

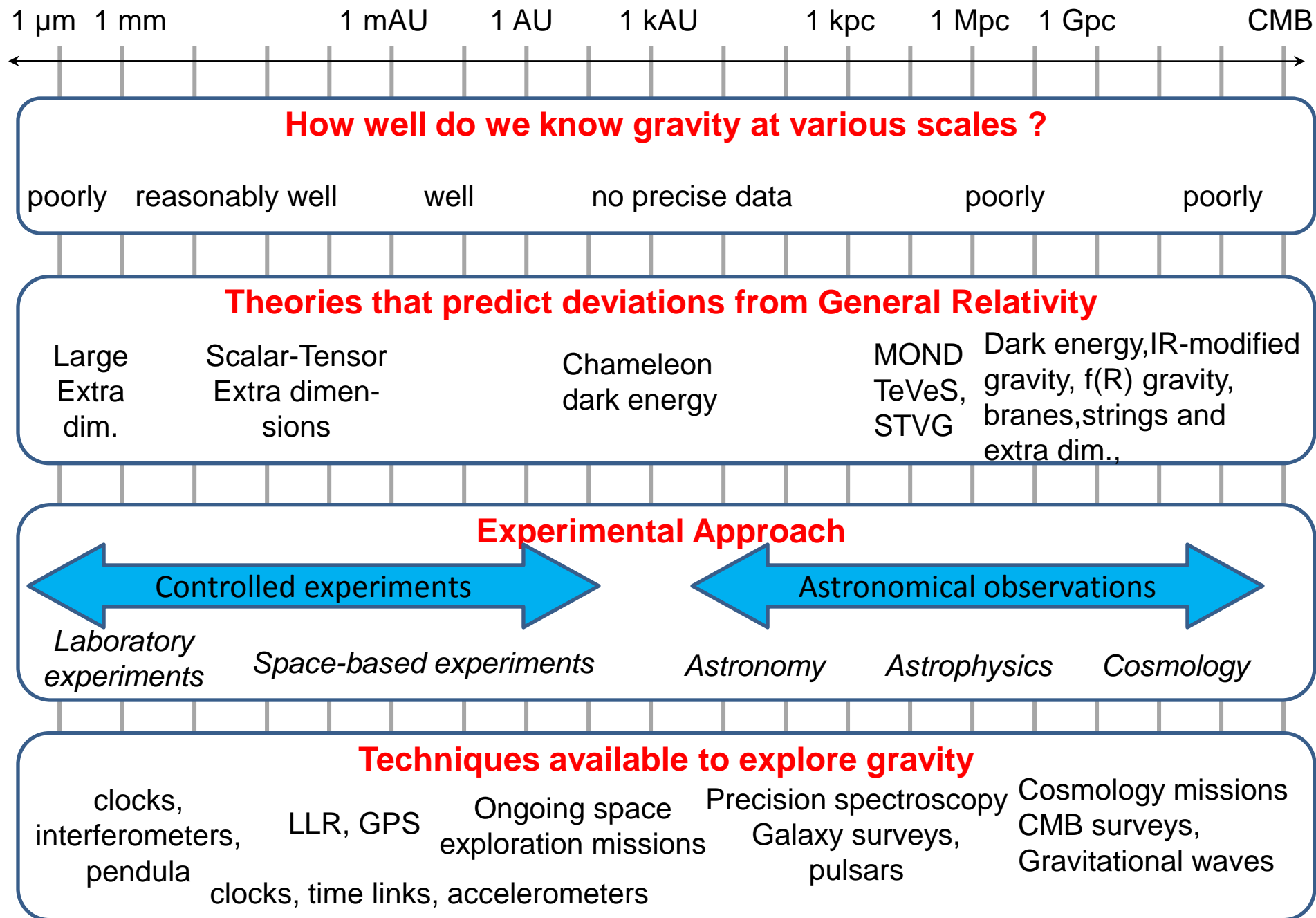
Deep-Space Navigation: a Tool to Investigate the Laws of Gravity

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Outline

- Laws of gravity in the solar system: observables, space probe dynamics, anomalies
- Cassini, Pioneer and the Pioneer anomaly
- Juno: Lense-Thirring at Jupiter
- Planned tests at Mercury with BepiColombo



At what level is General Relativity violated?

- In spite of the experimental success, there are strong theoretical arguments for violations of GR at some level.
- Unfortunately no reliable predictive, alternative theory has been proposed yet
- The theoretical uncertainties are so large that every experiment able to improve over previous tests is significant.
- Violations of GR from a single experiment will be accepted with great caution (if not skepticism). Confirmation with different techniques is essential.

Which tools are available?

- Geodesic motion of test masses (deep space probes, solar system bodies)
- Propagation of photons in a gravity field
- Measurements of angles, distances and velocities

Observables used in deep space navigation

Range (light travel time)

Phase comparison of modulation tones or codes in coherent radio links

Current accuracies :

1 - 3 m (incl. station bias)

0,2 m (BepiColombo Ka-band /multilink radio systems with wideband code modulation and delay calibration)

VLBI (angles)

Time delay at two widely separated ground antennas

Current accuracies:

$\approx 2-4$ nrad (Δ DOR)

(up to $\times 100$ better with phase referencing – but absolute accuracy limited by quasar position error)

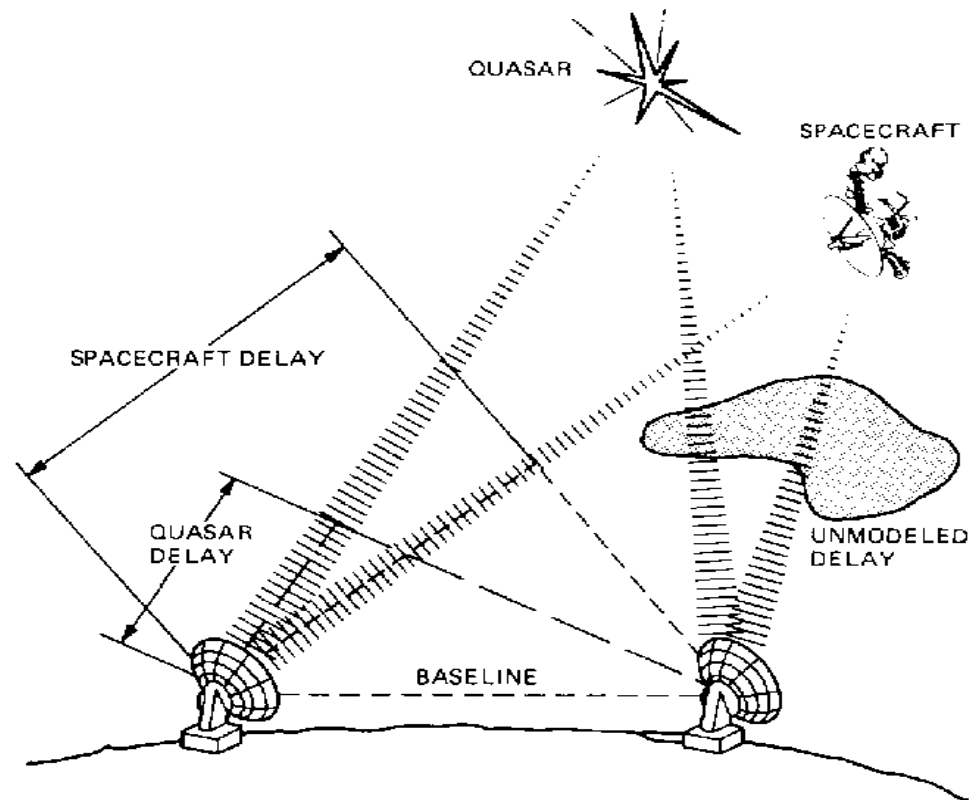
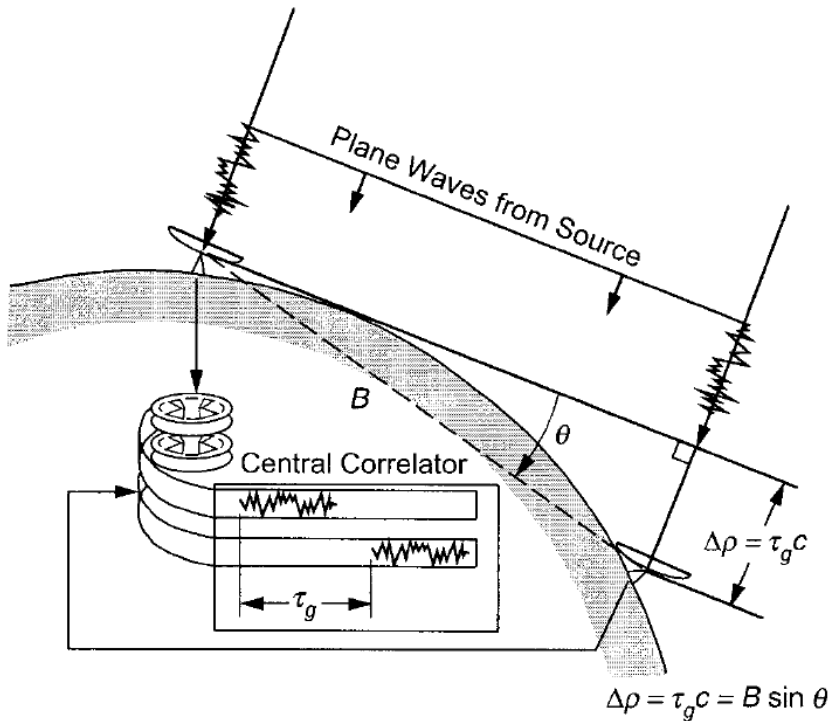
Range rate

Phase comparison (carrier) in coherent radio links

Current accuracies :

$3 \cdot 10^{-6}$ m/s @1000 s integr. times (Ka-band /multilink radio systems)

Angle measurements: Delta Differential One-way Ranging (Δ DOR)



Fighting Noise

- • **Uncertainties in the dynamical model (solar system ephemerides, asteroid masses)**
- • **Non-gravitational accelerations (onboard accelerometer)**
- • **Propagation noise (solar corona, interplanetary plasma, troposphere)**
- **Spacecraft and ground instrumentation**

Dynamical noise and non-gravs must be reduced to a level compatible with the accuracy of radio-metric measurements:

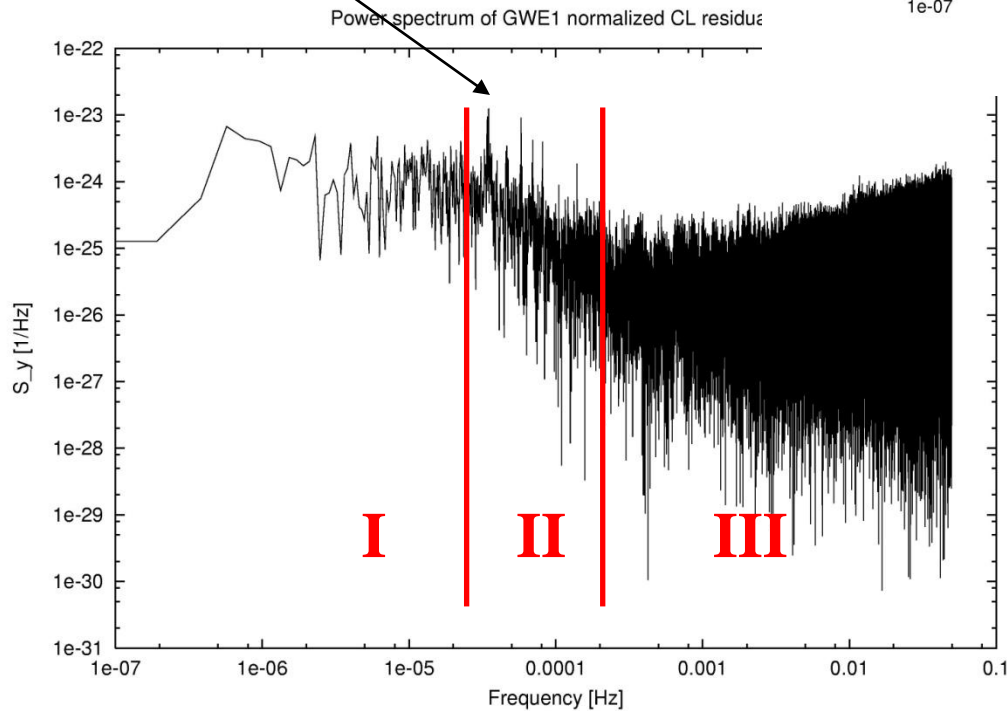
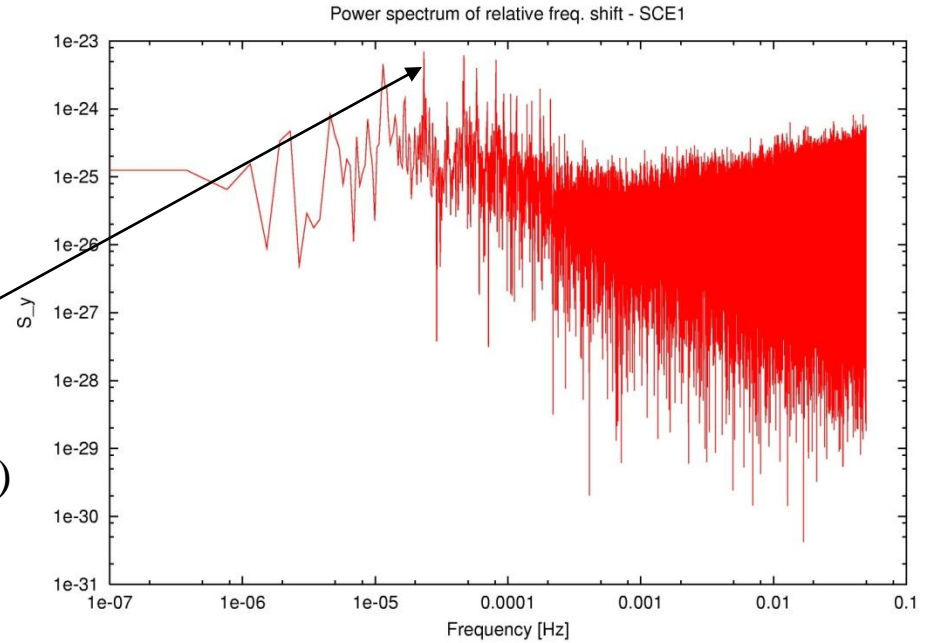
$$\sigma_a = \frac{1}{\tau} \sigma_v = 3 \times 10^{-8} \text{ cm s}^{-2} \quad \text{at } \tau = 10^4 \text{ s} \quad (\text{range rate})$$

$$\sigma_a = \frac{1}{\tau^2} \sigma_\rho = 1 \times 10^{-13} \text{ cm s}^{-2} \quad \text{at } \tau = 10^7 \text{ s} \quad (\text{range})$$

Power spectrum of
frequency residuals

Cassini 2002 SCE

Errors in solid tides models (1-2 cm)



Power spectrum of
frequency residuals

Cassini 2001 solar
opposition

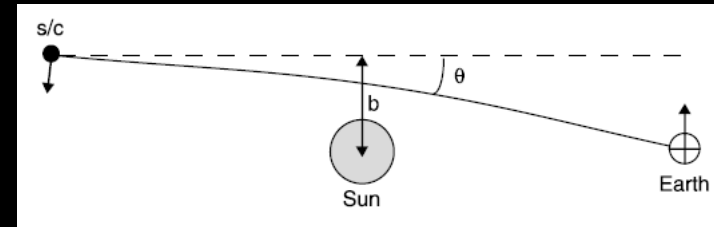
Tests based on propagation of photons

Deflection of light

$$\theta_{gr} = 2(1 + \gamma) \frac{M_{sun}}{b} = 4 \times 10^{-6} (1 + \gamma) \frac{R_{sun}}{b} \text{ rad}$$

**Main advantage:
short time scale !
[7-10 days]**

Solar Gravity



Time delay

$$\Delta t = (1 + \gamma) M_{sun} \ln \frac{l_0 + l_1 + l_{01}}{l_0 + l_1 - l_{01}}$$

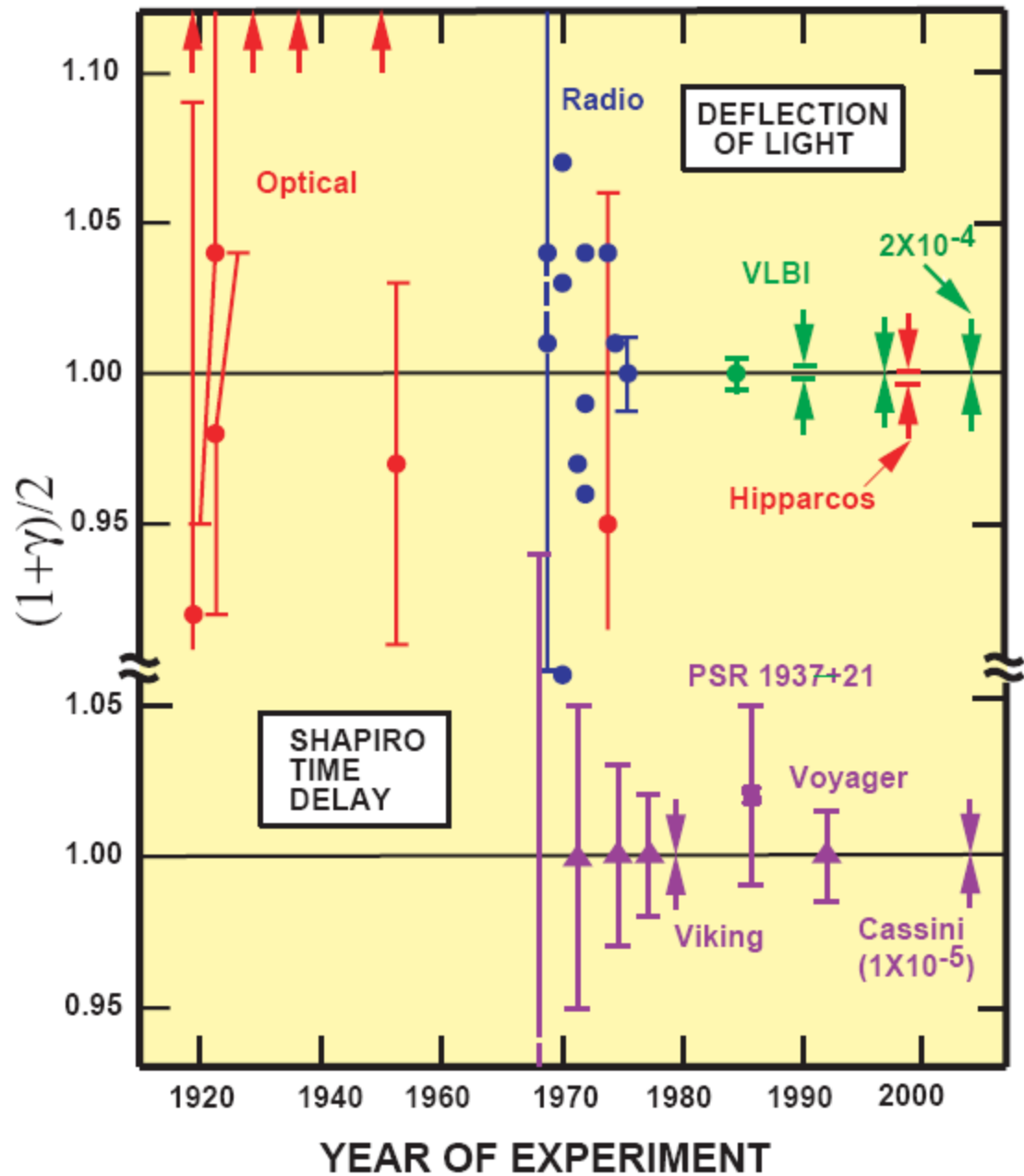
$\approx 70 \text{ km}$ for a grazing beam

Frequency shift

$$\frac{\Delta \nu}{\nu} = \frac{d}{dt} \Delta t \cong 4(1 + \gamma) \frac{M_{sun}}{b} \frac{db}{dt}$$

$\approx 8 \times 10^{-10}$ for a grazing beam

THE PARAMETER $(1+\gamma)/2$

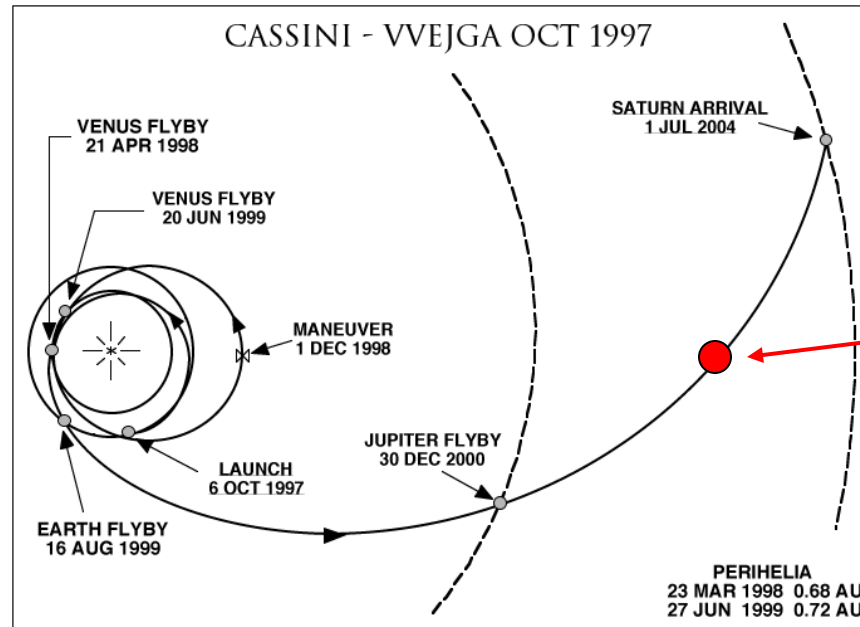


From:

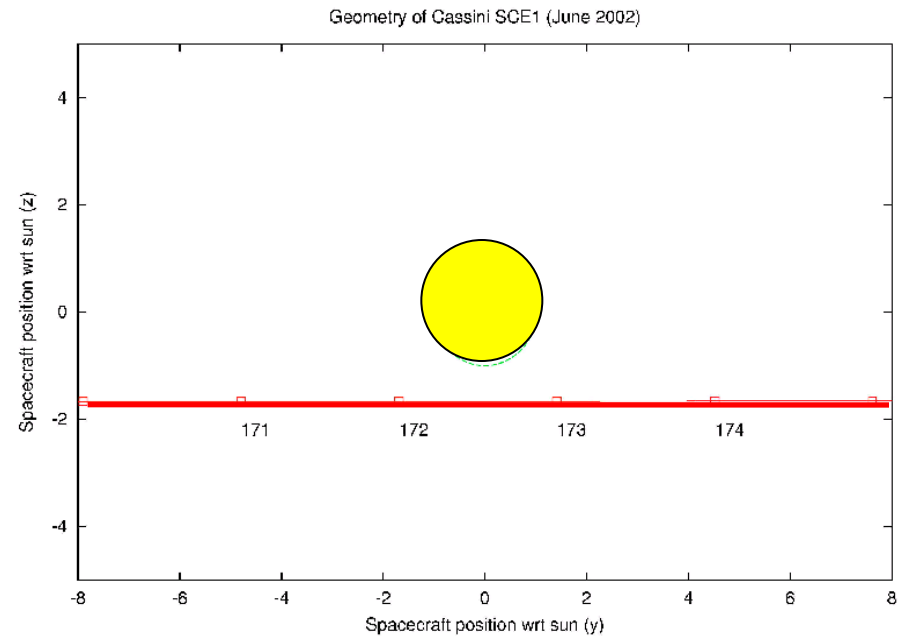
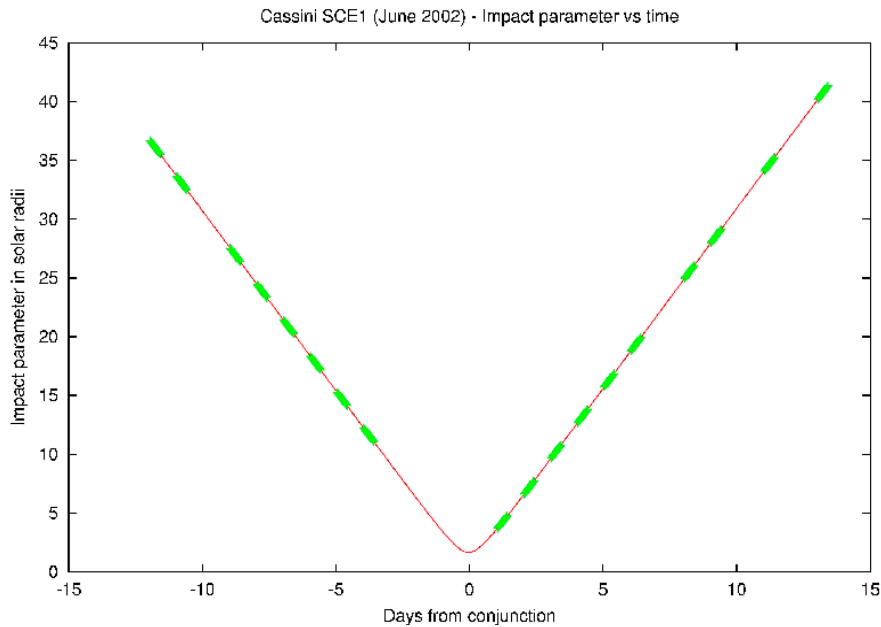
Clifford M. Will,
 "The Confrontation between General
 Relativity and Experiment",
 Living Rev. Relativity, 9, (2006), 3.
<http://www.livingreviews.org/lrr-2006-3>

The Cassini Solar Conjunction Experiment

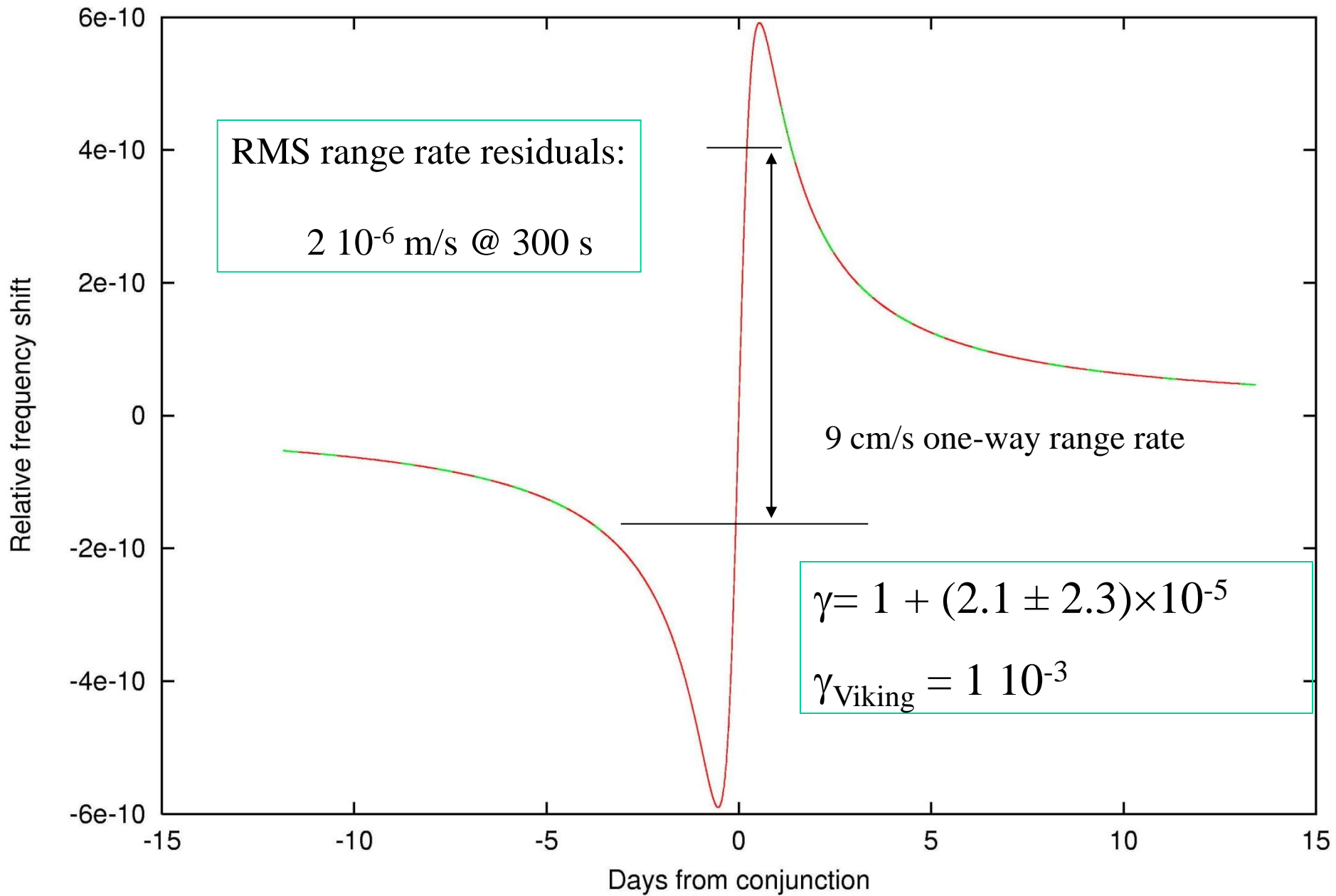




SCE1
 30 days
 coverage
 from DSN



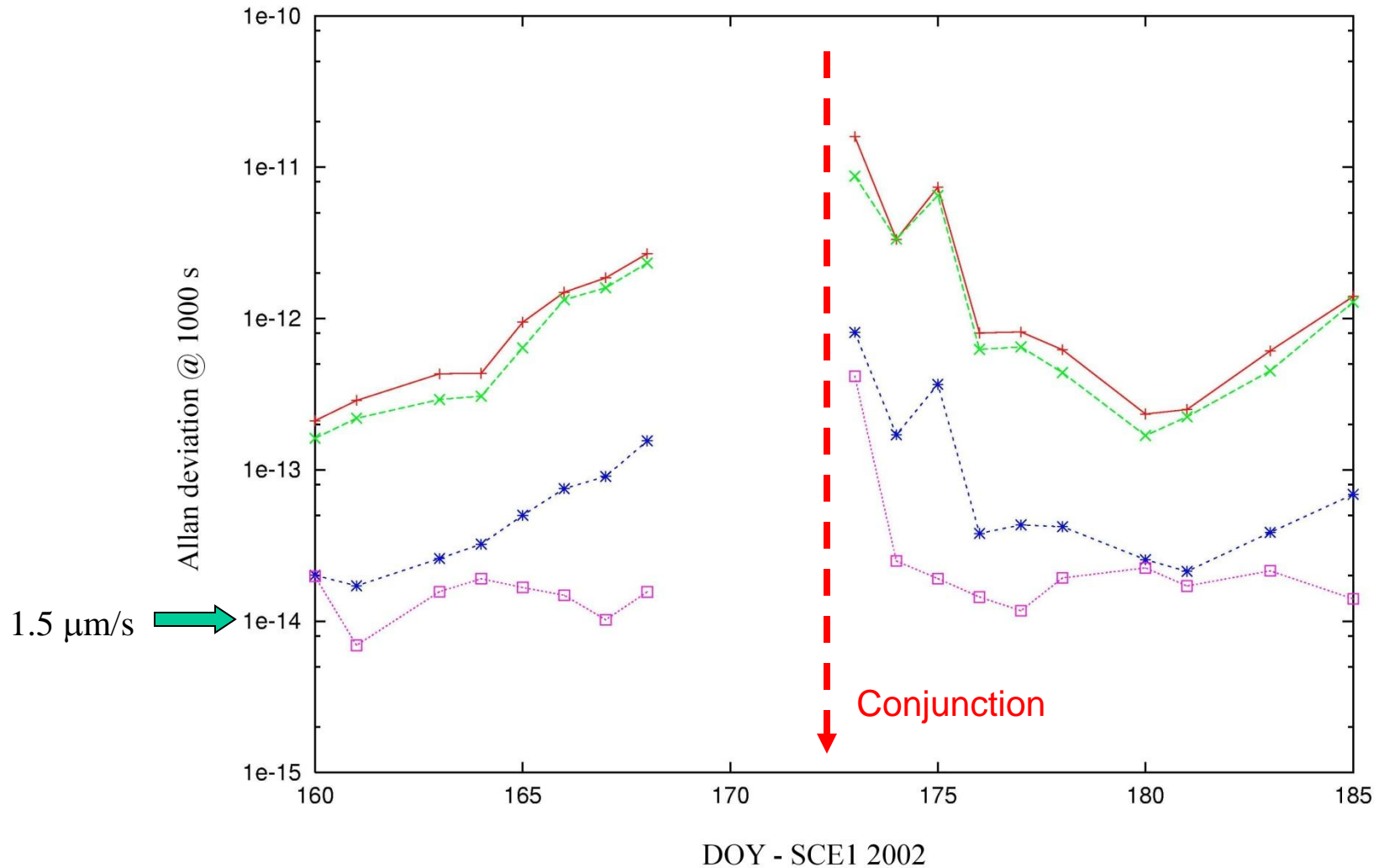
GR signal and GR signal + residuals (Cassini SCE1)



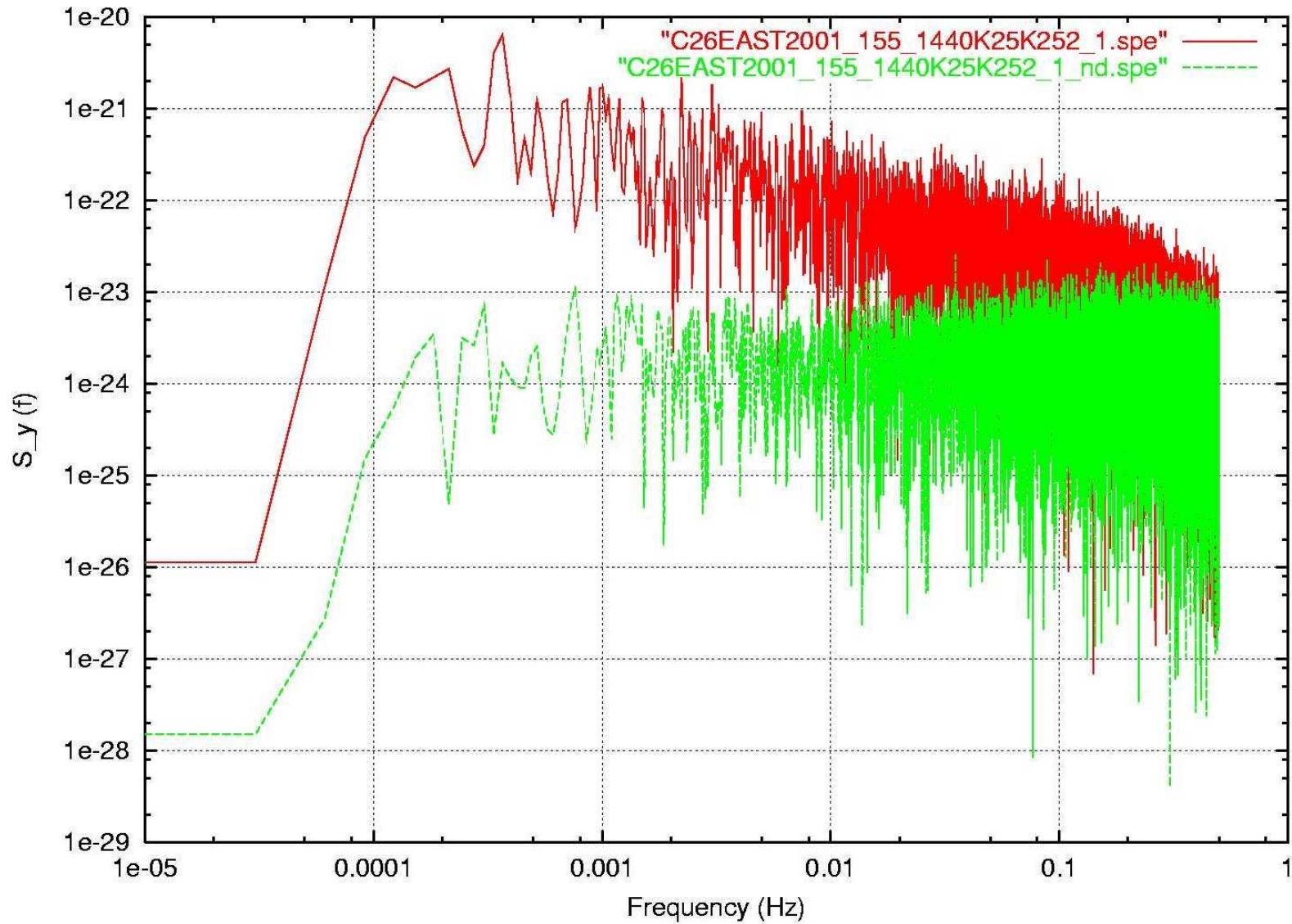
The trajectory of Cassini in the sky during SCE1



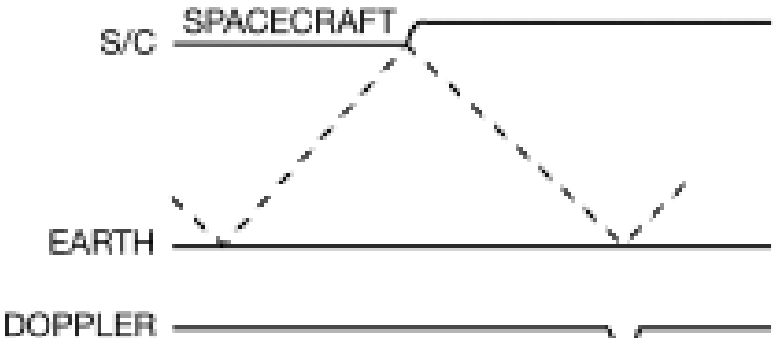
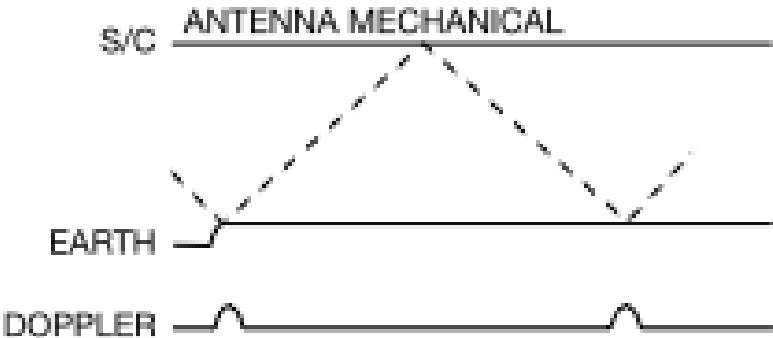
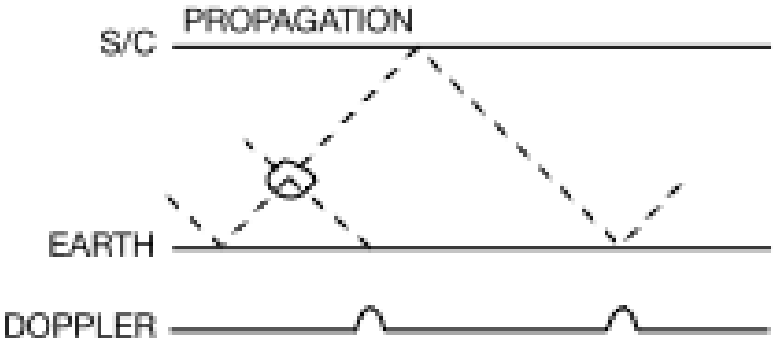
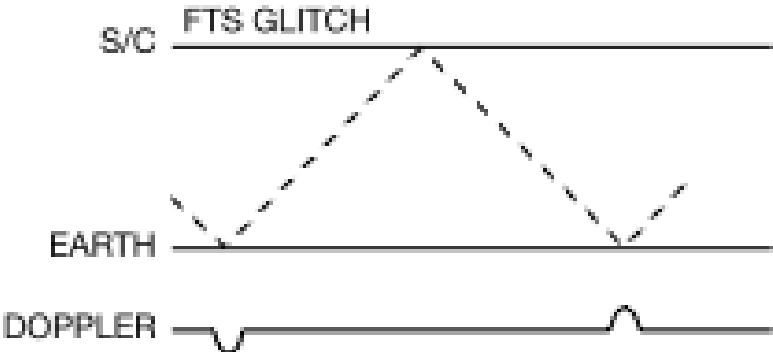
Plasma noise in the X/X, X/Ka, Ka/Ka links and the calibrated Doppler observable (daily Allan dev. @1000s, Cassini SCE1) Minimum impact parameter: $1.6 R_s$ (DOY 172)



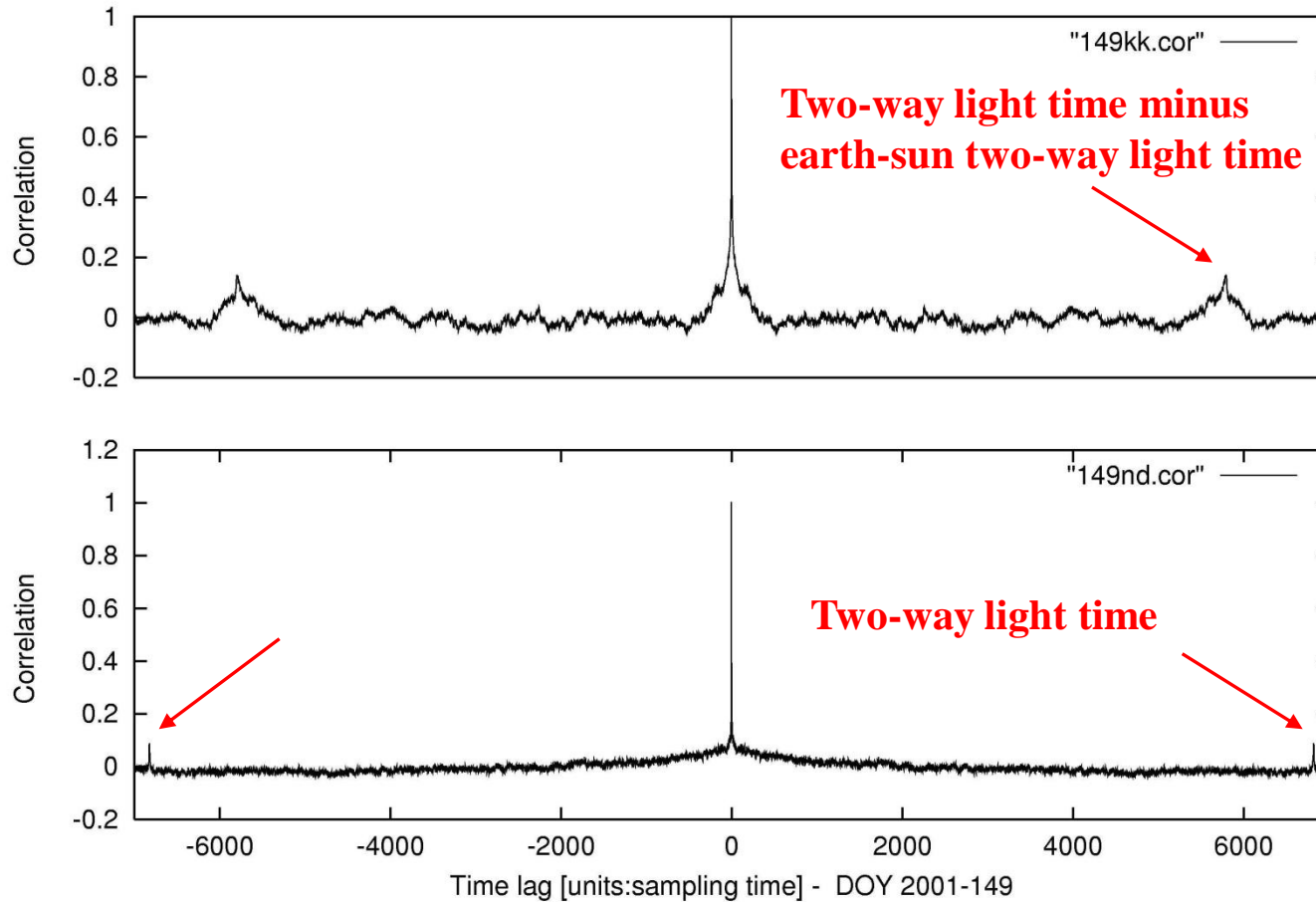
Power spectrum of relative frequency shift residuals



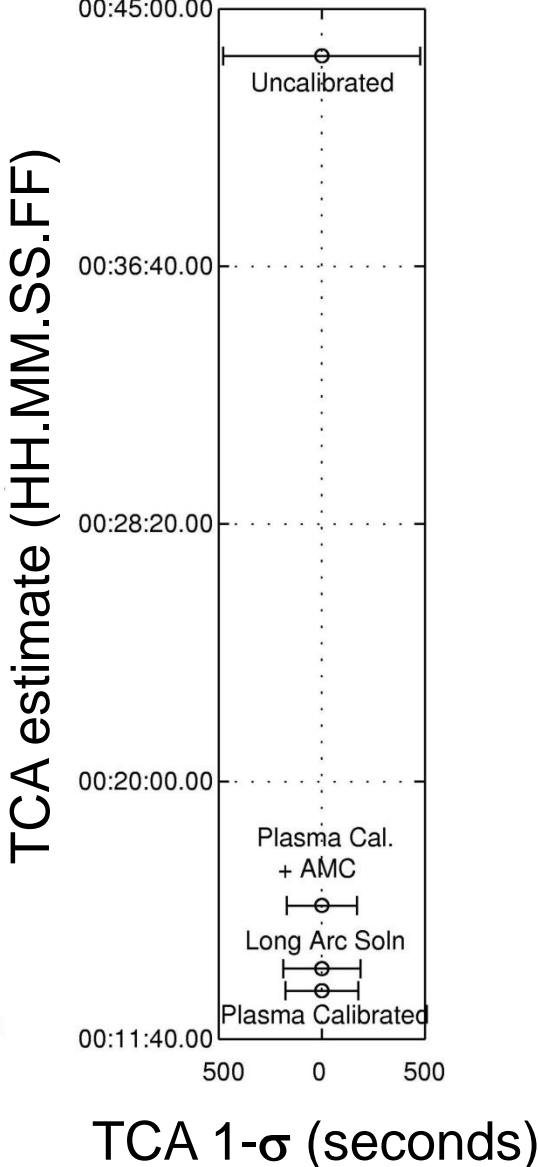
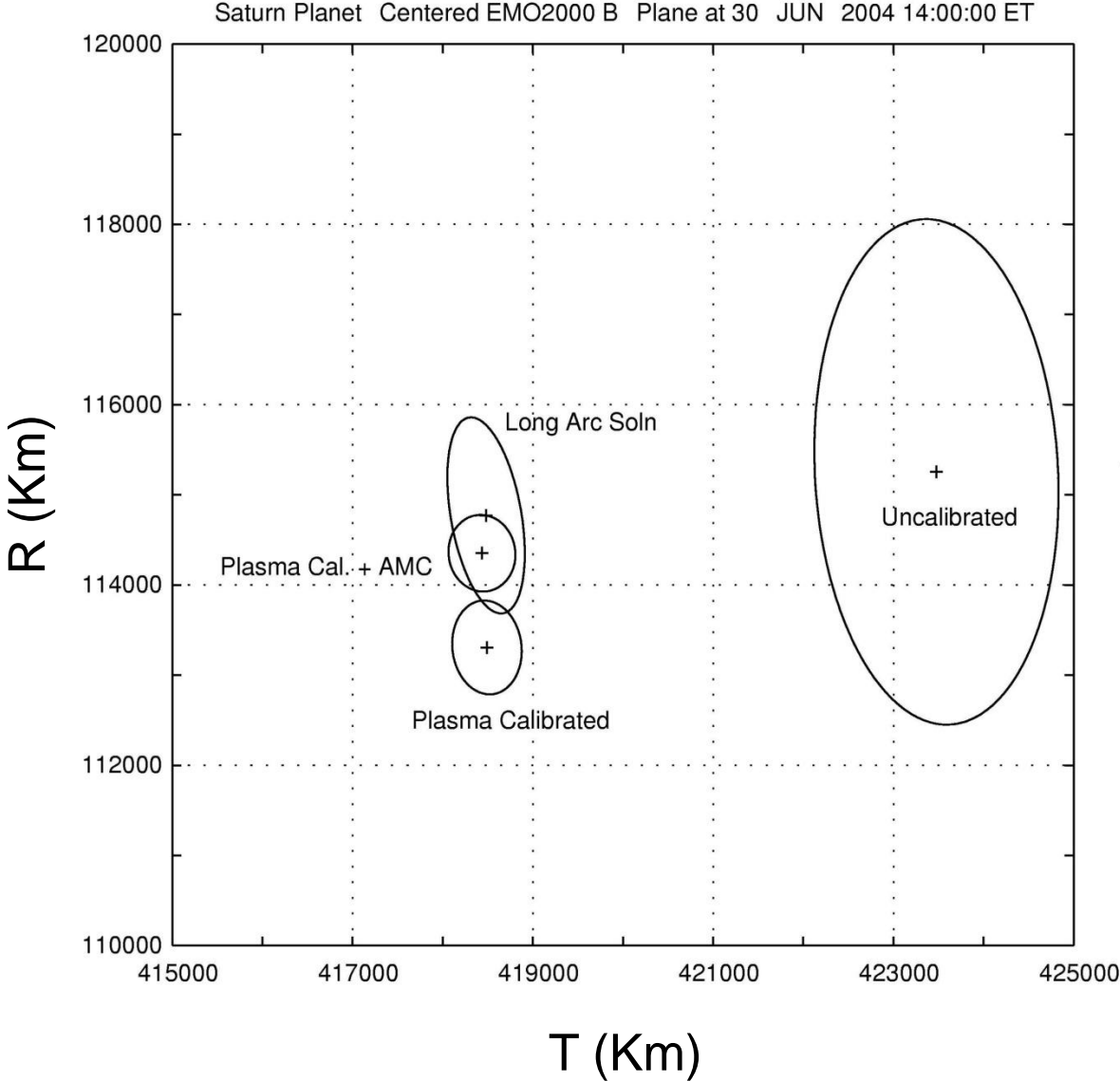
Noise Signatures in 2-way Doppler Link



ACF of Doppler residuals

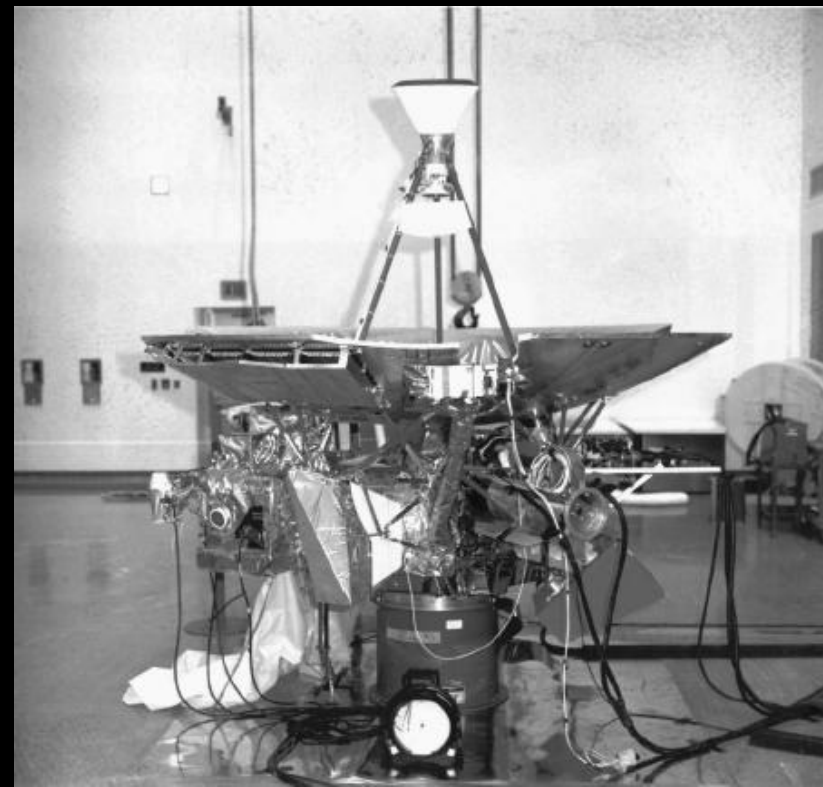


Saturn-centered B-plane plot of the Cassini orbital solutions

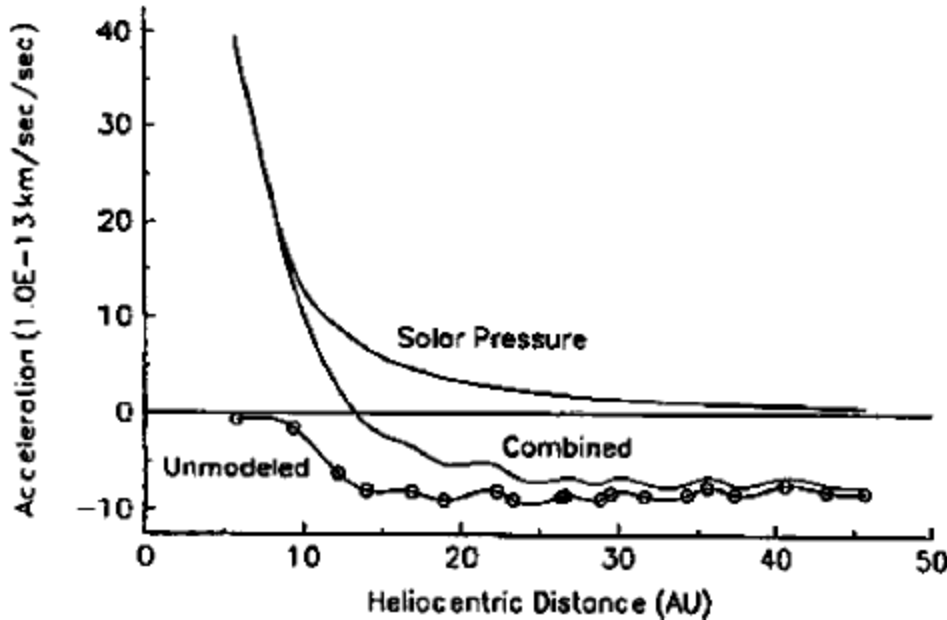


P.Tortora, L.Iess, J.J. Bordi, J.E. Ekelund, D. Roth, J. Guidance, Control and Dynamics, 27(2), 251 (2004)

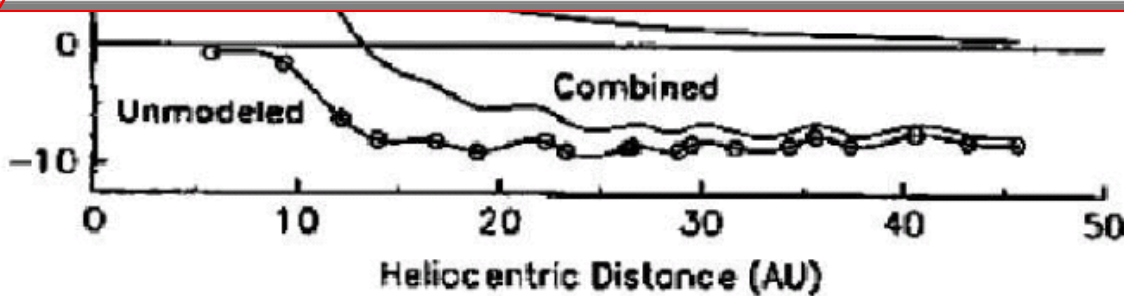
Pioneer anomaly - Facts




ACCELERATIONS ON PIONEER 10 AND 11
Positive Along Sun-Spacecraft Line



$$a_p = (8.74 \pm 1.33) \cdot 10^{-8} \text{ cm/s}^2$$

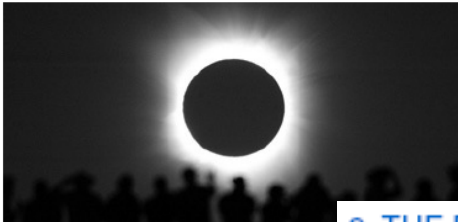


- 
- 1) Pointing toward the sun
 - 2) Almost constant

From Times Online
February 27, 2009

13 Unsolved scientific puzzles

Author Michael Brooks has investigated some of the most puzzling anomalies of modern science, those intractable problems that refuse to conform to the theories. Here he counts down the 13 strangest.



(David Gray/Reuters)

Michael Brooks

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TIMES RECOMMENDS

The other 12 things that do not make sense: missing mass, varying constants, cold fusion, life, death, sex, free will ...

2. THE PIONEER ANOMALY

RELATED LINKS

13 Things That Don't Make Sense by Michael Brooks

10 and 11, the mission head admitted that they had drifted off course. In every year of travel, the probes veer 8000 miles further away from their intended trajectory. It is not much when you consider that they cover 219 million miles a year; the drift is around 10 billion times weaker than the Earth's pull on your feet. Nonetheless, it is there, and decades of analysis have failed to find a straightforward reason for it. [Times Archive: Pioneer 11 arrival at Saturn, 1974](#)

Two spacecraft are flouting the laws of physics

In the 1970s NASA launched two space probes that have caused no end of headaches. About 10 years into the missions of Pioneer

centre of our culture

DOWNLOAD FREE COMIC



Before Da Vinci Code there was Rex Mundi

Pioneer anomaly: non-conventional hypotheses

- Dark matter
- Interplanetary dust
- Modified gravity
- ...

Yukawa-like force

$$F_Y(r) = -k \frac{e^{-r/\lambda}}{r^2}$$

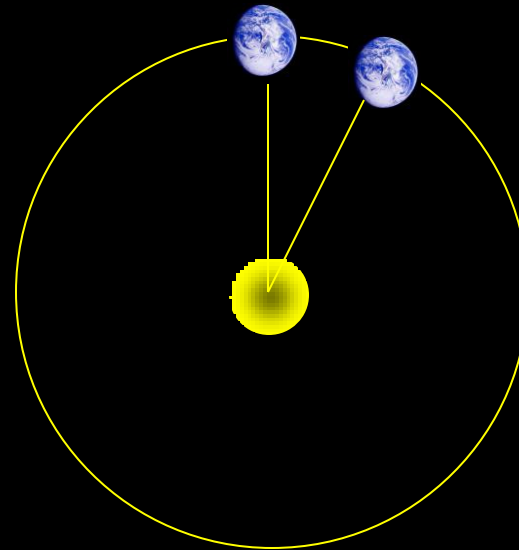
Phase referencing of Cassini:

$$a_p < 10^{-12} \text{ cm/s}^2 \quad (\text{Folkner et al., 2009})$$

PA would cause inconsistency in
planetary ephemerides

$$\omega = \tilde{\omega} \sqrt{1 + \frac{A_p r^2}{\mu}}$$

Corrections to planetary mean motion



For the Earth:

$$\Delta r = 200 \text{ km}$$

in one year!

Pioneer's RTG

(Radioisotope Thermoelectric Generators)

^{238}Pu → Half life = 88 y

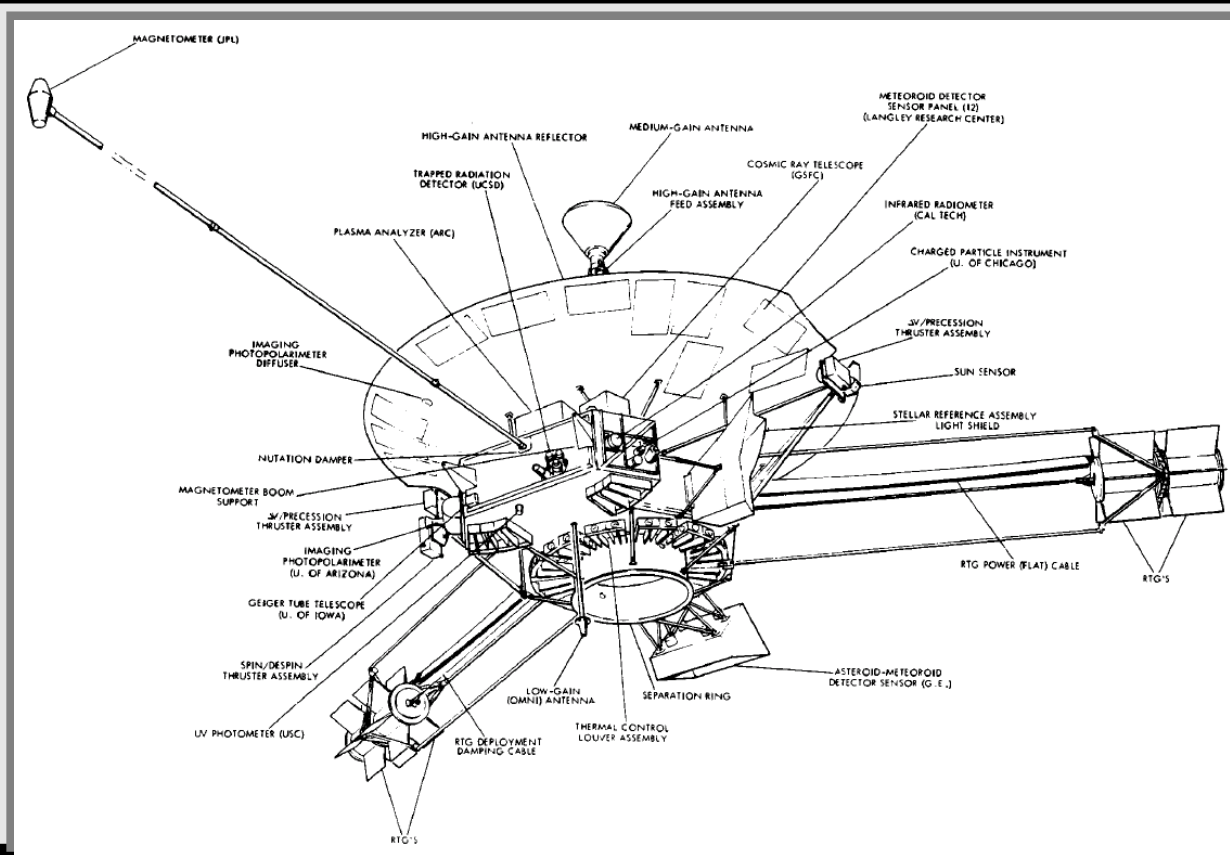
RTG thermal power = 2500 W

63 W, anisotropically radiated, would produce an acceleration equal to the “Pioneer anomaly”

This power is just 2,5 % of the total RTG power at launch (2500 W)

In 1991 RTG power was 20% lower (2000 W)

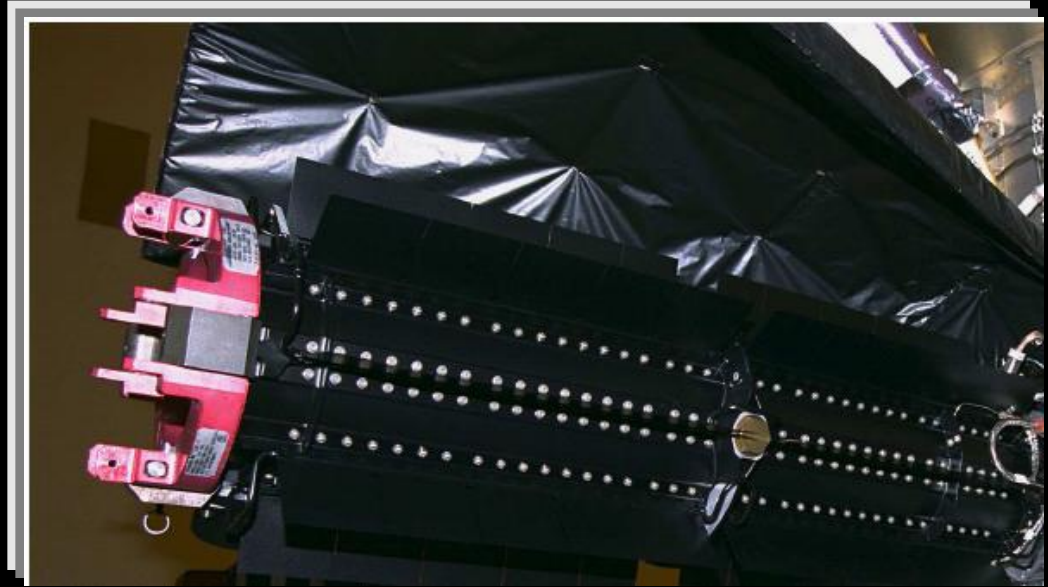
Acceleration is nearly constant!



Cassini's RTG

The 13 kW thermal emission is strongly anisotropic due to thermal shields

- RTG anisotropic emission is by far the largest non-gravitational acceleration experienced by the spacecraft during cruise and tour



$$a_{CAS} \approx 4.5 \cdot 10^{-7} \text{ cm/s}^2$$



$$a_{CAS} \approx 5a_p$$



- 30 % of total dissipated power must be radiated anisotropically

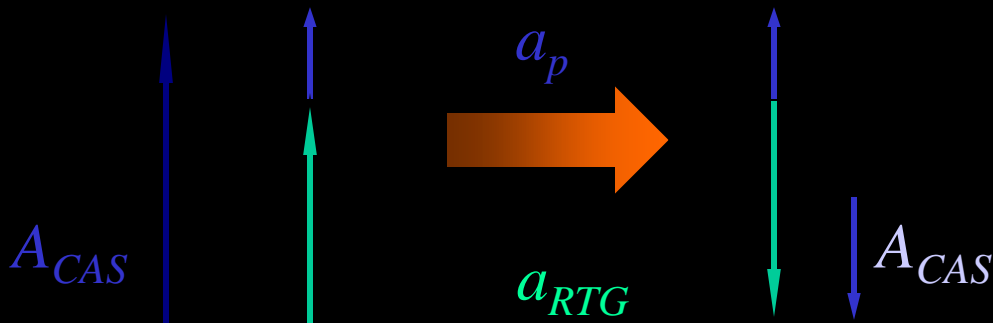
$$a_{CAS} = P_{\text{anisotropic}} / Mc$$



Is a_{CAS} hiding a “Pioneer anomaly”

Disentangling RTG and “Pioneer” acceleration

- Induce controlled orbital polarizations by orienting the spacecraft in different directions – Requires a undisturbed operations – Possible only in a the Post-Extended Mission



A 180 deg turn produces a $2 a_p$ variation of the total acceleration

- Exploit the large (2500 kg) mass decrease after SOI

$$a_c = a_{RTG} + a_{PA}$$

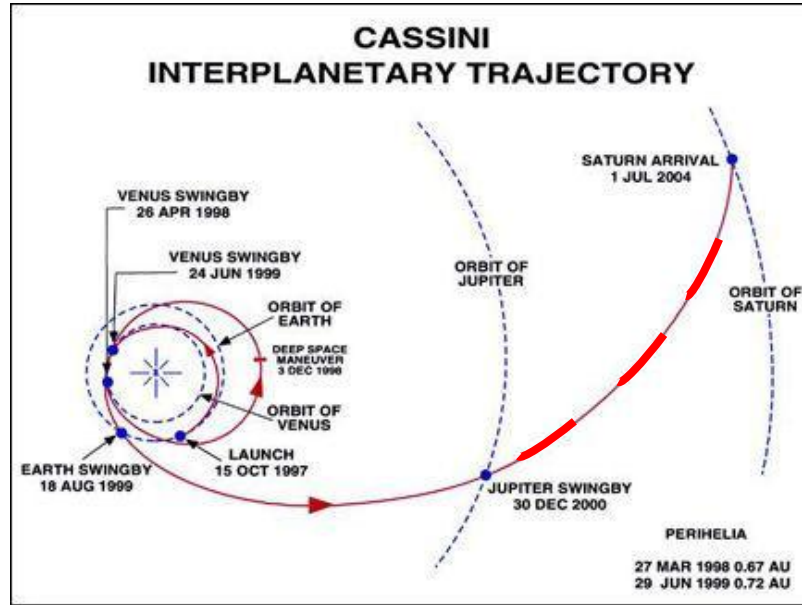
$$a_t = \frac{m_2}{m_1} a_{RTG} + a_{PA}$$

$$\sigma_{PA} = \left(\frac{m_2}{m_1} \sigma_t + \sigma_c \right) \left(\frac{m_1}{m_2} - 1 \right)^{-1}$$

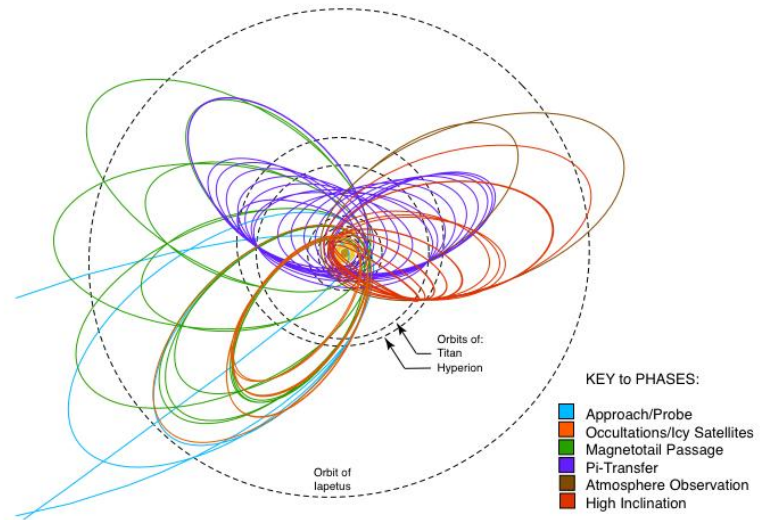
$$\sigma_c / a_c \cong 4\% \quad \text{well determined!}$$



Cassini Cruise and Tour Phases

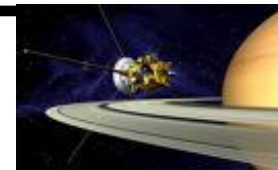


CASSINI MISSION SATELLITE TOUR



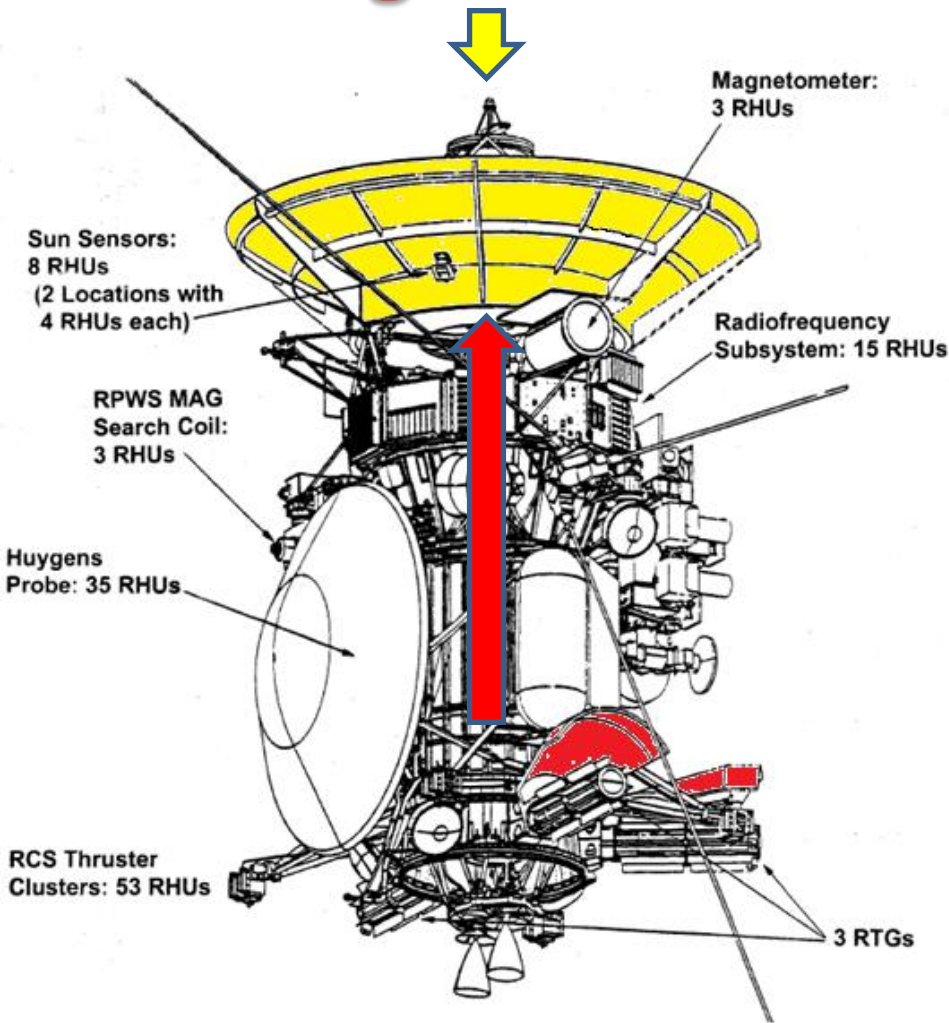
SOI: 1 July 2004

Spacecraft mass decreased from 4.6 tons to 2.8 tons after SOI/PRM/Huygens release





Non-gravitational accelerations – cruise



Leading sources:

■ RTG

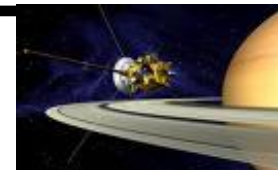
■ Solar radiation pressure $\left(\frac{A}{M} \cong 0.0023 \text{ m}^2 \text{ kg}^{-1} \right)$

At the epoch of the first radio science experiment (6.65 AU, Nov. 2001):

$$\frac{a_{RTG}}{a_{SP}} \cong \frac{3 \times 10^{-12} \text{ km s}^{-2}}{5 \times 10^{-13} \text{ km s}^{-2}} = 6$$

The two accelerations are nearly aligned (within 3°) and highly correlated. Disentangling the two effects was complicated by variations of HGA thermo-optical coefficients.

HGA thermo-optical properties have been inferred by temperature readings of two sensors mounted on the HGA back side





Solar pressure acceleration

Thermal Equilibrium

 Infinite thermal conductivity

 α spec value=0.15



$$\varepsilon = \frac{\alpha\Phi}{\sigma T^4}$$



Thermal emission properties

 are mostly unaffected by

 radiation and outgassing



$$\alpha = \frac{\varepsilon\sigma T^4}{\Phi}$$

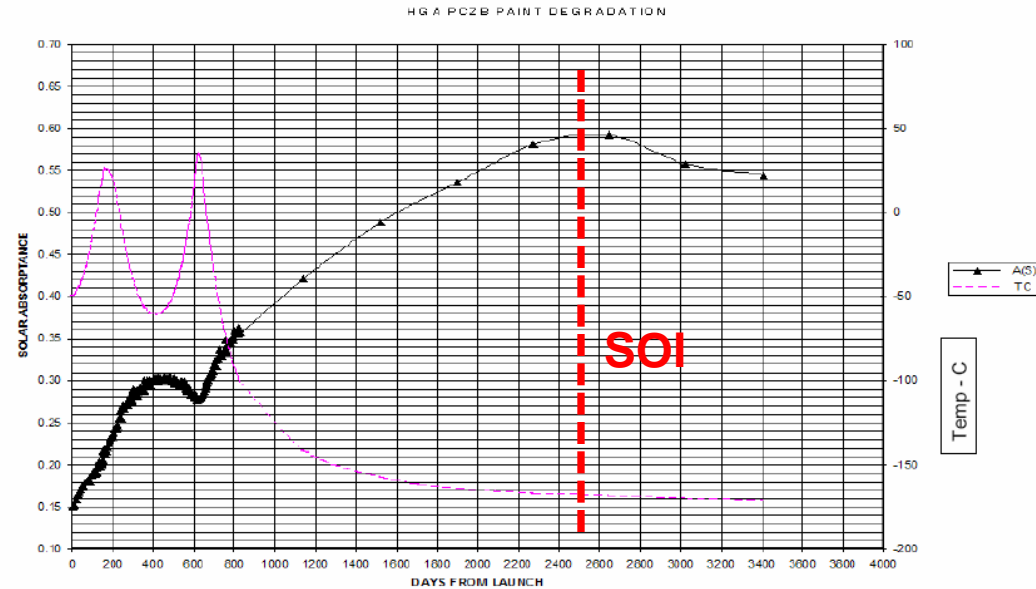


Specular reflectivity neglected

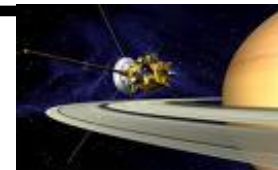
 Lambertian diffuse reflectivity



$$F_{SP} \propto A \frac{(5 - 2\alpha)}{3}$$



Source: S.C. Clark (JPL)

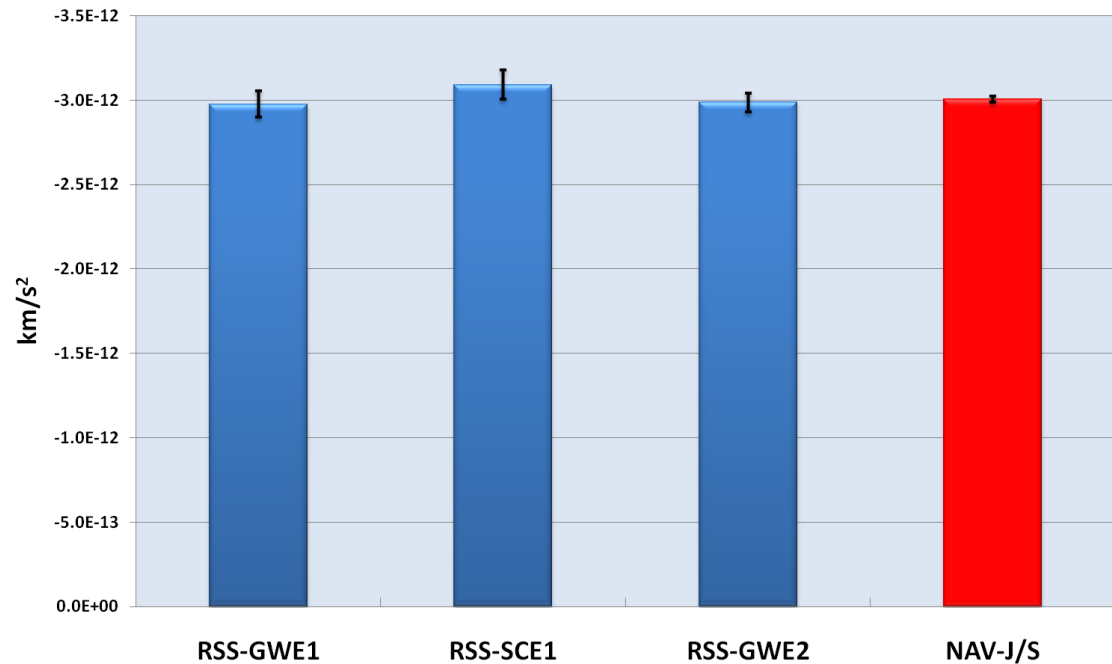




RTG – radial component estimates

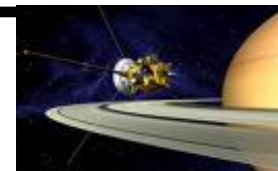
GWE1	• $(-2.98 \pm 0.08) \times 10^{-12} \text{ km s}^{-2}$
SCE1	• $(-3.09 \pm 0.08) \times 10^{-12} \text{ km s}^{-2}$
GWE2	• $(-2.99 \pm 0.06) \times 10^{-12} \text{ km s}^{-2}$
J/S	• $(-3.01 \pm 0.02) \times 10^{-12} \text{ km s}^{-2}$

RTG radial acceleration - cruise



4.3 kW of net thermal emission required
 (30% of total RTG power- 13 kW)

$$P = aMc$$





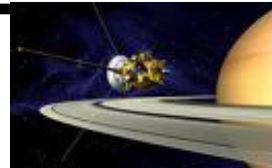
Searching for anomalous accelerations

The non-gravitational acceleration experienced by Cassini in the radial direction can in principle hide a Pioneer-like effect ($a_{Cas} \approx 3 \cdot a_{Pioneer}$). This can be assessed by comparing the non-gravitational accelerations after a large mass decrease

$$a_{rad} = \underbrace{a_{RTG} \left(\frac{m_0}{m_i} \right)}_{\text{Residual}} + \underbrace{a_{SP} \left(\frac{m_0}{m_i} \right) \left(\frac{r_0}{r_i} \right)^2}_{\text{Accounted for}}$$

$$\left. \begin{aligned} m_c a_c &= F_{RTG} \\ m_t a_t &= F_{RTG} \end{aligned} \right\}$$

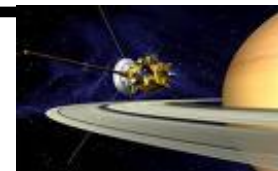
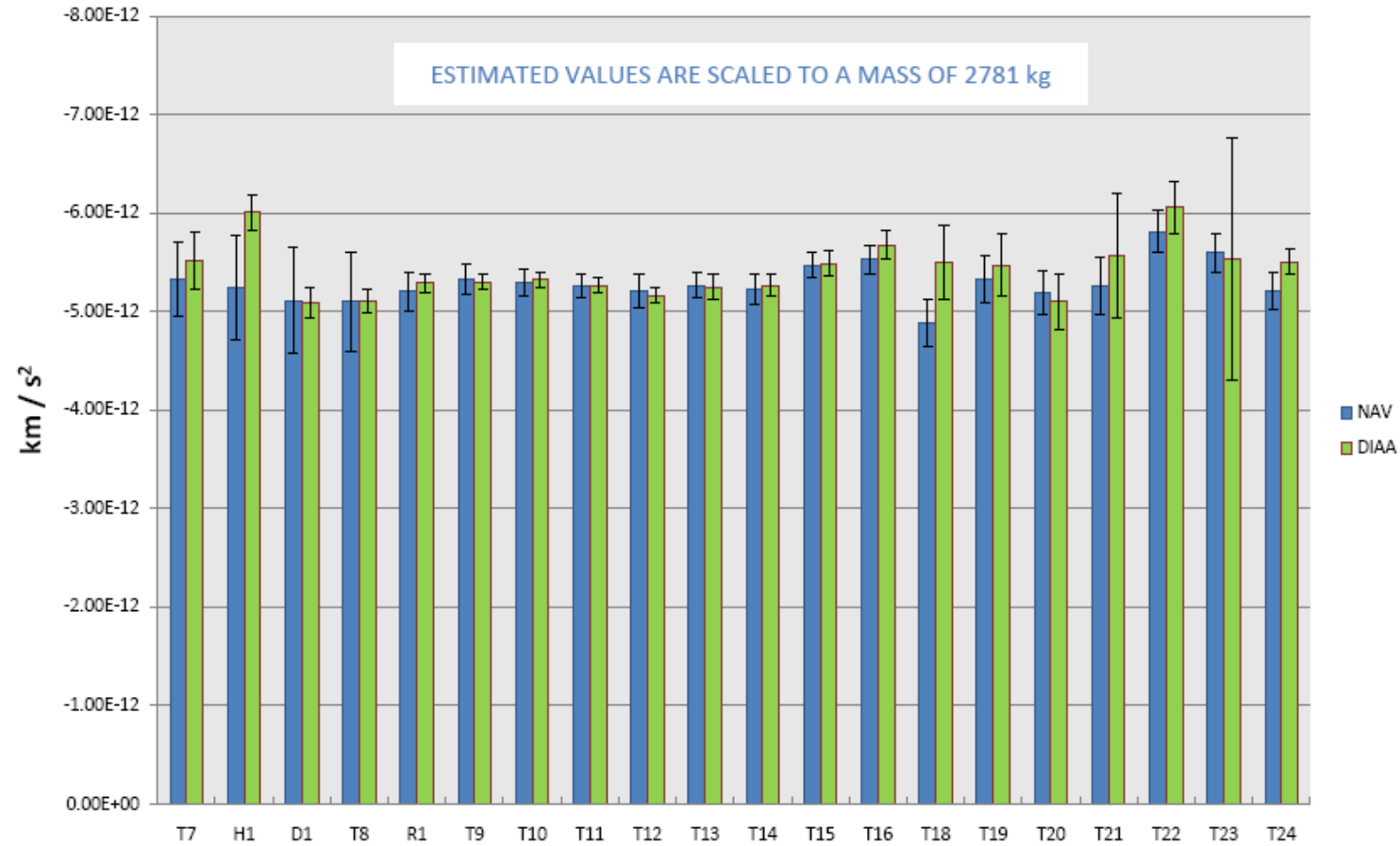
If the radial force experienced by Cassini is due only to RTG anisotropic thermal emission, the acceleration must be inversely proportional to the mass.





MA and NAV estimates

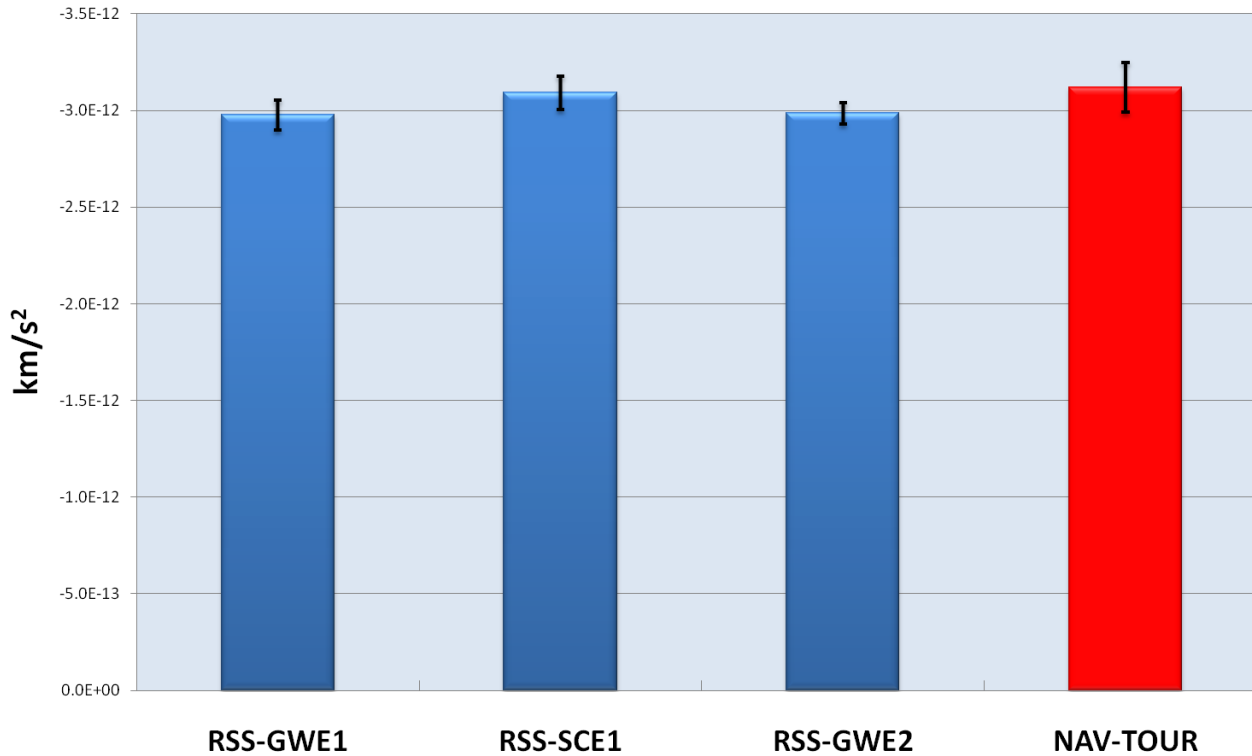
RTG - Radial Acceleration





Cruise-tour comparison

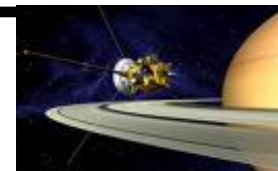
RTG - cruise and tour estimates



$$(-3.12 \pm 0.12) \times 10^{-12} \text{ km s}^{-2}$$



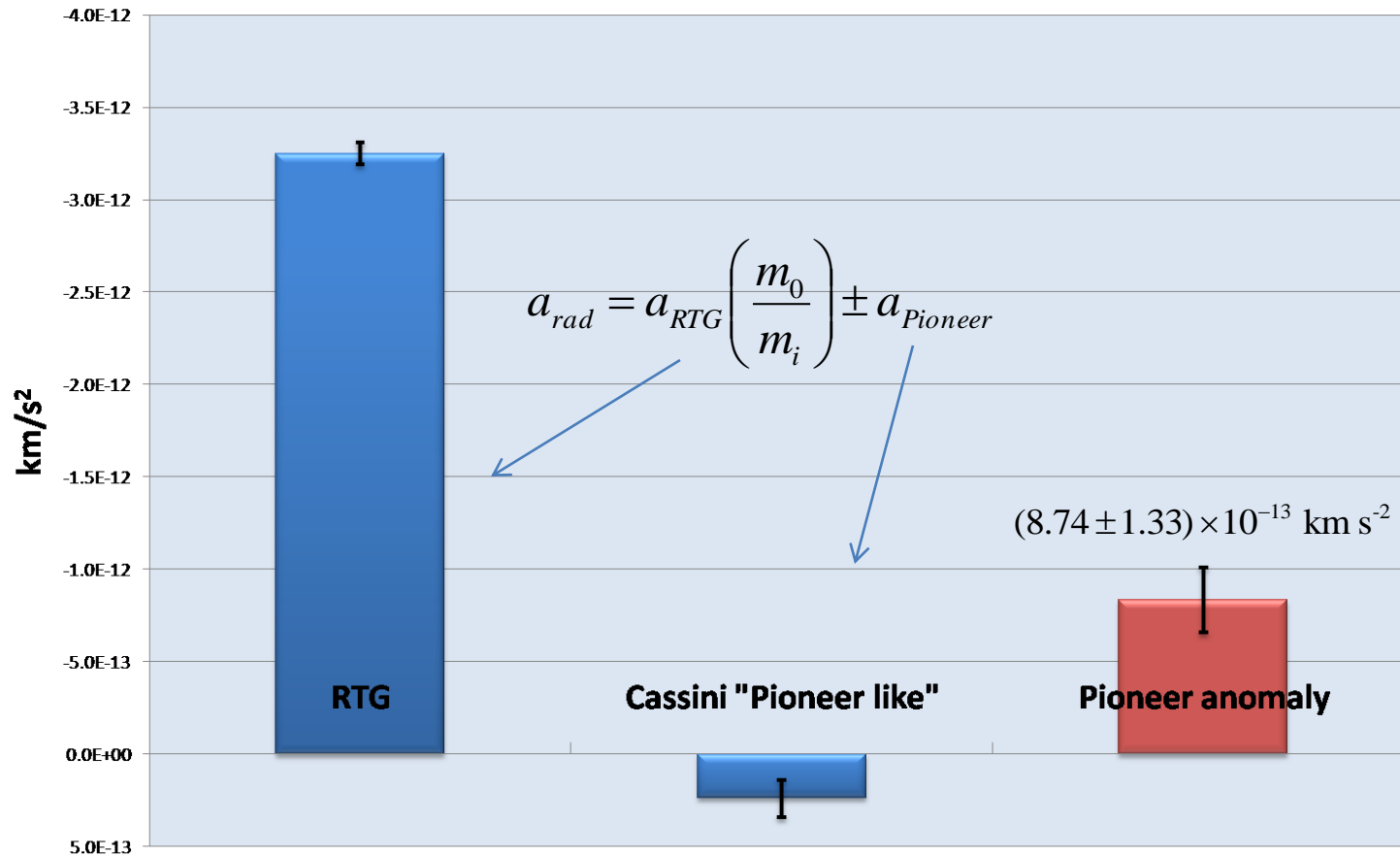
**Weighted mean value of
NAV estimates up to T49
(Dec. 2008)
61 independent
solutions (data arcs
spanning intervals of at
least 1.5 revs)**



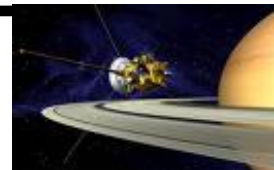


Upper limit to anomalous acceleration

RTG and Pioneer accelerations



(Di Benedetto and less, 20° International Symposium on Space Flight Dynamics, 2009)



Flyby anomaly

Appears only during Earth flybys of deep space probes.

No anomaly during planetary and satellite flybys

Effects: impossibility to fit simultaneously inbound and outbound arcs.

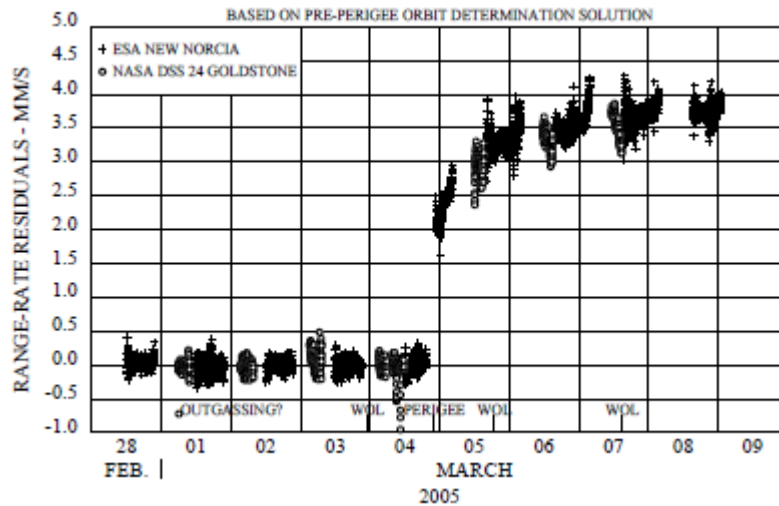
Solving for an impulsive burn at pericenter allows a global fit

Parameter	GLL-I	GLL-II	NEAR	Cassini	Rosetta	M'GER
Date	12/8/90	12/8/92	1/23/98	8/18/99	3/4/05	8/2/05
H (km)	960	303	539	1175	1956	2347
ϕ (deg)	25.2	-33.8	33.0	-23.5	20.20	46.95
λ (deg)	296.5	354.4	47.2	231.4	246.8	107.5
V_f (km/s)	13.740	14.080	12.739	19.026	10.517	10.389
V_∞ (km/s)	8.949	8.877	6.851	16.010	3.863	4.056
DA (deg)	47.7	51.1	66.9	19.7	99.3	94.7
I (deg)	142.9	138.7	108.0	25.4	144.9	133.1
α_i (deg)	266.76	219.35	261.17	334.31	346.12	292.61
δ_i (deg)	-12.52	-34.26	-20.76	-12.92	-2.81	31.44
α_o (deg)	219.97	174.35	183.49	352.54	246.51	227.17
δ_o (deg)	-34.15	-4.87	-71.96	-4.99	-34.29	-31.92
M_{SC} (kg)	2497	2497	730	4612	2895	1086
ΔV_∞ (mm/s)	3.92	-4.6	13.46	-2	1.80	0.02
σ_{V_∞} (mm/s)	0.3	1.0	0.01	1	0.03	0.01
Equation (1) (mm/s)	4.12	-4.67	13.28	-1.07	2.07	0.06

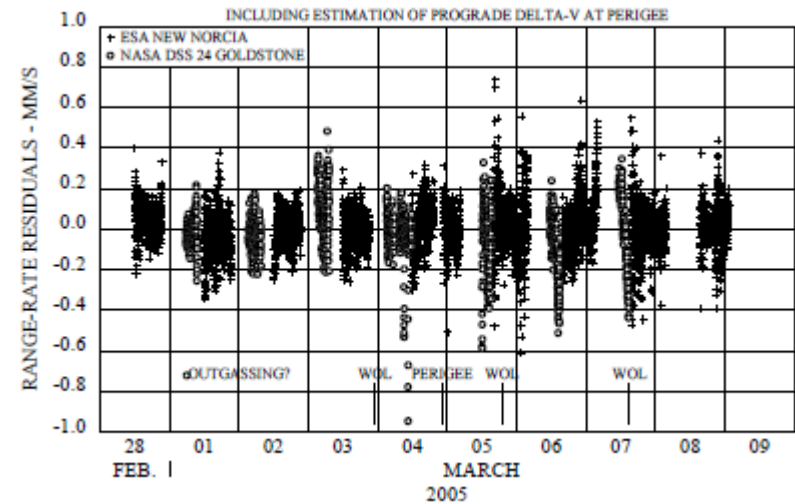
From Anderson et al., 2008

Flyby anomaly

Post-perigee data zero-weighted



Solving for prograde delta-V



From Morley and Budnik, 2006

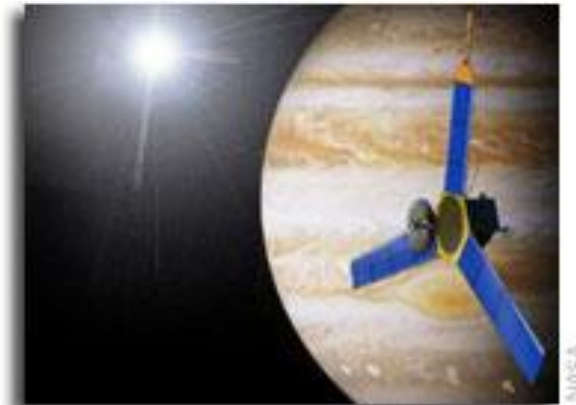
New physics?

Errors in the model used in the OD codes? (It is the same in all SW used in deep space navigation!)

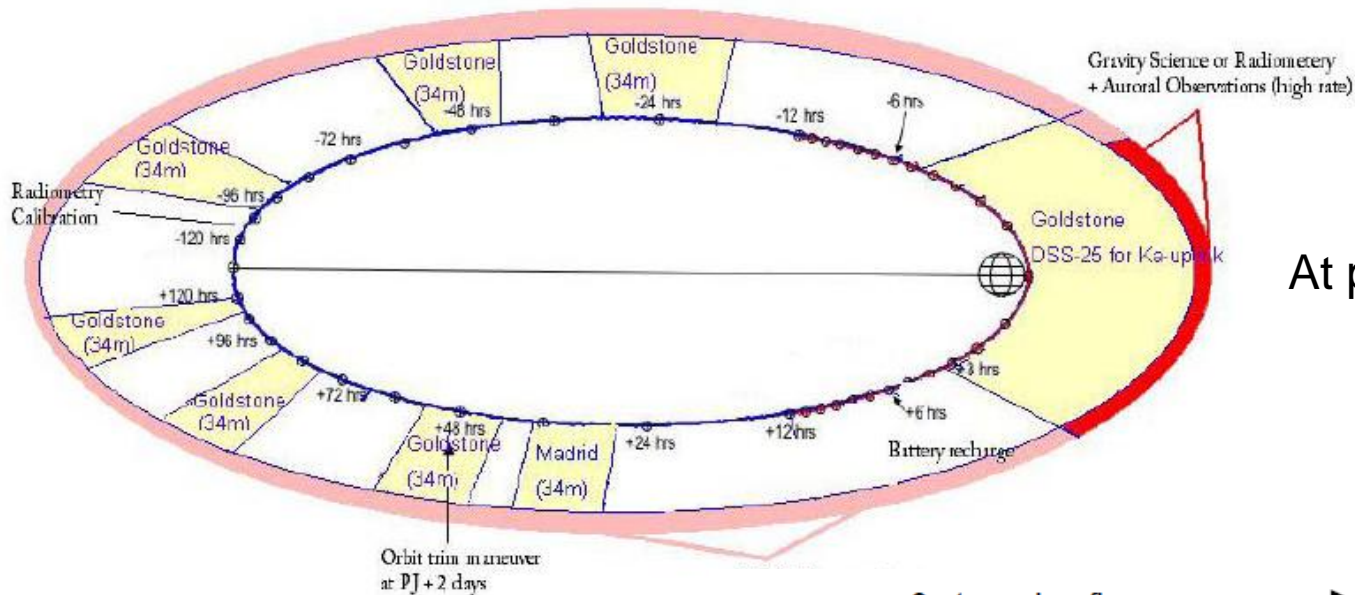
NASA Juno Mission

- ***New Frontiers* mission**
- **Investigation of atmosphere, magnetosphere and interior of Jupiter**

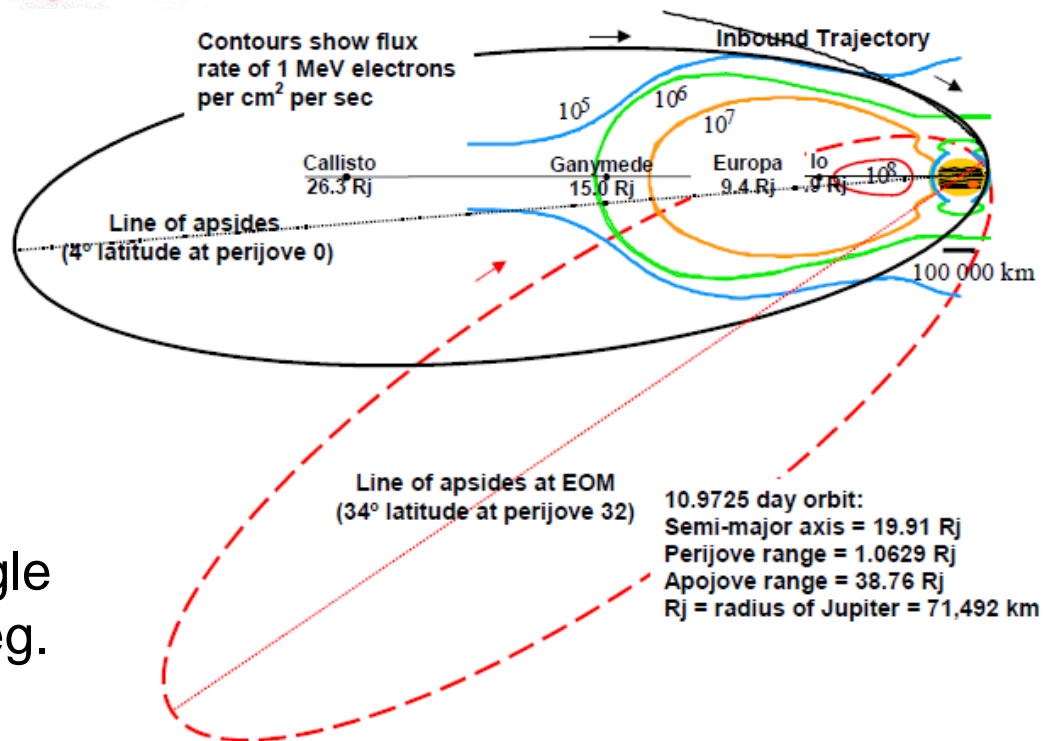
Launch	August 2011
Jupiter Orbit Insertion (JOI)	August 2016
Mission duration	1 year (32 orbits)
Orbit inclination	Polar (90°)
Orbit eccentricity	0.9466
Orbit period	11 days
Pericenter altitude	5000 km
Spacecraft mass @ Jupiter	1300 kg
Power	Solar arrays (54 m ²)
Attitude control	Spin stabilized



At pericenter, $v = 70$ km/s



At pericenter, $v = 70 \text{ km/s}$



Juno:
 orbit geometry and tracking
 At arrival orbit is nearly face on, then Earth view angle increases up to nearly 30 deg.

Lense-Thirring Precession of Juno

$$\ddot{\mathbf{r}} = \frac{(1+\gamma)\mu_J}{c^2 r^3} \left| \frac{\mathbf{J}}{m_J} \right| \left[\frac{3}{r^2} (\mathbf{r} \times \mathbf{v})(\mathbf{r} \cdot \hat{\mathbf{P}}) + (\mathbf{v} \times \hat{\mathbf{P}}) \right]$$

$$\frac{d\mathbf{S}}{dt} = \boldsymbol{\Omega} \times \mathbf{S} \quad \boldsymbol{\Omega} = \frac{(1+\gamma)G}{2c^2 r^3} \left[-\mathbf{J} + \frac{3(\mathbf{J} \cdot \mathbf{r}) \mathbf{r}}{r^2} \right]$$

$\frac{\mathbf{J}}{m_J}$ = Jupiter specific angular momentum

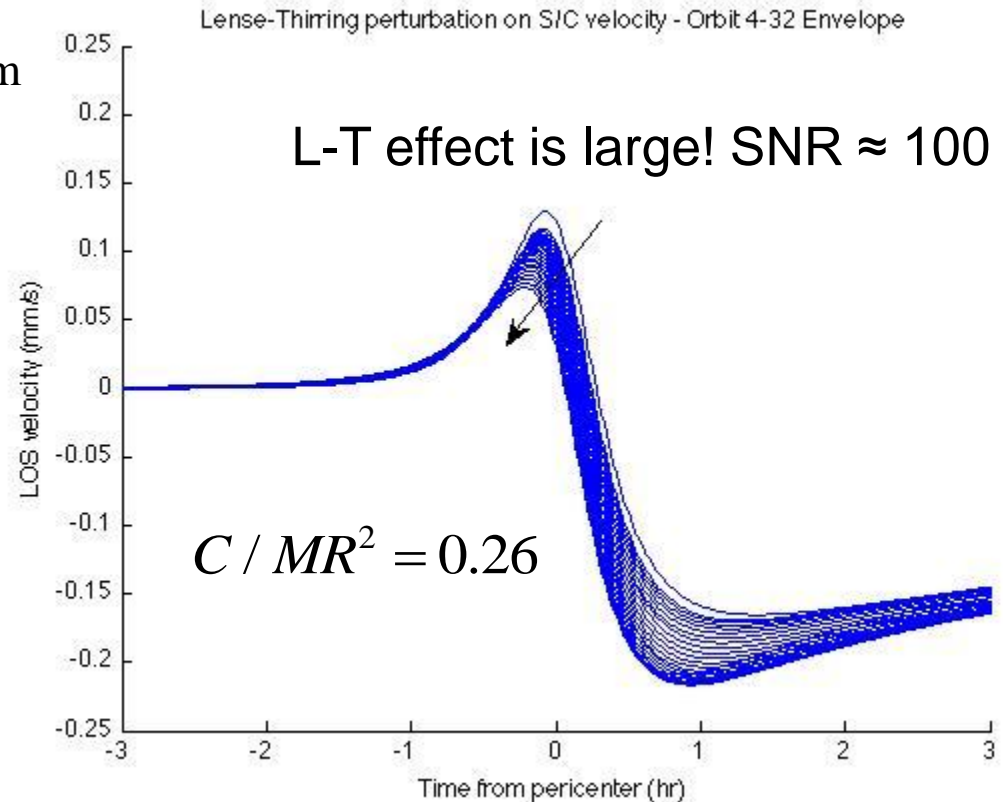
\mathbf{r} = Jupiter-S/C position vector

\mathbf{v} = S/C velocity relative to Jupiter

$\hat{\mathbf{P}}$ = direction of the Jupiter spin axis

μ_J = Jupiter gravitational constant

(proposed by Iorio, 2008)



BepiColombo: ESA's mission to Mercury

Launch: Ariane 5 (2014)

Solar Electric Propulsion

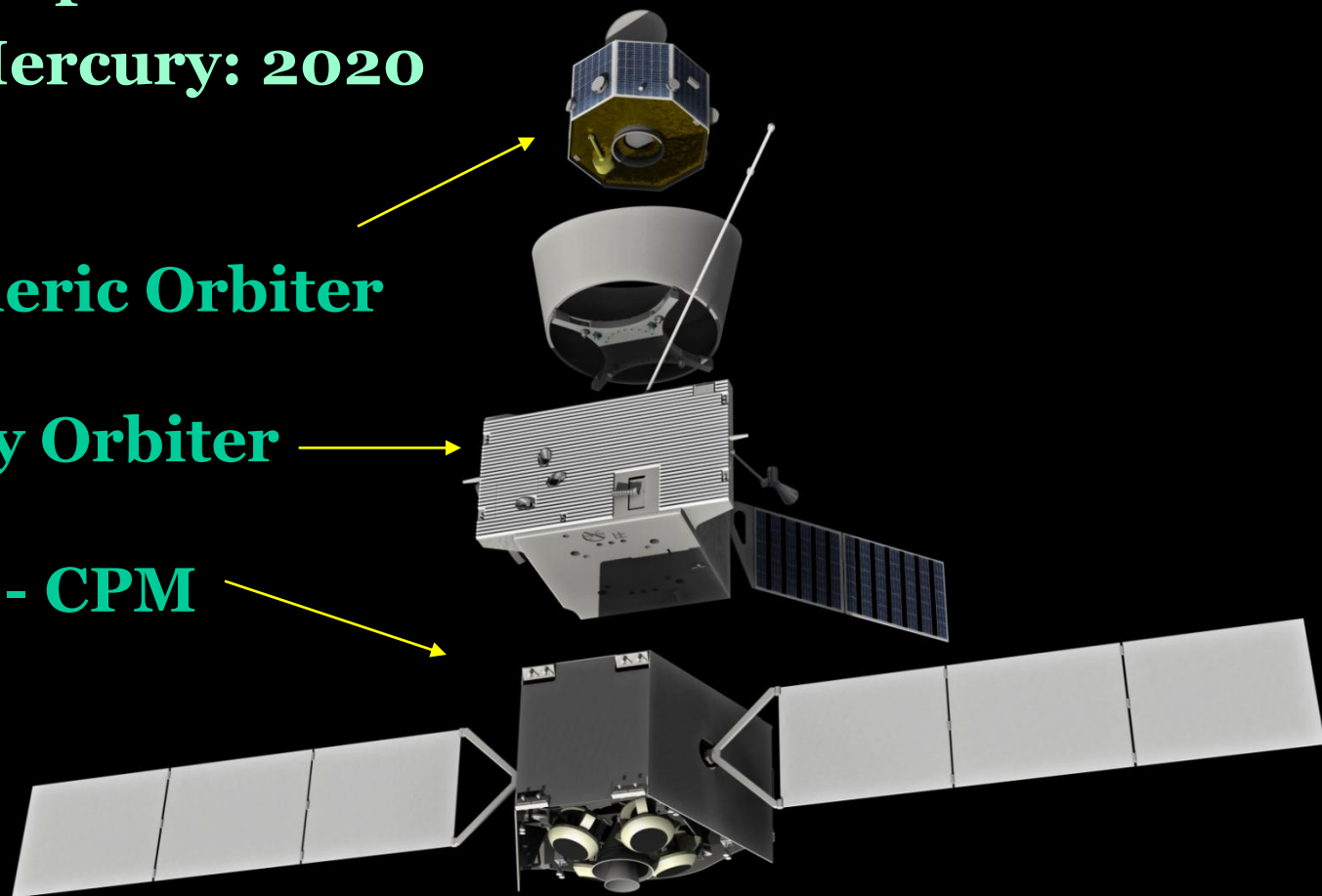
Chemical Propulsion

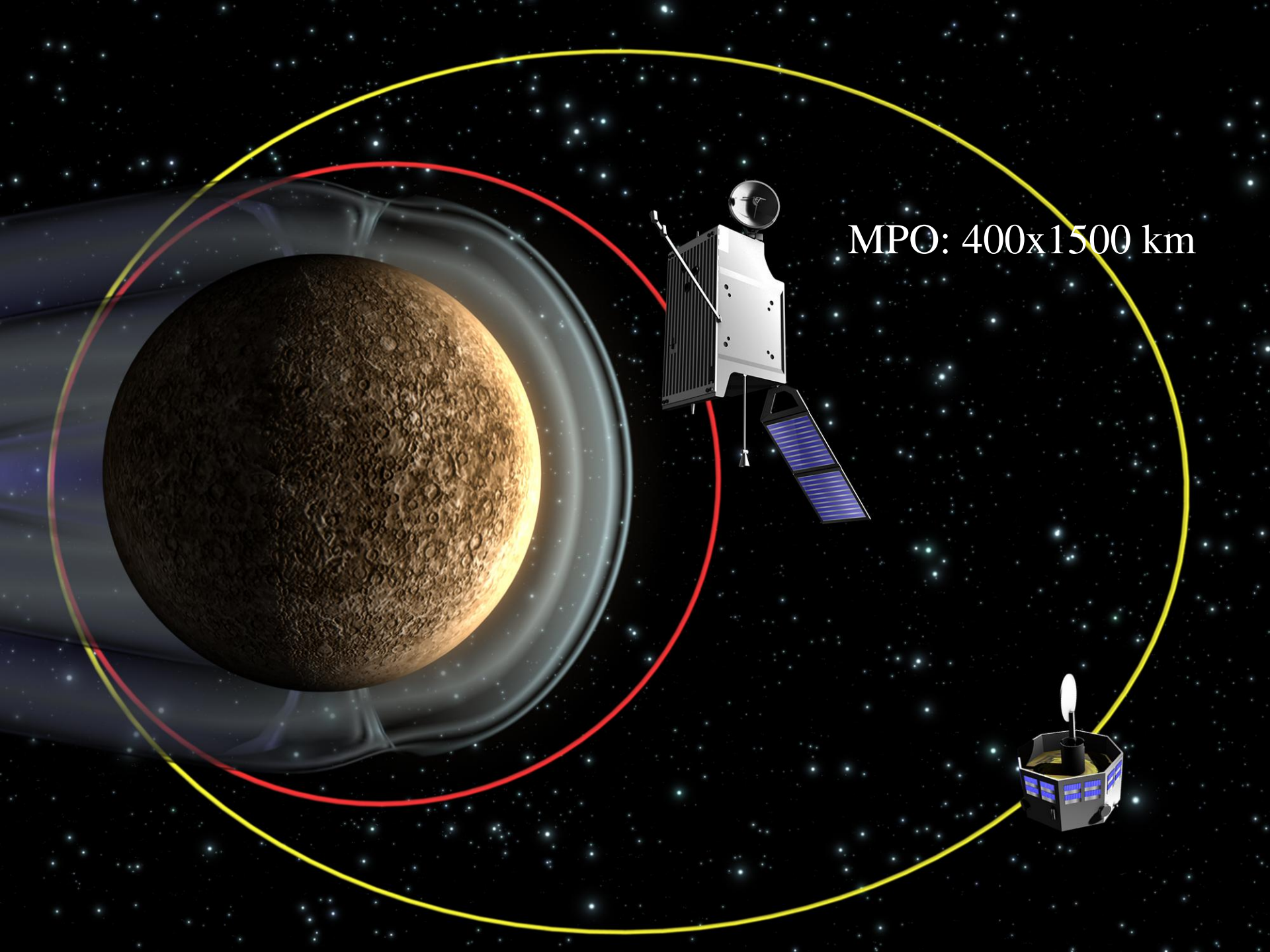
Arrival at Mercury: 2020

Magnetospheric Orbiter

Planetary Orbiter

SEPM - CPM



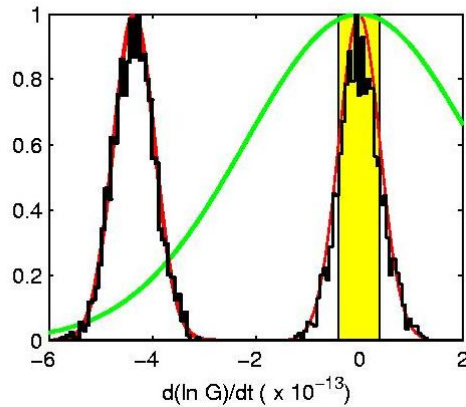
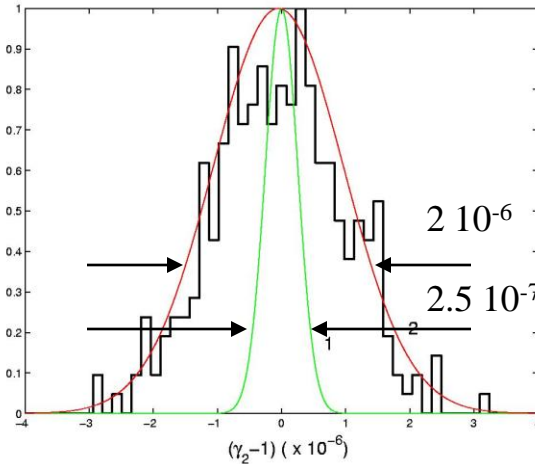
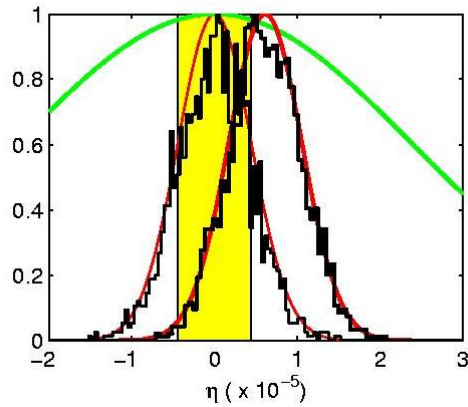
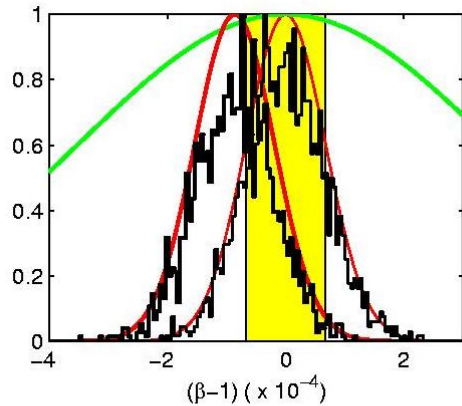


MPO: 400x1500 km

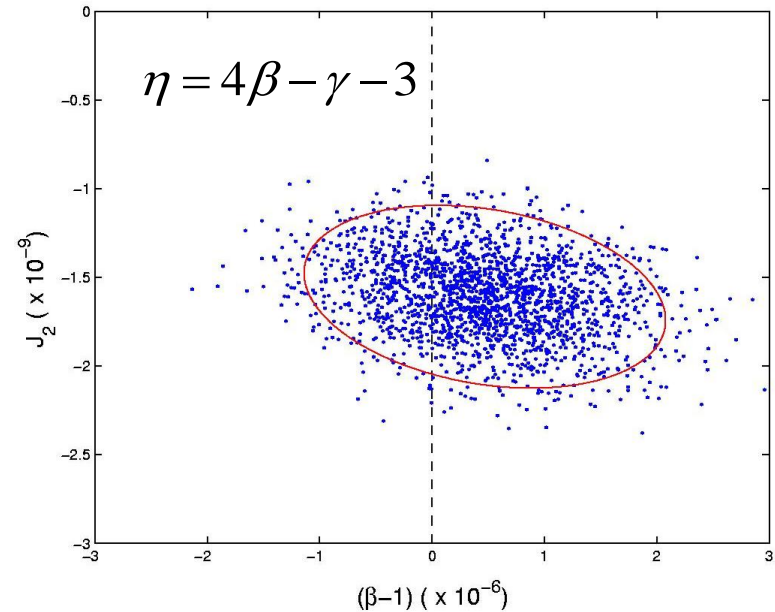
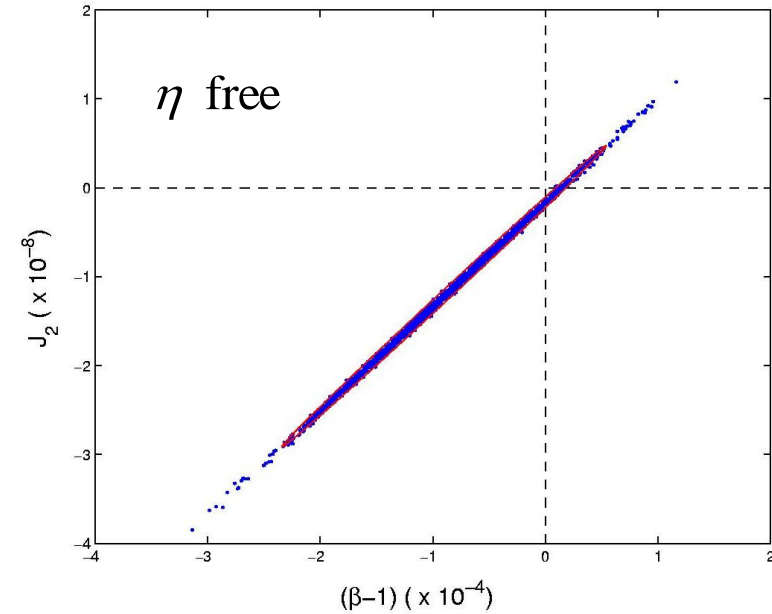
2000 simulations of 1y experiment

No preferred frame – η free

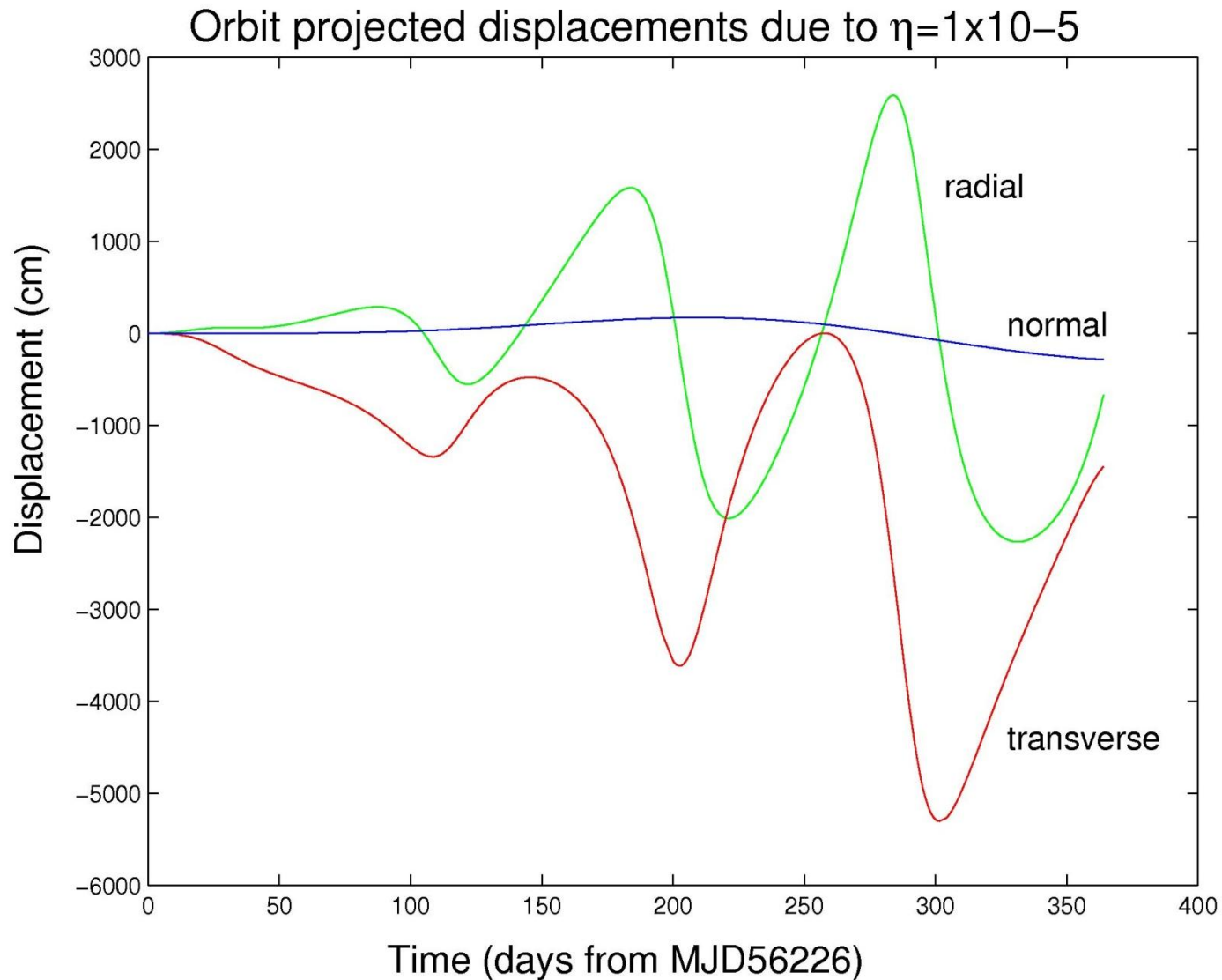
Cruise SCE \longrightarrow



Correlation ellipses



The effect of SEP violations on the Earth-Mercury distance



Parameter	Present accuracy	MORE
γ	2×10^{-5}	2×10^{-6}
β	1×10^{-4}	2×10^{-6}
η	5×10^{-4}	8×10^{-6}
J_2^\odot	4×10^{-8}	2×10^{-9}
\dot{G}/G	$9 \times 10^{-13} \text{ yr}^{-1}$	$3 \times 10^{-13} \text{ yr}^{-1}$

factor of 50
discrepancy
with Bender et al.
(2007)

Current accuracies of selected PN parameters and values expected from the BepiColombo MORE experiment. Metric theories of gravity with no preferred frame effects are assumed.

Milani et al. Phys. Rev. D, **66**, 082001 (2002).

Prospects for future missions

- **Free-flying spacecraft**
- Subject to stray accelerations and uncertainties in the masses of solar system bodies
- Onboard accelerometers of limited use (unless LISA class or better): must be bias-free and work to very low frequencies
- **Planetary orbiters**
- Tied to central body, nearly immune to stray accelerations
- Subject to uncertainties in the masses of solar system bodies
- **Planetary landers**
- Immune to stray accelerations, but subject to the effects of rotational dynamics (and again to unmodelled accelerations from asteroids)
- Planetary rotation is of paramount interest to geophysics; opportunity for synergies

Final remarks

- Advances in solar system tests of gravity have been painfully slow.
- So far, progress has relied upon piggy-back experiments (Viking, Cassini, BepiColombo, GAIA)
- Progress has been made in ruling out claims of violations of GR at solar system scales.
- Lacking a predictive theoretical framework for violations of GR, space agencies are not willing to invest on dedicated missions.
- In addition, any experiment claiming a violation will not be immediately accepted! Concurrence of different measurements is crucial.
- However, cosmological evidence for a new physics should boost the experimental efforts also at solar system level. Indeed, violations at cosmological scales will almost surely affect laws of gravity at short scales, maybe with detectable effects in classical tests.

Additional material

Jupiter's metric with Newtonian quadrupole correction:

$$ds^2 = \left(1 - 2\frac{GM_J}{rc^2} + J_2 \frac{GM}{rc^2} \left(\frac{R_J}{r} \right)^2 (3\cos^2 \theta - 1) \right) dt^2 - \left(1 + 2\frac{GM_J}{rc^2} - J_2 \frac{GM}{rc^2} \left(\frac{R_J}{r} \right)^2 (3\cos^2 \theta - 1) \right) (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2)$$

At Juno's pericenter ($r \approx R_J$), the correction due to the quadrupole is simply of order J_2 . Both the monopole time delay and relativistic Doppler shift must therefore contain a correction of the same order.

Only order of magnitude estimates beyond this point. N. Ashby has carried out more precise calculations.

Two-way monopole time delay and relativistic Doppler shift:

$$\Delta t = 4 \frac{R_g}{c} \ln \frac{l_0 + l_1 + l_{12}}{l_0 + l_1 - l_{12}} = 4.6 \times 10^{-8} \text{ s} = 13.8 \text{ m}$$

$$\left(\frac{\Delta f}{f} \right)_0 \cong 8 \frac{R_g}{cb} \frac{db}{dt} \approx 8 \frac{R_g}{b} \frac{v}{c} = 3.4 \times 10^{-11} \quad \left(\frac{\Delta f}{f} \right) = \frac{d}{dt} \Delta t$$

b = impact parameter = 74000 km

v = velocity at pericenter = 60 km/s

R_g = gravitational radius of Jupiter = 1.5 m

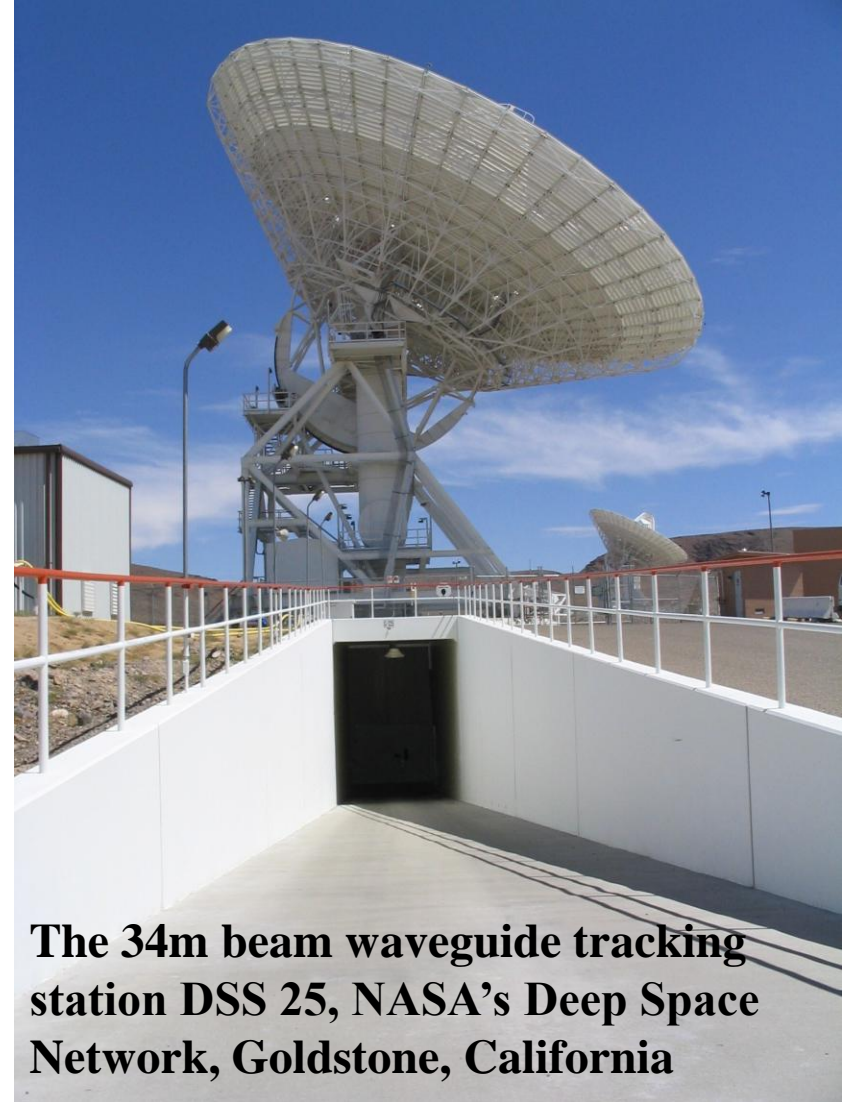
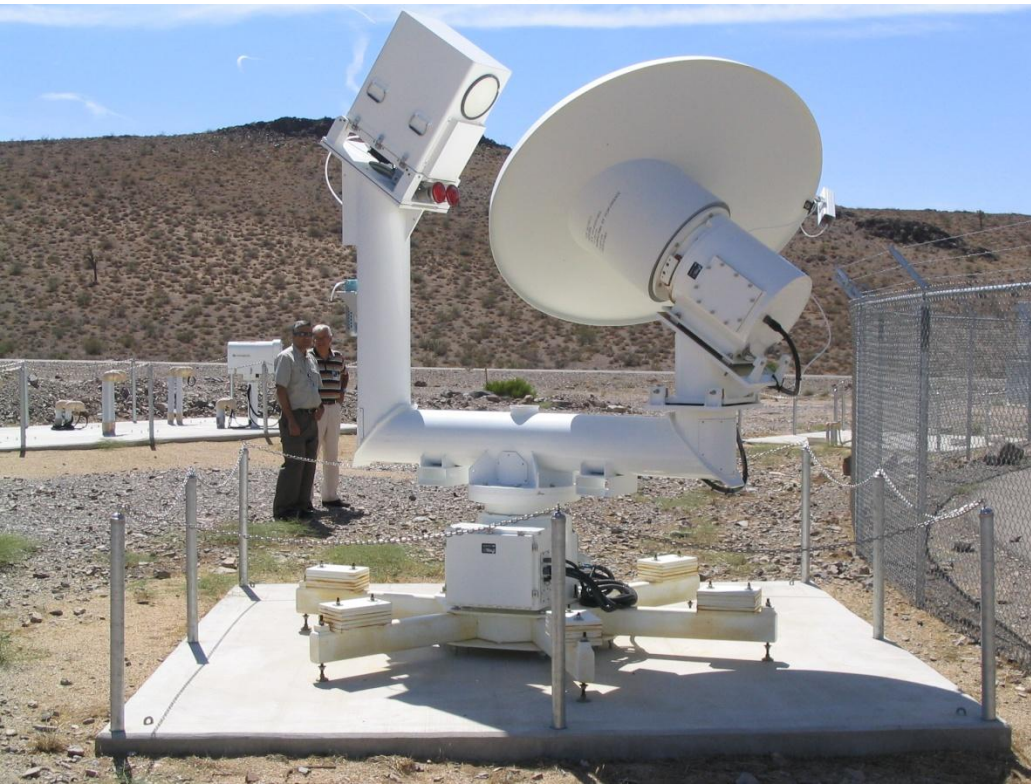
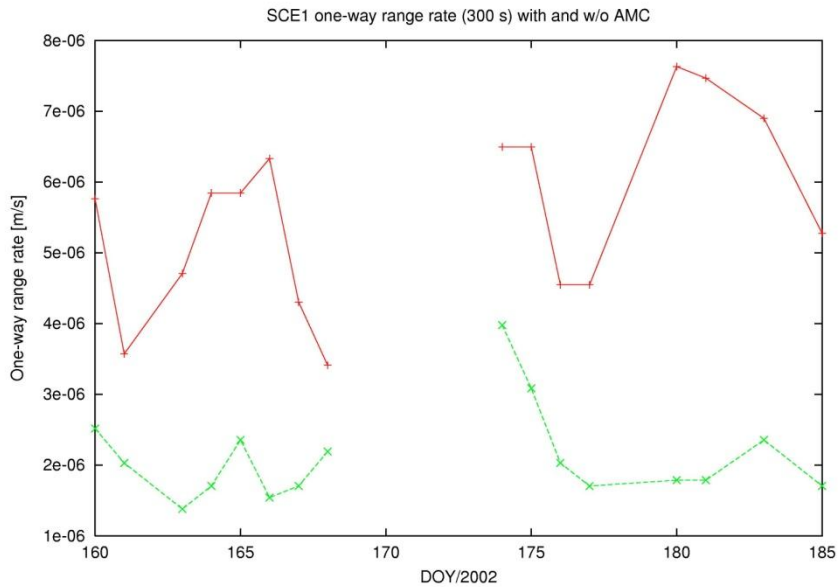
This Doppler shift is only a factor of 6 smaller than the one experienced by Cassini during SCE1!

Two-way quadrupole relativistic Doppler shift:

$$\left(\frac{\Delta f}{f}\right)_2 \cong J_2 \left(\frac{\Delta f}{f}\right)_0 = 6.8 \times 10^{-13}$$

about a factor of 70 larger than the measurement error. The effect is asymmetric across pericenter and mimics a Newtonian J3. Note that the correction to the light time is below the accuracy of current ranging systems.

The effect is large and must be accounted for in the OD software (currently it is not).



The 34m beam waveguide tracking station DSS 25, NASA's Deep Space Network, Goldstone, California

The Advanced Media Calibration System for tropospheric dry and wet path delay corrections.

Precession of Jupiter's spin axis

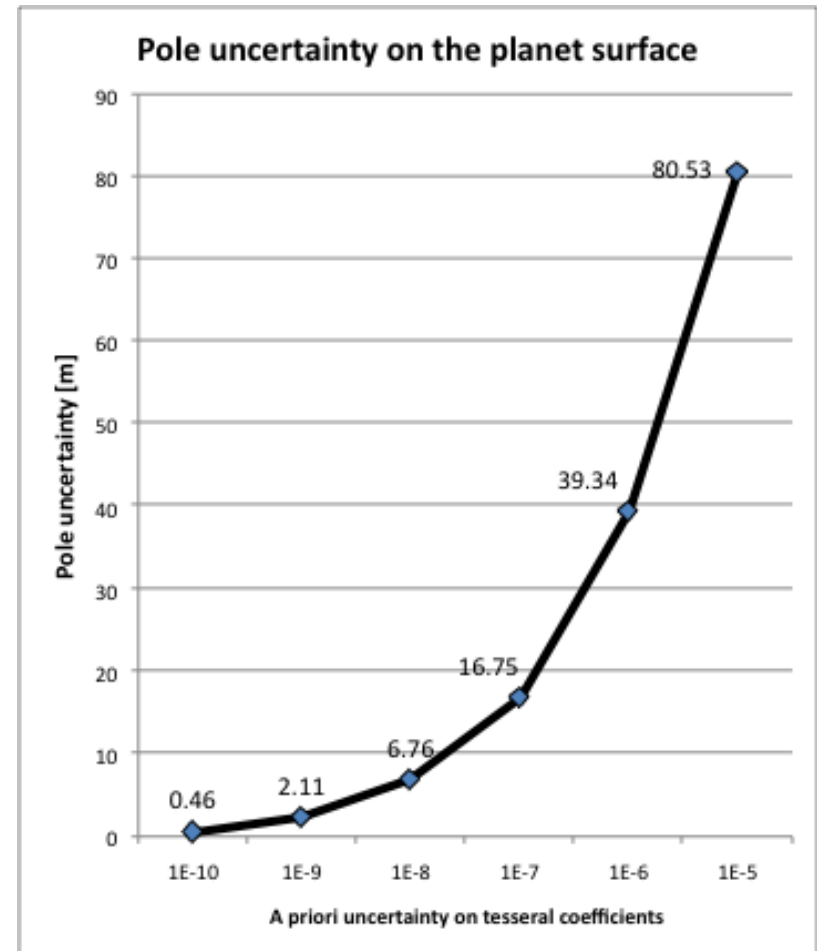
Provides also the angular momentum of Jupiter !

- The quadrupole and the inertia tensors share the same eigenvectors.

$$Q = \frac{\sqrt{5}}{3} MR^2 \begin{bmatrix} -(C_{20} - \sqrt{3}C_{22}) & \sqrt{3}S_{22} & \sqrt{3}C_{21} \\ \sqrt{3}S_{22} & -(C_{20} + \sqrt{3}C_{22}) & \sqrt{3}S_{21} \\ \sqrt{3}C_{21} & \sqrt{3}S_{21} & 2C_{20} \end{bmatrix}$$

$$Q = \frac{1}{3} I \cdot Tr(\mathfrak{I}) - \mathfrak{I}$$

- By diagonalizing the quadrupole tensor one computes the principal axes of inertia and their associated uncertainties.



Lense-Thirring Precession of Juno

- Option 1: assume GR is true and estimate Jupiter's angular momentum from L-T
- Option 2: assume GR is true and combine estimates of LT and pole precession in an improved solution for Jupiter's angular momentum
- Option 3: combine estimates of LT and pole precession solving simultaneously for the L-T parameter and Jupiter's angular momentum

Caveat: how separable is L-T from other effects, e.g. accelerations due to zonal harmonics?

Corrently L-T at Jupiter is not modelled in any OD software.