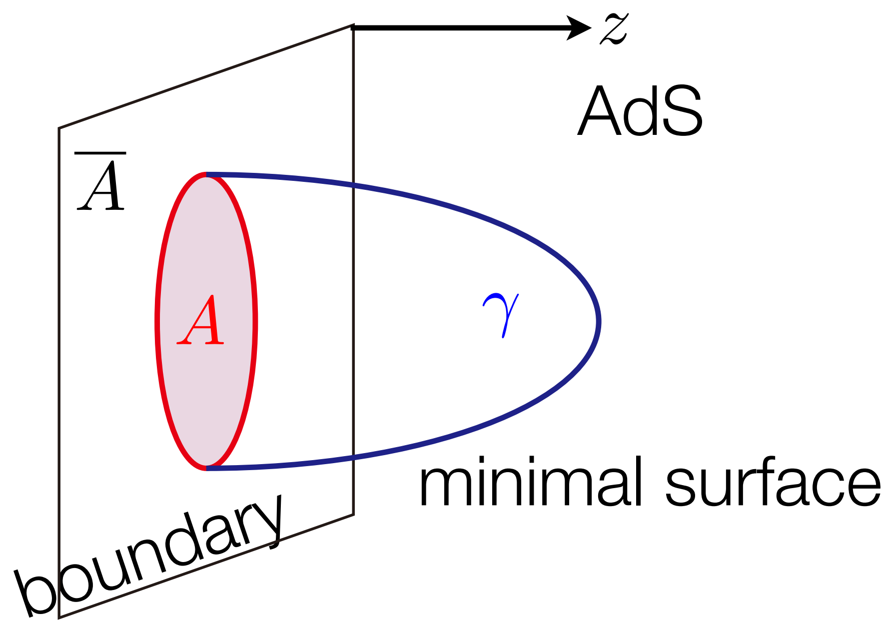
$$S_P = -\mathrm{tr}(\rho_P \log \rho_P) = 0$$

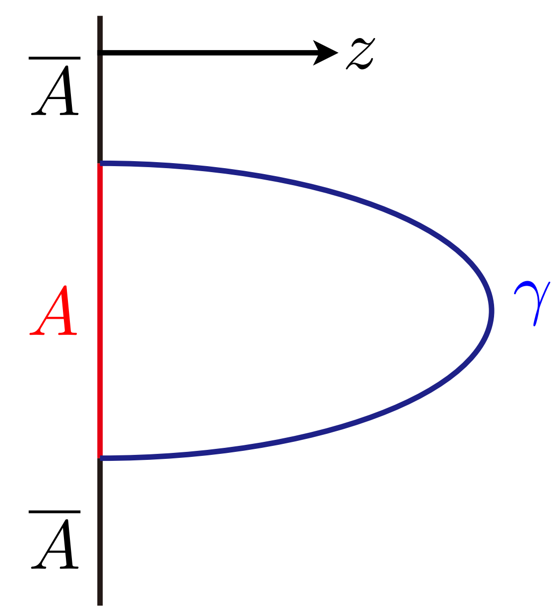
So the entanglement entropy works as an order parameter.

Holographic entanglement entropy

AdS/CFT correspondence (Holography)

Field theory on boundary	Gravity theory in bulk
Partition function	$Z_{\rm CFT} = e^{-S_{\rm grav}}$
Wilson loop	$\langle W \rangle = e^{-{\rm Area}(\gamma)}$
Entanglement entropy	$S_A = \frac{1}{4G_N}{\rm Area}(\gamma)$





[Ryu-Takayanagi, PRL 96 (2006) 181602]

EPR = ER conjecture

ER bridge

[Maldacena-Susskind, arXiv:1306.0533]

Consider the eternal AdS-Schwarzschild black hole.
There are two boundaries and two CFTs.

$$H_{L,R}|n\rangle_{L,R} = E_n|n\rangle_{L,R}$$

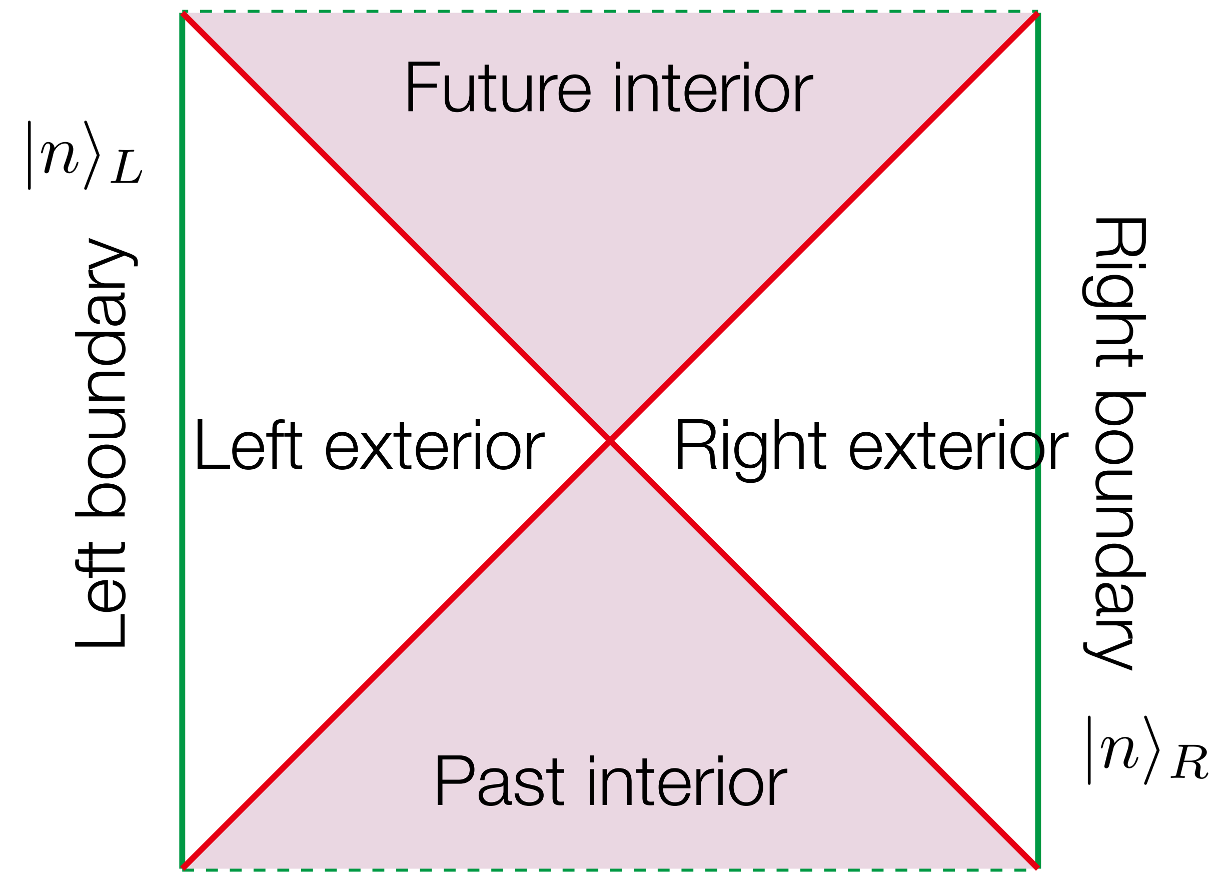
This eternal BH is described by the entangle state,

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2}|n\rangle_L \otimes |n\rangle_R$$

We can interpret this state in two ways:

1. a single black hole in thermal equilibrium

[Israel, Phys.Lett.A57 (1976) 107]

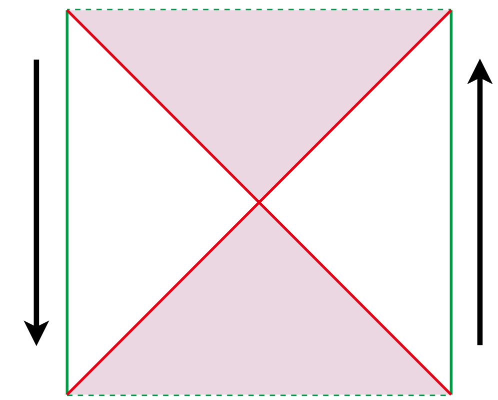


Penrose diagram

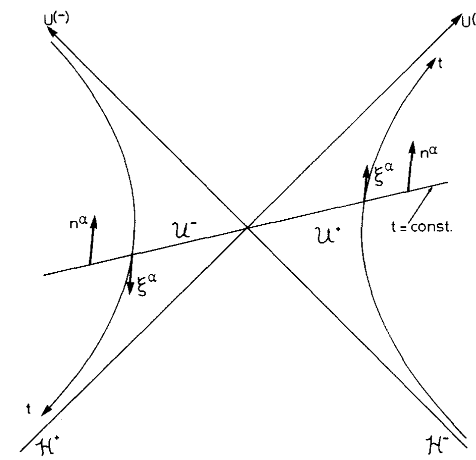
2. two black holes in disconnected spaces with a common time

[Maldacena, hep-th/0106112]

1. a single black hole in thermal equilibrium



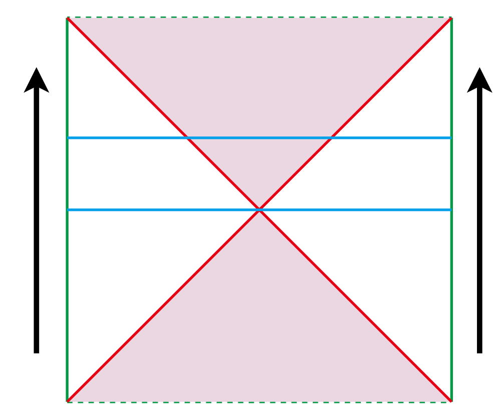
Define a fictitious thermofield Hamiltonian, $H_{\mathrm{tf}} = H_R - H_L$, which generates boosts.



$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |n\rangle_L \otimes |n\rangle_R$$

is an eigenvector of H_{tf} with eigenvalue zero.

2. two black holes in disconnected spaces with a common time



The time evolution is generated by $H = H_R + H_L$

$$|\Psi(t)\rangle = \sum_n e^{-\beta E_n/2} e^{-2iE_n t} |\overline{n}\rangle_L \otimes |n\rangle_R$$

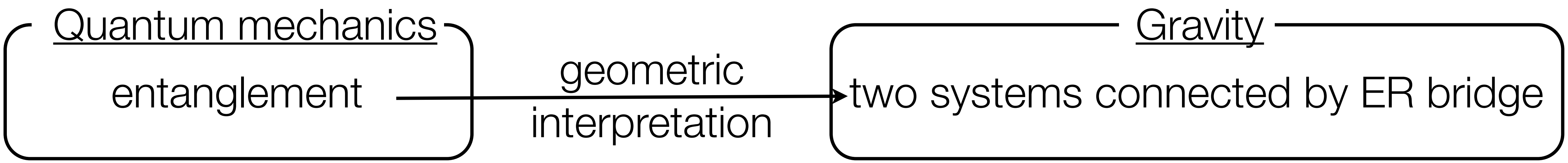
The state $|\Psi\rangle$ describes the two black holes at a specific instant, $t = 0$.

Even though the two black holes exist in separate non-interacting worlds, their geometry is connected by an Einstein-Rosen bridge (a wormhole).

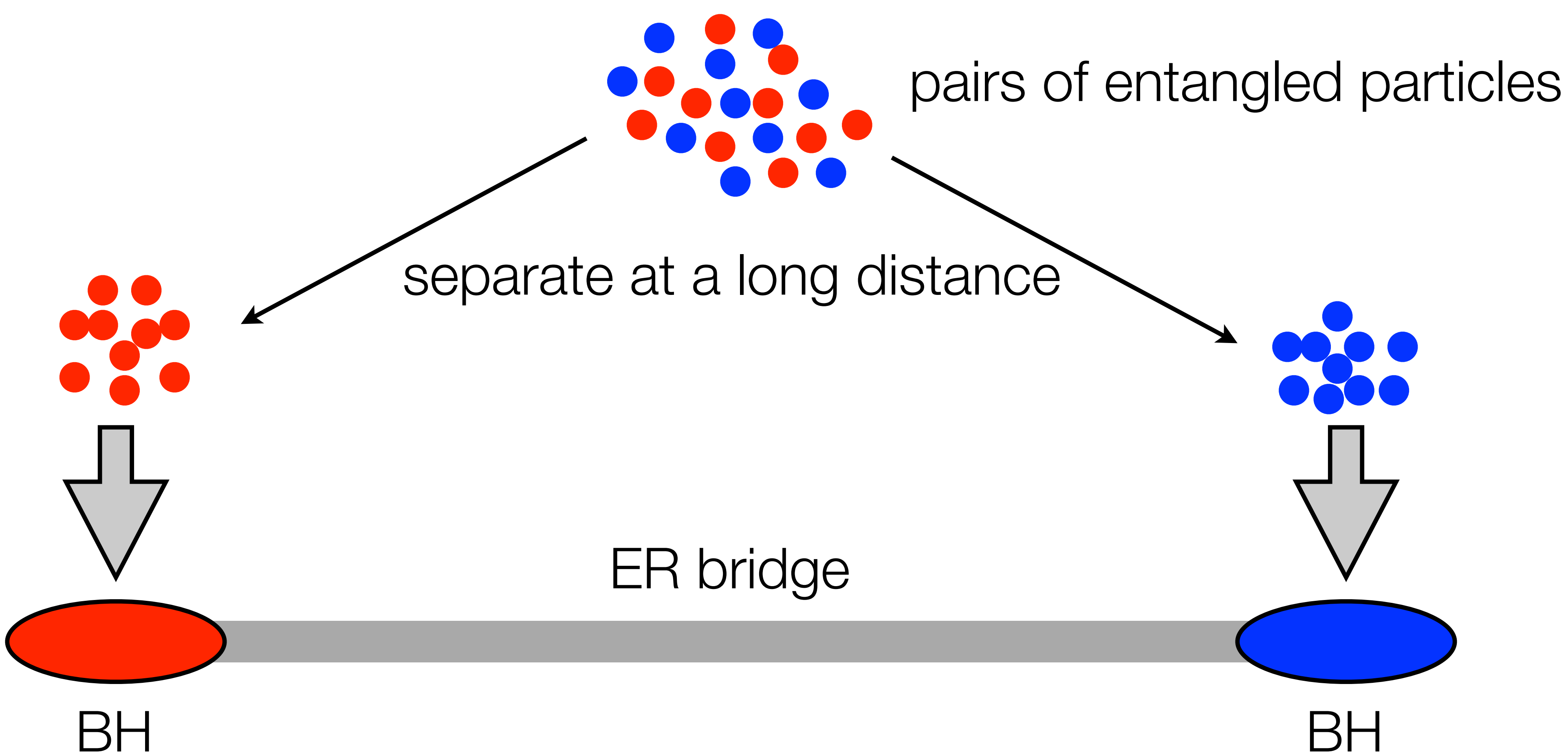
[Einstein-Rosen, Phys.Rev. 48 (1935) 73]

EPR = ER conjecture

From the example of eternal black hole, we studied



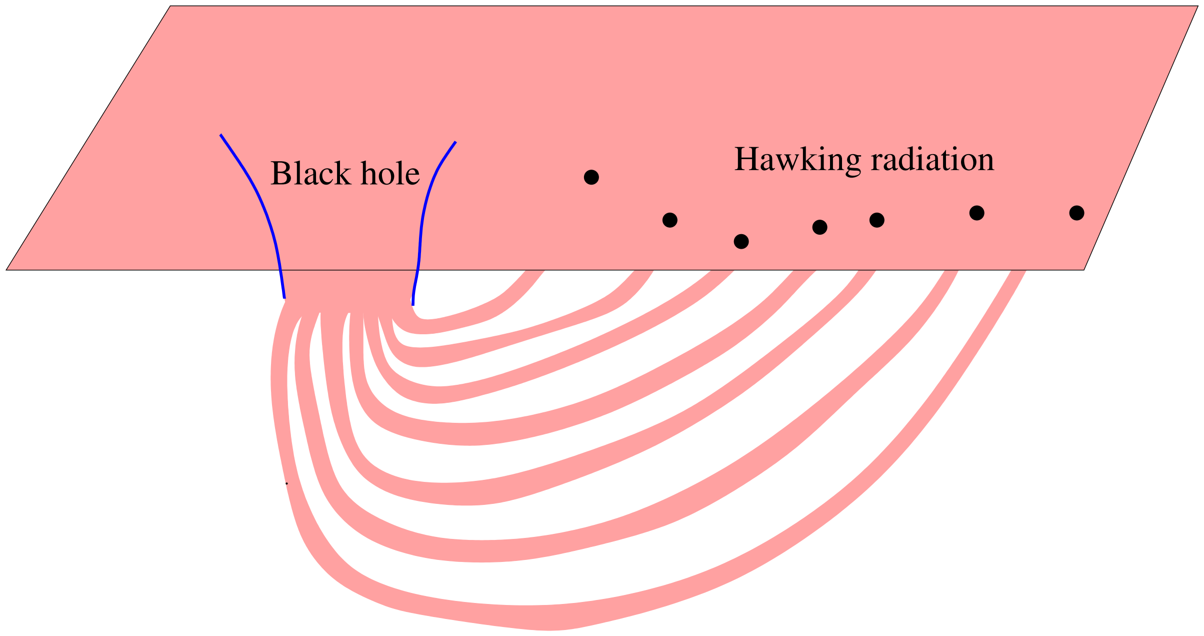
By extending this concept, Maldacena and Susskind conjectured



Further extension

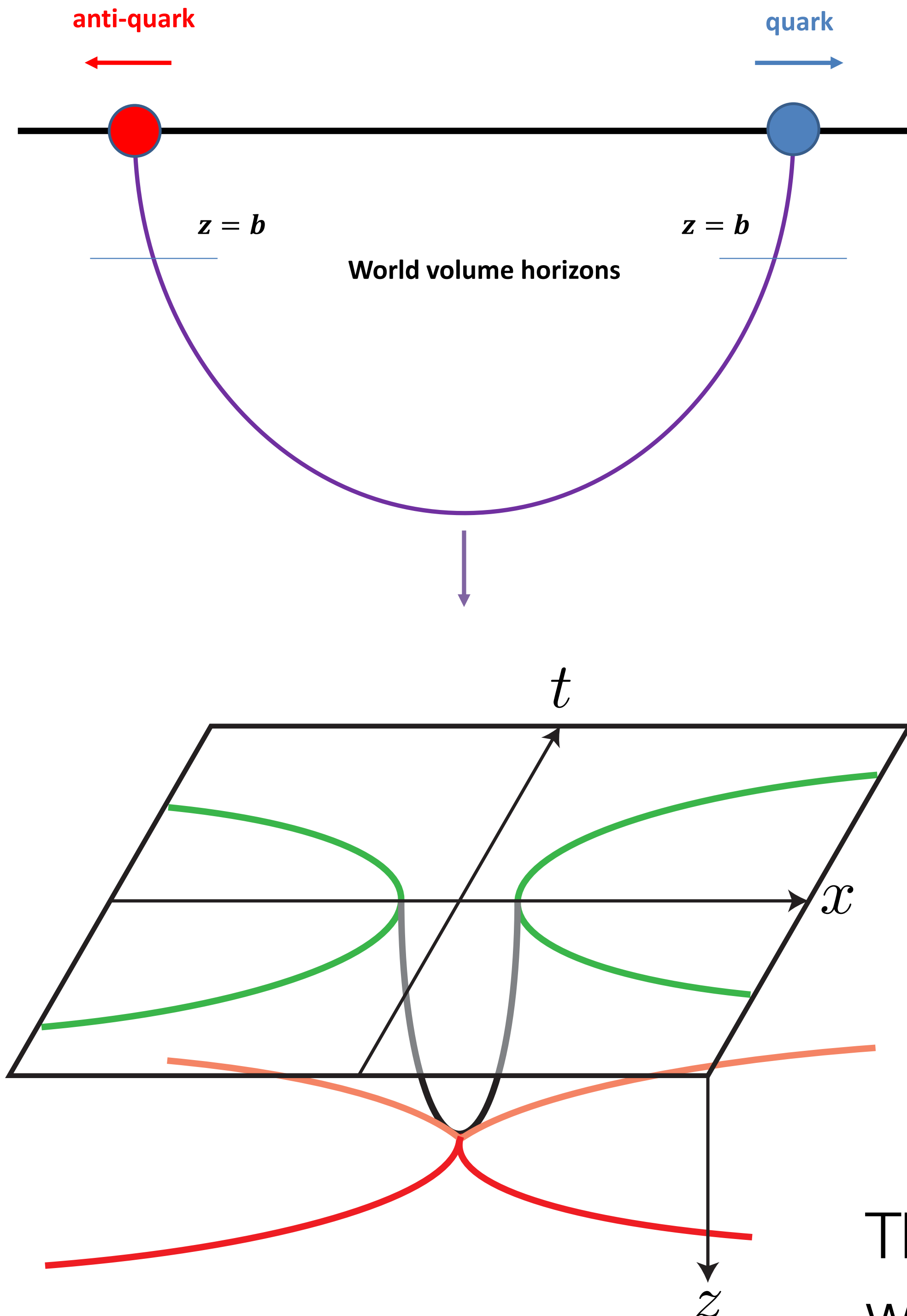
Even for an entangled pair of particles, in a quantum theory of gravity, there must be a Planckian bridge between them.

ER bridge



Accelerating quark and anti-quark

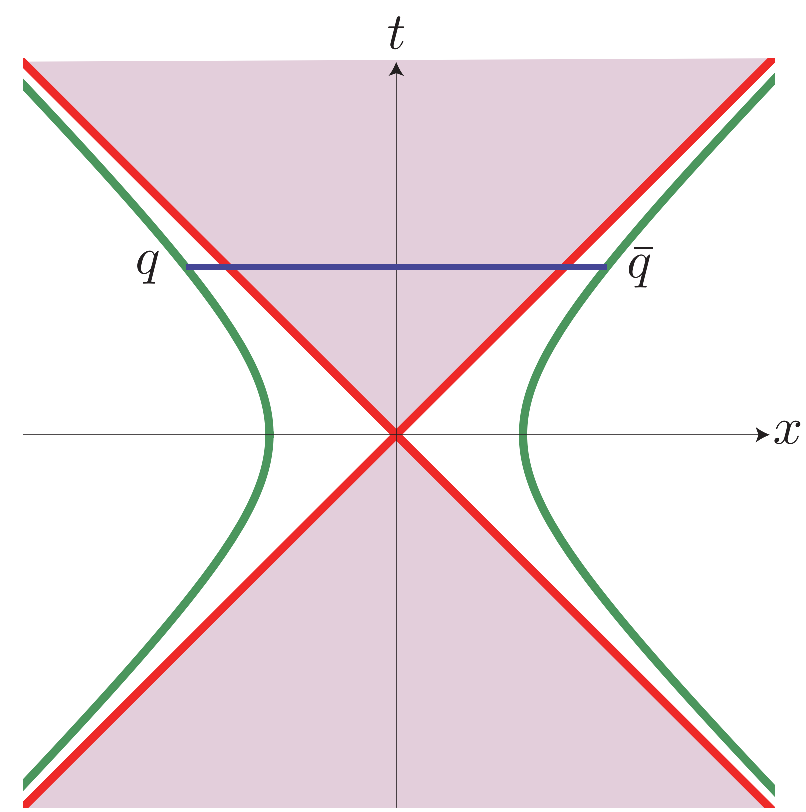
The holographic surface of accelerating quark and anti-quark



$$x^2 = t^2 + b^2 - z^2$$

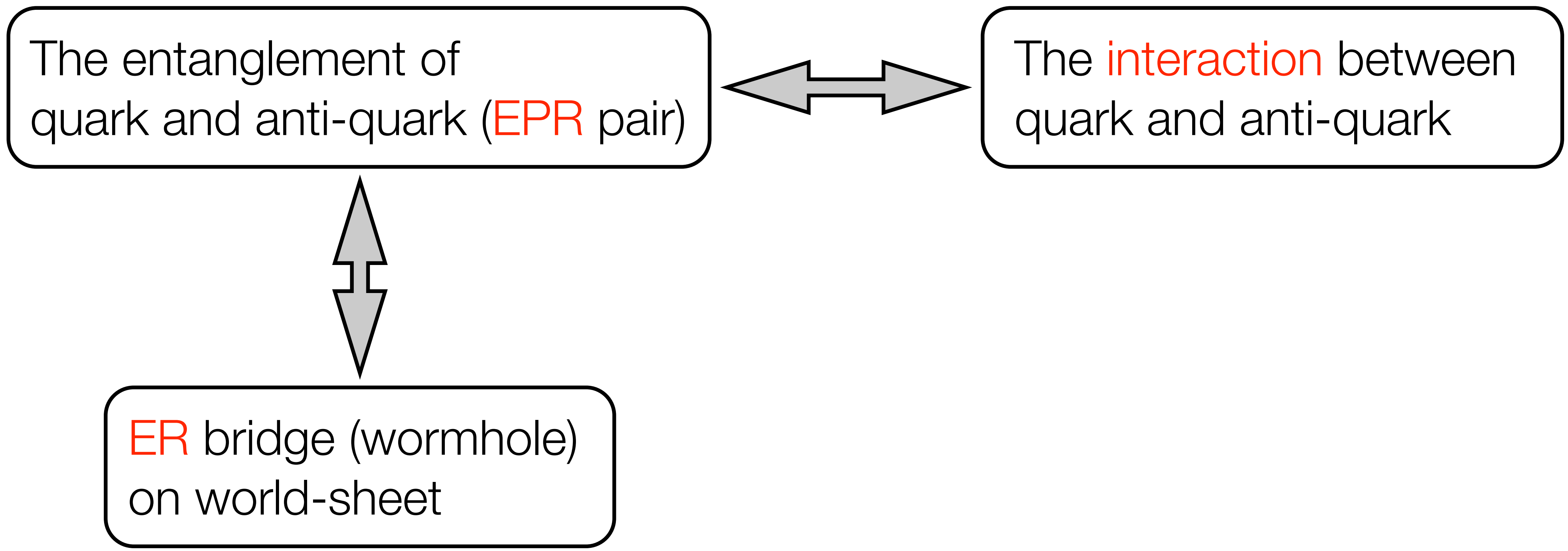
[Xiao, PLB 665 (2008) 173]

The trajectories of quark and anti-quark are causally disconnected on the world-sheet.



The quark and anti-quark are entangled by the wormhole that the open string goes through.

[Jensen-Karch, PRL 111 (2013) 211602]



Do other interacting particles also have ER bridge on world-sheet?

Fortunately, we know the minimal surface in AdS that describes a gluon-gluon scattering.

Gluon scattering

Minimal surface solution for gluon scattering

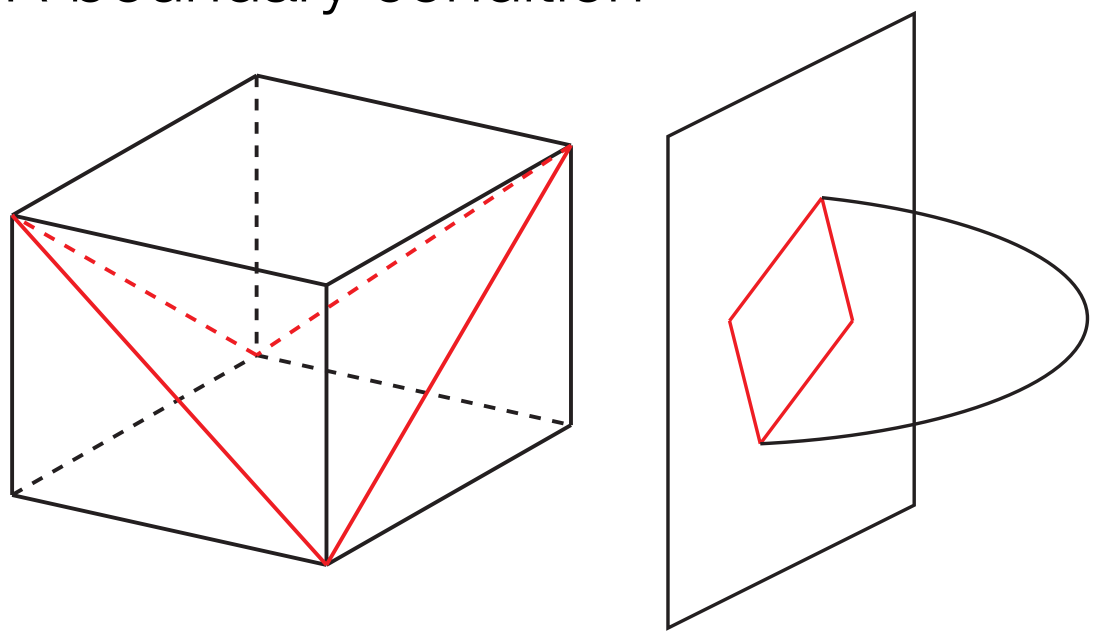
AdS_5 (momentum space)

[Alday-Maldacena, JHEP 0706 (2007) 064]

$$ds^2 = \frac{R^2}{r^2}(\eta_{\mu\nu}dy^\mu dy^\nu + dr^2)$$

$$\Delta y^\mu = 2\pi k^\mu$$

$r = 0$ IR boundary condition



The solution of Nambu-Goto action

$$y_0 = \frac{\alpha\sqrt{1+\beta^2}\sinh u_1 \sinh u_2}{\cosh u_1 \cosh u_2 + \beta \sinh u_1 \sinh u_2},$$

$$y_1 = \frac{\alpha \sinh u_1 \cosh u_2}{\cosh u_1 \cosh u_2 + \beta \sinh u_1 \sinh u_2},$$

$$y_2 = \frac{\alpha \cosh u_1 \sinh u_2}{\cosh u_1 \cosh u_2 + \beta \sinh u_1 \sinh u_2},$$

$$y_3 = 0,$$

$$r = \frac{\alpha}{\cosh u_1 \cosh u_2 + \beta \sinh u_1 \sinh u_2},$$

Mandelstam variables:

$$-s(2\pi)^2 = \frac{8\alpha^2}{(1-\beta)^2},$$

$$-t(2\pi)^2 = \frac{8\alpha^2}{(1+\beta)^2}.$$

AdS_5 (momentum space)

$\downarrow$ "T-dual" transformation: $\partial_m y^\mu = \frac{R^2}{z^2}\epsilon_{mn}\partial_n x^\mu \,, \quad z = \frac{R^2}{r}$

AdS_5 (position space)

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

The Alday-Maldacena solution is mapped to

$$x_0 = -\frac{R^2}{2\alpha}\sqrt{1+\beta^2}\sinh u_+ \sinh u_- \,,$$

$$x_+ := \frac{x_1 + x_2}{\sqrt{2}} = -\frac{R^2}{2\sqrt{2}\alpha}\left[(1+\beta)u_- + (1-\beta)\cosh u_+ \sinh u_-\right] \,,$$

$$x_- := \frac{x_1 - x_2}{\sqrt{2}} = \frac{R^2}{2\sqrt{2}\alpha}\left[(1-\beta)u_+ + (1+\beta)\sinh u_+ \cosh u_-\right] \,,$$

$$x_3 = 0 \,,$$

$$z = \frac{R^2}{2\alpha}\left[(1+\beta)\cosh u_+ + (1-\beta)\cosh u_-\right]$$

where $u_\pm := u_1 \pm u_2$. For later convenience, we introduce

$$X_\mu := \frac{\alpha}{R^2}x_\mu \quad (\mu = 0, +, -, 3) \,, \quad Z := \frac{\alpha}{R^2}z \, (\geq 1)$$

Causal structure on world-sheet

The induced metric on world-sheet [SS-Sin, to appear]

$$ds^2_{\rm ws} = R^2\left(g_{++}du_+^2 + 2g_{+-}du_+du_- + g_{--}du_-^2\right)$$

$$g_{++} = \frac{4(1+\beta)^2\sinh^2 u_+ + 4(1+\beta^2) - \left[(1+\beta)\cosh u_+ - (1-\beta)\cosh u_-\right]^2}{2\left[(1+\beta)\cosh u_+ + (1-\beta)\cosh u_-\right]^2}\,,$$

$$g_{+-} = \frac{2(1-\beta^2)\sinh u_+ \sinh u_-}{\left[(1+\beta)\cosh u_+ + (1-\beta)\cosh u_-\right]^2}\,,$$

$$g_{--} = \frac{4(1-\beta)^2\sinh^2 u_- + 4(1+\beta^2) - \left[(1+\beta)\cosh u_+ - (1-\beta)\cosh u_-\right]^2}{2\left[(1+\beta)\cosh u_+ + (1-\beta)\cosh u_-\right]^2}\,.$$

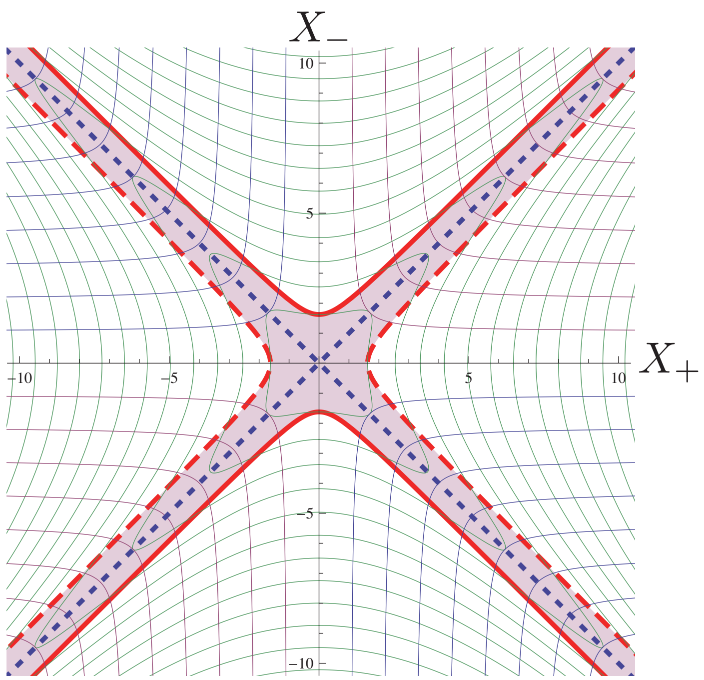
Horizons

$$g_{++} = 0: \quad (1-\beta)\cosh u_- = (1+\beta)\cosh u_+ + 2\sqrt{(1+\beta)^2\sinh^2 u_+ + 1 + \beta^2}$$

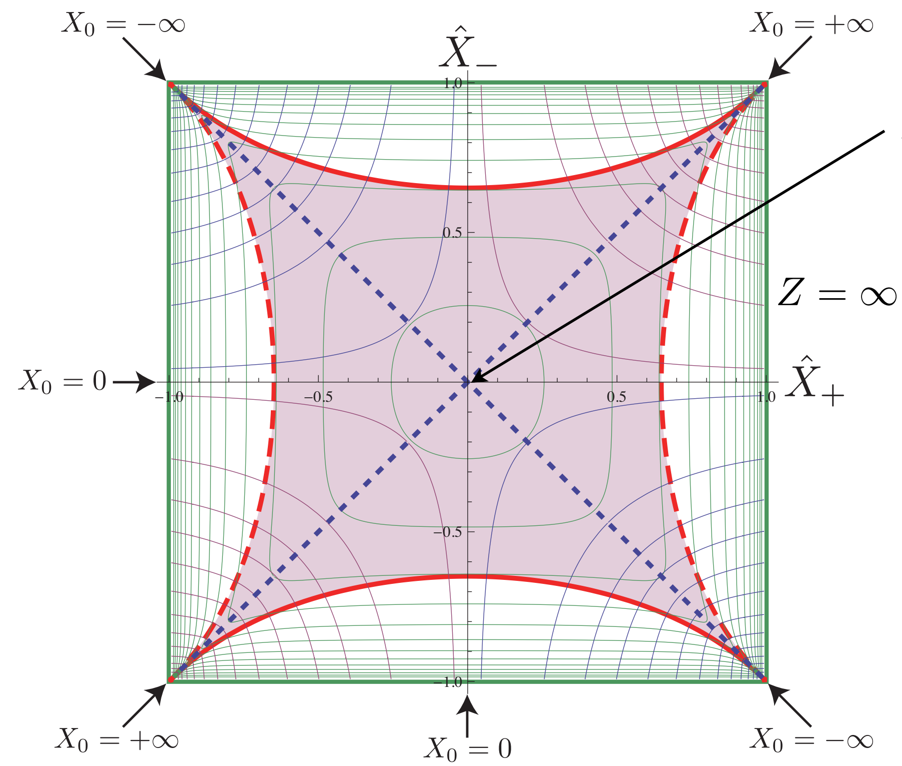
$$g_{--} = 0: \quad (1+\beta)\cosh u_+ = (1-\beta)\cosh u_- + 2\sqrt{(1-\beta)^2\sinh^2 u_- + 1 + \beta^2}$$

$\boxed{0 \leq \beta < 1}$ $\quad X_\pm \in (-\infty, +\infty)$ $\quad \hat{X}_\pm := \frac{2}{\pi} \arctan X_\pm \in [-1, 1]$

$\beta = 0$



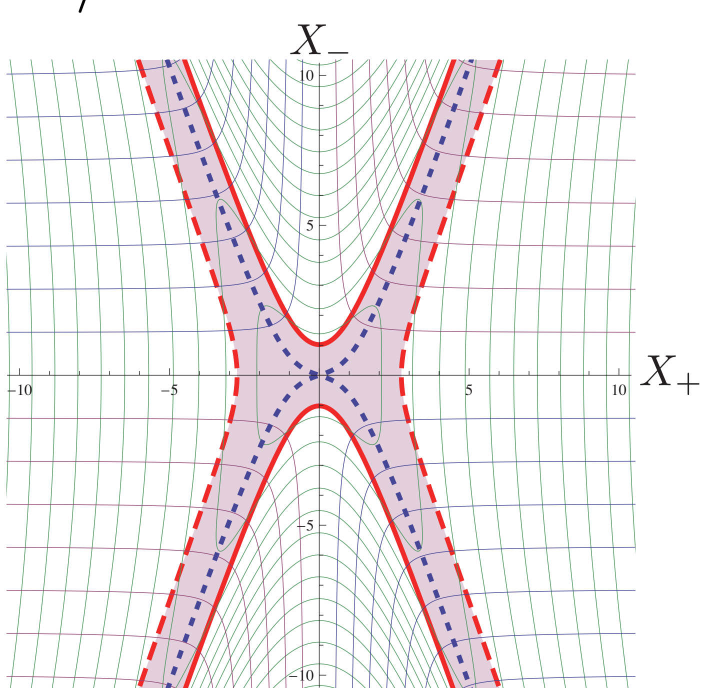
(a)



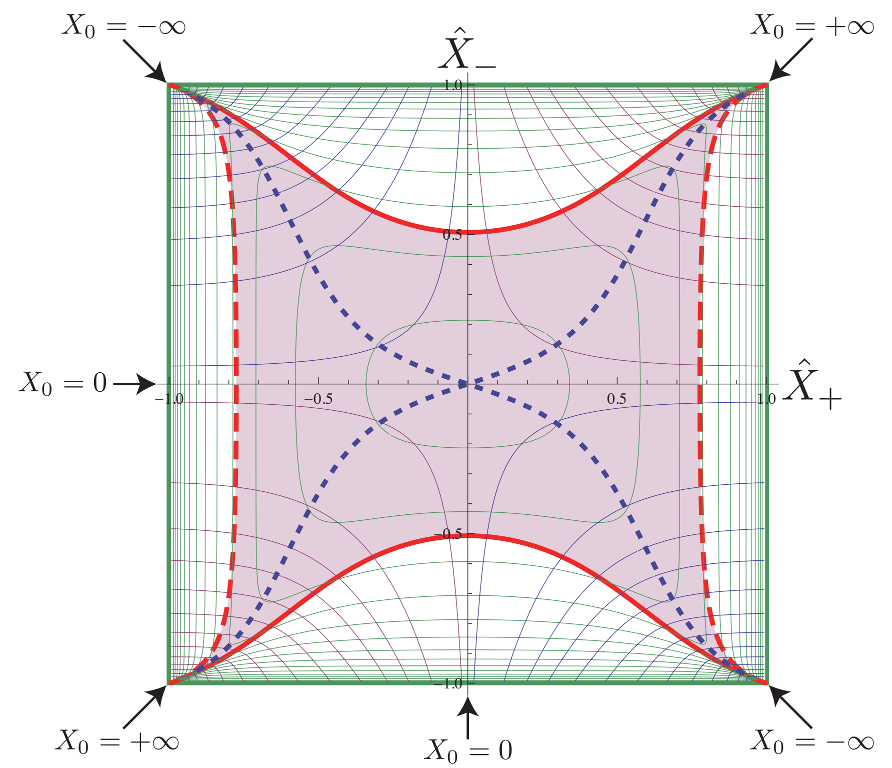
(b)

red: $g_{--} = 0$,
dashed red: $g_{++} = 0$,
dotted blue:
$g_{++} = g_{--}$

$\beta = 1/2$



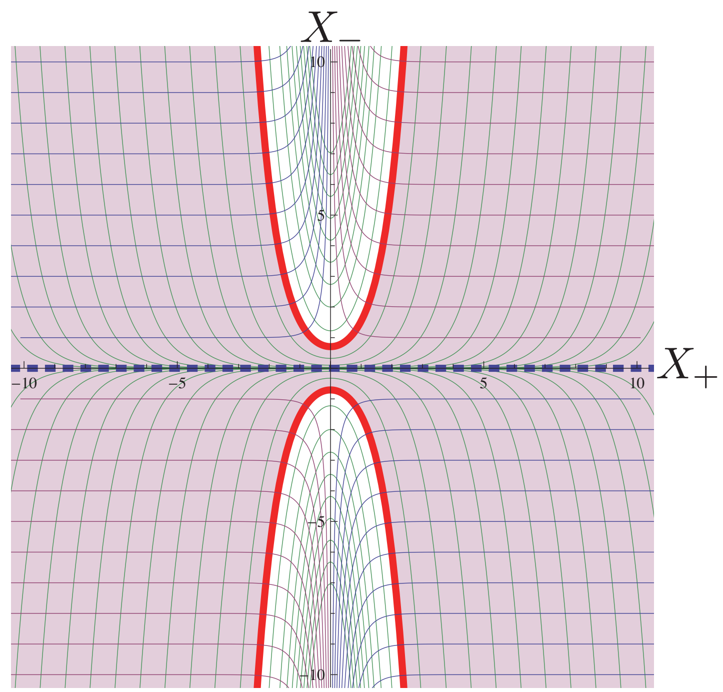
(a)



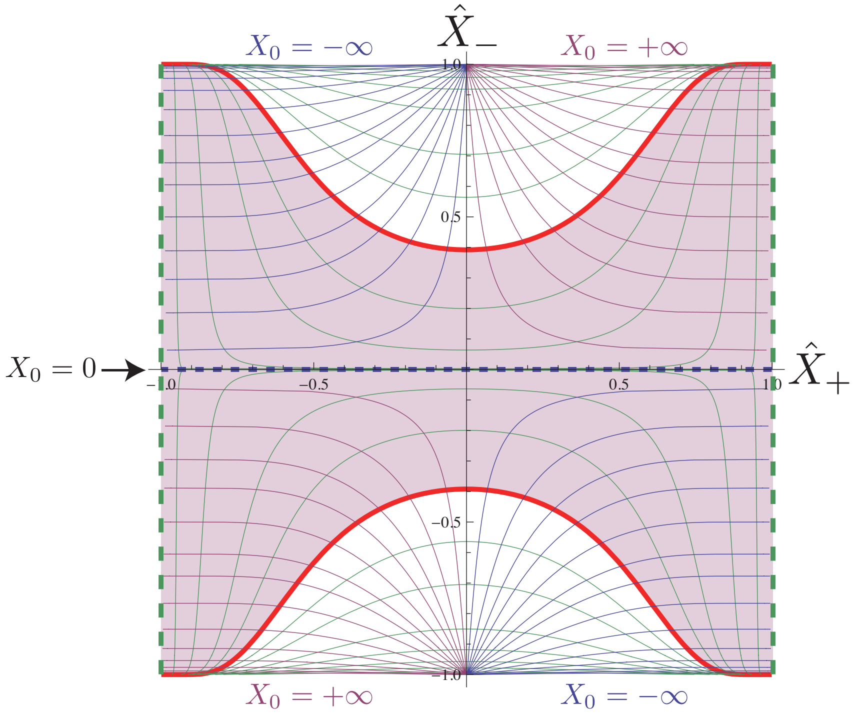
(b)

$\beta = 1$ Regge limit: $-s \to \infty$ with $-t$ fixed.

$$X_0 = -\frac{1}{\sqrt{2}} \sinh u_+ \sinh u_- \,, \quad X_+ = -\frac{1}{\sqrt{2}} u_- \,, \quad X_- = \frac{1}{\sqrt{2}} \sinh u_+ \cosh u_- \,,$$
$$X_3 = 0 \,, \quad Z = \cosh u_+ \,.$$



(a)

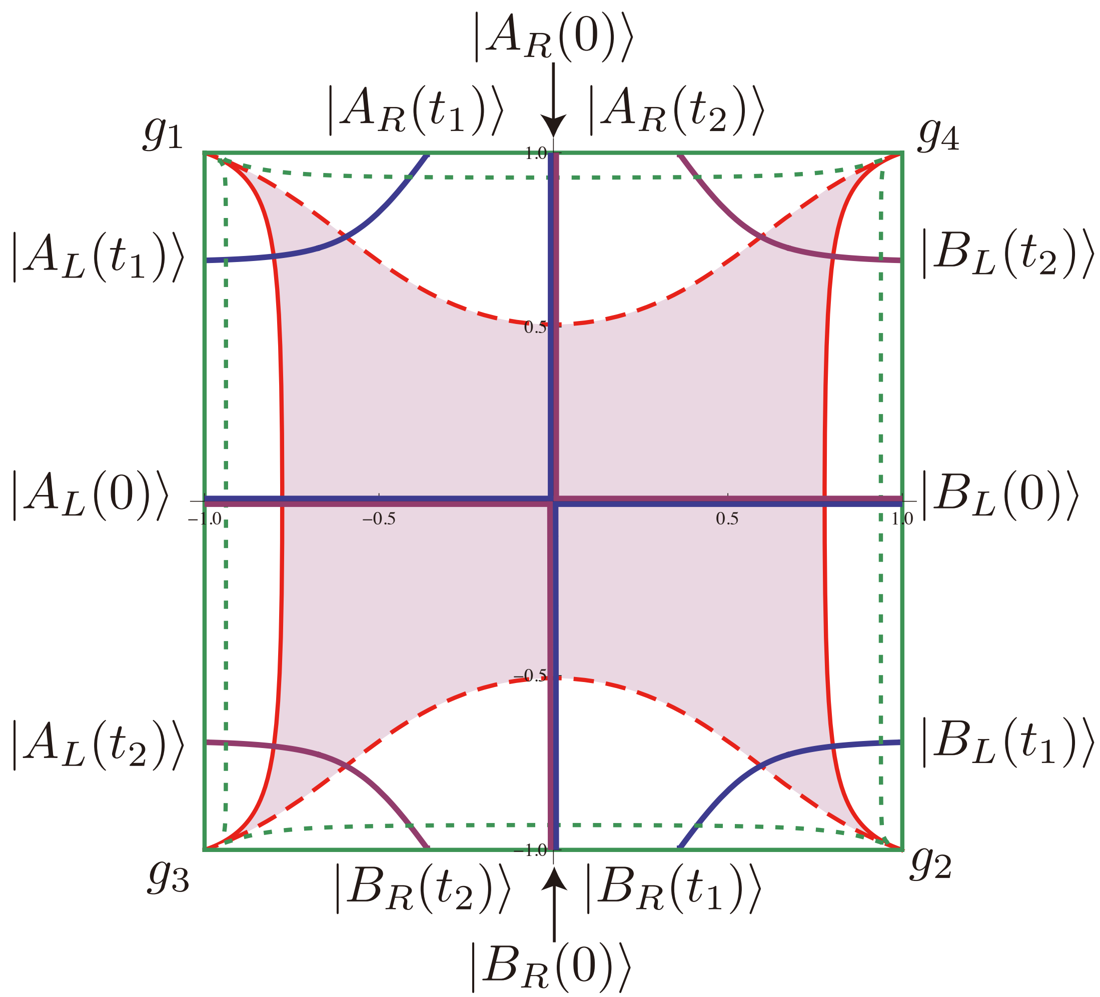


(b)

While g_{++} is positive definite, g_{--} is negative in $\cosh u_+ > \sqrt{2}$.

EPR = ER = Interaction?

The entanglement is given by a wormhole.



Incoming gluons:

$$|g_1(t_1)\rangle\rangle = |A_L(t_1)\rangle \otimes |A_R(t_1)\rangle$$

$$|g_2(t_1)\rangle\rangle = |B_L(t_1)\rangle \otimes |B_R(t_1)\rangle$$

Outgoing gluons:

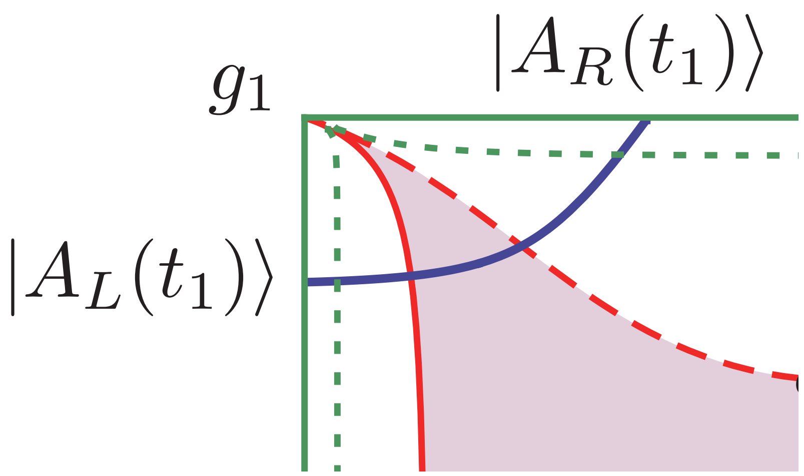
$$|g_3(t_2)\rangle\rangle = |A_L(t_2)\rangle \otimes |B_R(t_2)\rangle$$

$$|g_4(t_2)\rangle\rangle = |B_L(t_2)\rangle \otimes |A_R(t_2)\rangle$$

The entanglement is induced by gluonic interaction.

There are two ways to see entanglement.

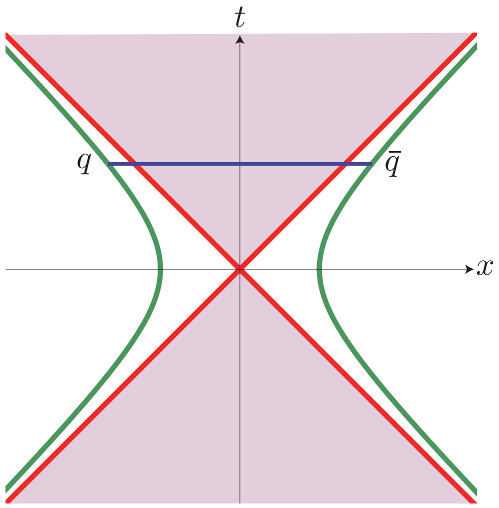
1. Internal entanglement



$$|g_1(t_1)\rangle\rangle = |A_L(t_1)\rangle \otimes |A_R(t_1)\rangle$$

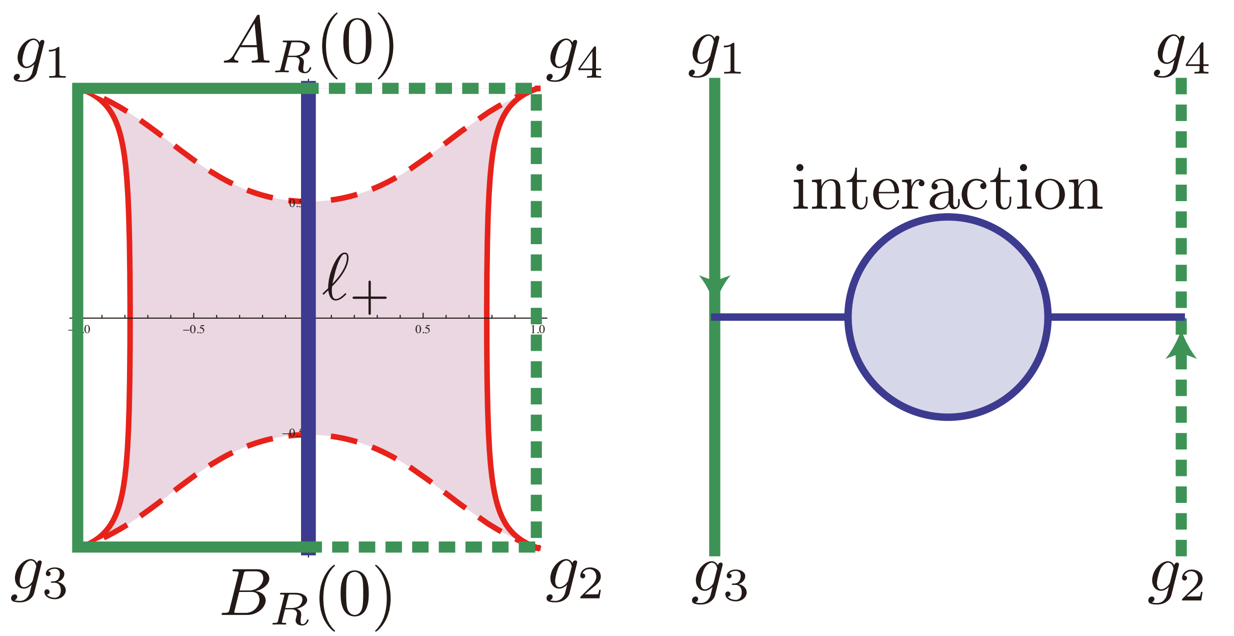
The open string endpoints in each gluon are entangled by the open string going through the wormhole.

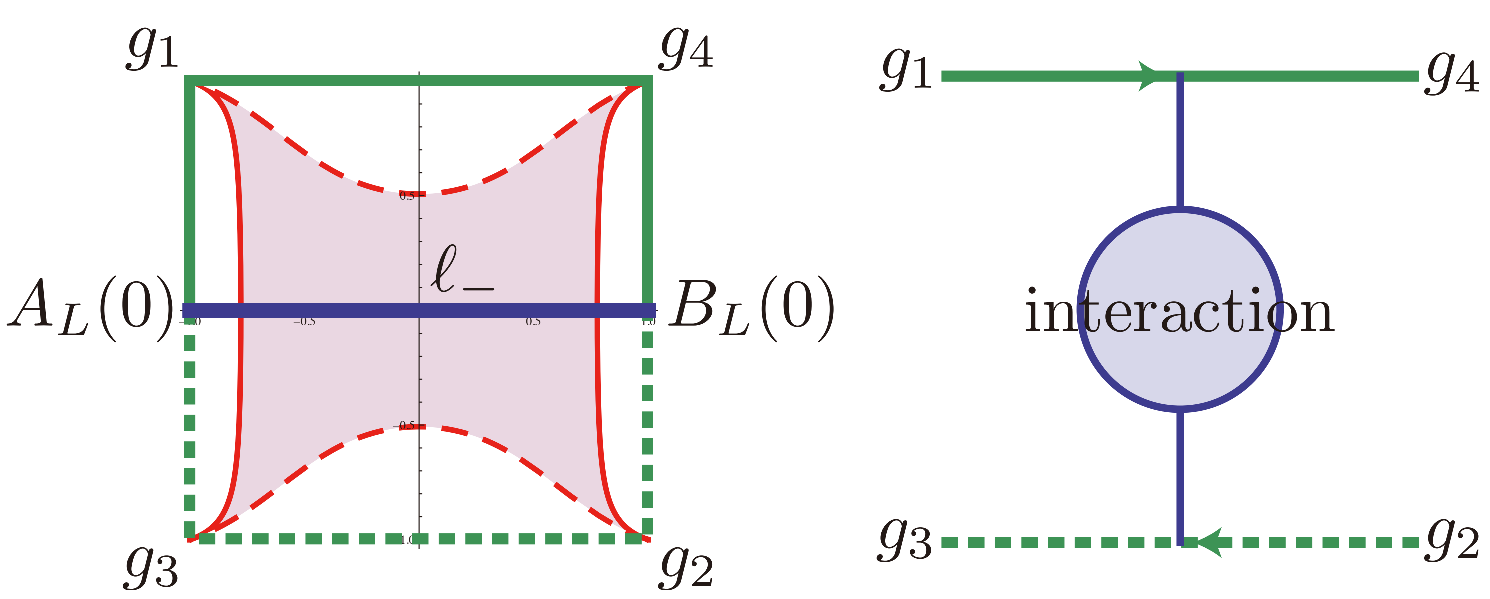
This is in the same way as the entanglement of quark and anti-quark.



2. Entanglement of gluons

There are two channels.





How can we measure the entanglement of gluons?

i) (naively) log of scattering amplitude

The scattering amplitude corresponds to the Wilson loop which is given by the area of minimal surface.

$$\mathcal{A} \sim e^{-\text{Area}}, \quad S \sim \log \mathcal{A} = \frac{\sqrt{\lambda}}{2\pi}\left(\log \frac{1+\beta}{1-\beta}\right)^2 + (\text{divergent part})$$

ii) the length between boundaries at the contacting points

We consider

$$\ell_+(\beta) = R\int_{-u_{+\infty}}^{+u_{+\infty}} du_+ \sqrt{g_{++}}\Big|_{u_-=0}, \quad \ell_-(\beta) = R\int_{-u_{-\infty}}^{+u_{-\infty}} du_- \sqrt{g_{--}}\Big|_{u_+=0}$$

where we introduced the cutoff, z_∞ $(\to \infty)$.

$$2\frac{\alpha z_\infty}{R^2} = (1+\beta)\cosh u_{+\infty} + 1 - \beta = (1-\beta)\cosh u_{-\infty} + 1 + \beta$$

$$\ell_\pm(\beta) = R\left[\sqrt{6}\log\frac{2\alpha z_\infty}{R^2} + \sqrt{6}\log\frac{1}{1\pm\beta} + \mathcal{O}\left(\frac{1}{z_\infty}\right)\right]$$

$$S \sim \ell_+(\beta) + \ell_-(\beta) = R\left[\sqrt{6}\log\frac{1}{1-\beta^2} + (\text{divergent part})\right]$$

Anyway, S diverges at the Regge limit, $\beta = 1$ and vanishes at $\beta = 0$.

EPR = ER = Interaction