

TEST OF MODELS FROM POLARIZATION EXPERIMENTS  
EXAMPLE: THE RULE  $\Delta J = 1$  in  $0^{-\frac{1}{2}+} \rightarrow 1^{-\frac{3}{2}+}$ .

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ABSTRACT

In the space of observed polarization parameters,  $\mathcal{D}$  the domain predicted by the model must be a subdomain of  $D$ , the polarization domain predicted by general conservation laws (e.g. angular momentum, parity, isospin, etc.). We recall the shape of  $D$  for usual high-energy experiments producing spin 1 or 3/2 particles and some general model predictions in those cases. As a new illustration we present an analysis of the world data on  $\pi^+p^+ \rightarrow \rho^0\Delta^{++}$  or  $\omega^0\Delta^{++}$  when 19 polarization parameters are observed. It strongly favors the rule  $\Delta J = 1$  between baryon states.

INTRODUCTION

This contribution to the conference does not deal with polarized beams or polarized targets. However, I thought fit to accept the invitation to contribute a half-hour talk in order to propagandize about some of the work on polarization that Doncel, Minnaert and I have done during the last ten years.<sup>1</sup>

The study of differential cross-sections in high energy physics has revealed simple and fundamental laws for the dependence on energy and on momentum transfer. Similarly, since all these reactions involve spinning particles, the study of polarization effects may allow the discovery of simple and fundamental laws for the dependence on angular momentum transfer.

A new physical law or model can be tested by an experiment only if, for this experiment, they have stronger implications than those derived from fundamental invariance principles. In the present literature, where results of polarization measurements of hadrons with spin greater than  $\frac{1}{2}$  are given, the consequence of angular momentum and parity conservation seemed generally to be ignored. In the same manner that energy momentum conservation defines a domain for the energy momentum of the final particles (i.e., the phase space) of a reaction, angular momentum and parity conservation define a polarization domain  $D$  for the observed polarization parameters.

The model to be tested must predicts for these parameters a subdomain  $\mathcal{D}$  of  $D$ . The value of the test will depend on how much the experimental data yield points near  $\mathcal{D}$ , with experimental errors small with respect to  $D$ .

As an example of such a study we have analyzed all available data on polarization correlations in hadronic reactions of the type  $0^- \frac{1}{2}^+ \rightarrow 1^- \frac{3}{2}^+$ . Reactions of this type are among the most complicated measurements presently performed in high-energy physics: at least 19 significant polarization parameters can be measured. So we feel that this analysis has some value as an example. Moreover, it yields an interesting physical result: while the change of spin from the initial to the final baryon could be obtained by both angular momentum transfers  $\Delta J = 1$  (dipole) and  $\Delta J = 2$  (quadrupole), we find that the experimental data strongly suggest a pure  $\Delta J = 1$  transition from the fundamental baryon octet to the first decuplet.

### THE POLARIZATION DOMAIN

We recall here the essential steps necessary to the determination of the polarization domain: for more details we refer to our previous publications<sup>2-3</sup>.

i) The polarization states of two particles of spin  $j_1$  and  $j_2$  (here  $j_1 = 1$ ,  $j_2 = \frac{3}{2}$ ) is described by a density matrix  $\rho$ , i.e., a  $n \times n$ , ( $n = (2j_1+1)(2j_2+1)$ ), Hermitian ( $\rho^* = \rho$ ), positive ( $\rho \geq 0$ ), trace one matrix. Such matrices form a  $n^2-1$ , convex self-dual domain in the  $n^2$  dimensional Euclidean space  $E$  of  $n \times n$  Hermitian matrices, whose scalar product is  $(\rho_1, \rho_2) = \text{tr} \rho_1 \rho_2$ .

ii) When  $n_0 = \pi_i(2j_i+1)$  for initial particles (here = 2) is smaller than  $n$  (which is here 12), angular momentum conservation implies that rank  $\rho = n_0$  when all momenta and polarization of the final particles are completely determined (which is not the case here, see iv).

iii) Parity conservation may impose on  $\rho$  to be in a linear subspace of  $E$ . This is the case, for instance, when:

"The reaction is parity conserving, the initial particles are unpolarized; only three linearly independent momenta are observed." (a)

(as it is in the case here). Then the initial state is invariant by reflection through the reaction plane and the final state must have the same symmetry. The corresponding conditions have been expressed very neatly in ref. 4. Here  $\rho$  must be in a 72-dimensional subspace of the 144-dimensional space  $E$ .

iv) Most often the polarization measurement is partial, i.e., one observes only the orthogonal projection of the point of  $E$  representing  $\rho$ , on a linear subspace of  $E$ . This is the case when the polarization is observed through the angular distribution of a parity conserving two-body decay: only the "alignment" is measured (here the quadrupolar polarizations for  $\rho$ ,  $K^*$ ,  $\phi$ ,  $\Delta$ . It happens to be also the case for the three-body decay of the  $\omega$ ). However, if the  $\frac{3}{2}^+$  baryon is a  $Y^*$ , its polarization can be completely determined by the sequential decays  $Y^* \rightarrow \pi + \Lambda$ ,  $\Lambda \rightarrow \pi + N$ ; then one can measure 47 polarization parameters. We know only of one experiment where this has been done<sup>5</sup>. For all other data, 19 polarization parameters are essentially measured<sup>6</sup>, so  $D_0$  is 19-dimensional. No condition is

left on the rank of  $\rho$ . For an infinite precision measurement of the momenta of the final particles  $D_0$  would not be convex. But actual experiments, to improve the statistics use large bins in  $t$ , the momentum transfer, so the polarization domain  $D$  is more realistically the convex hull of  $D_0$ .

If the polarization of one particle only is measure, the polarization domain is three-dimensional and is respectively a cone for the spin 1 meson<sup>7</sup> and a sphere for the spin 3/2 baryon<sup>8,9</sup>.

### THE $\Delta\vec{J} = 1$ RULE

This rule is contained in more specific models. For instance, the Stodolsky-Sakurai model<sup>10</sup> for the reactions  $0^- \frac{1}{2}^+ \rightarrow 0^- \frac{1}{2}^+$  implies  $\Delta\vec{J} = 1$  since it assumes that the reaction is dominated by a  $M_1$  (magnetic dipole) transition<sup>11</sup>. The rule  $\Delta\vec{J} = 1$  is also a consequence of the quark model, and therefore of SU(6); indeed in this model the lowest octet and decuplet are in a same supermultiplet, i.e., the quarks are in the same space-state (s-state) and the spin change from  $\frac{1}{2}^+$  to  $\frac{3}{2}^+$  during the reaction is only due to the spin flip of one of the quarks; the spin flip of a spin  $\frac{1}{2}$  particle creates a pure  $\Delta\vec{J} = 1$  angular momentum transfer.

For the reaction we study here, the rule  $\Delta\vec{J} = 1$  makes no prediction on the observable separate polarizations<sup>12</sup>. In the 19-dimensional observable domain  $D$  of joint polarization, it predicts an 8-dimensional subdomain  $\mathcal{D}$  which is in the intersection of  $D$  by a 13-dimensional linear subspace  $E_T$  (For details and proofs see ref. 14). By partial integration of phase space, corresponding to large bins in  $t$  (the momentum transfer) in actual experiments, only the 6 linear relations which determined  $E_T$  can be tested.<sup>15</sup>

The natural way to test them is to project all data on  $E_T^\perp$ , the 6-dimensional vector subspace of  $E$  orthogonal to  $E_T$ . Instead of a 6-dimensional figure, the 3 lowest drawings on Figure 1 show the projection of the domain  $D$  and the experimental data on three mutually orthogonal two-planes  $x_1 y_1$ ,  $x_2 y_2$ ,  $x_3 y_3$ ; the theoretical point  $T_p$  orthogonal projection of  $E_T$  on  $E_T^\perp$ , projects respectively on  $A = Q$  of  $x_1 y_1$  and on the origins  $O$  of  $x_2 y_2$ ,  $x_3 y_3$ . The grouping of the experimental points around the theoretical one, corresponding to the rule  $\Delta\vec{J} = 1$  is impressive<sup>17</sup>; the few abnormal points have rather large errors. Of course there must be correlations among the three projections on  $x_i y_i$  ( $i = 1, 2, 3$ ) of an experimental point. Indeed, if the projection on  $x_1 y_1$  falls on  $A = Q$ , then angular momentum and parity conservation already requires the projection on  $x_3 y_3$  to be 0 and that on  $x_2 y_2$  to be inside the dotted circle. In the same Fig. 1, the upper two diagrams correspond to a simplified test that we proposed<sup>18</sup> in 1973. We consider the project of  $D$  on the three-dimensional space  $E_D$  spanned by the observed diagonal matrices in transversity quantization (i.e., bimultipole components  $T_{00}^{22}$ ,  $T_{00}^{02}$ ,  $T_{00}^{20}$ ). The projection of  $D$  on  $E_D$  is the self-dual tetrahedron ABCD. The rule  $\Delta\vec{J} = 1$  predicts the line segment AQ in the face ACD. The experimental points are well grouped on AQ. Their distribution on AQ depends on different physical mechanisms. For instance one pion exchange or pure unnatural

parity exchange (in  $\pi p \rightarrow \rho \Delta$ ) corresponds to the point Q. All reactions  $\pi p \rightarrow \rho \Delta$  and  $K p \rightarrow K^* \Delta$  give data near Q for all energies between 3 and 13 GeV and for not too large momentum transfer. This is not the case for respectively  $\omega^0$  or  $\phi$  production. Figure 2 gives a more detailed analysis of an experiment<sup>19</sup> of  $\omega^0$  production at 13 GeV.

We refer again to our other references (and especially 14) for more details. We consider our study as just an example of what can be done to analyze polarization measurements of resonances in high-energy physics.

#### ACKNOWLEDGMENT

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#### FOOTNOTES AND REFERENCES

1. Some of this work does include polarized beams or polarized targets, e.g., "Amplitude reconstruction for usual quasi two-body reactions with unpolarized or polarized target". 97 pages CERN preprint 74-7 to appear in *Fortschritte der Physik*.
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11. This model predicts a unique polarization, represented by the south pole of the Doncel sphere; such a polarization is not possible on the forward and backward directions--indeed, the cross section must then vanish. The model is well verified at not too high energy, see e.g. M.G. Doncel, Second International Winter Meeting in Fundamental Physics 1974 (Instituto de Estudios Nucleares, Madrid).
12. For  $Y^*$ , the polarization can be completely measured and the polarization domain is 7-dimensional. The  $\Delta J = 1$  rule predicts the absence of  $L = 3$  polarization multipole, so the model subdomain is 4-dimensional. Some inconclusive data has been published for  $0^- \frac{1}{2}^+ \rightarrow 0^- Y^*$ ; for a critical study see Ref. 13.

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14. We have submitted to Nuclear Physics B a more detailed paper, "A selection rule on angular momentum transfer in reactions of the type  $0^{-\frac{1}{2}+} \rightarrow 1^{-\frac{3}{2}+}$ ".
15. These relations were first given in ref. 16 where they are called "Conditions A".
16. A. Biatas, K. Zalewski, Nucl. Phys. B6 (1968) 465.
17. For a full list of references, see ref. 14. It is mainly  $\pi^+p^+ \rightarrow \rho^0\Delta^{++}$  or  $\omega^0\Delta^{++}$ , with also some  $K^{\pm}N$  relations, between 3 and 13 GeV.
18. M. Doncel, L. Michel, P. Minnaert, "The polar angle distribution in point decay of spin 1 and  $\frac{3}{2}$  resonances" CERN/PhII/Phys. 73-39, communication to the second Aix-en-Provence Conference on Elementary Particles (1973).
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TRANSFER OF ANGULAR MOMENTUM  $J = 1$

WORLD DATA : 50 POINTS (T)

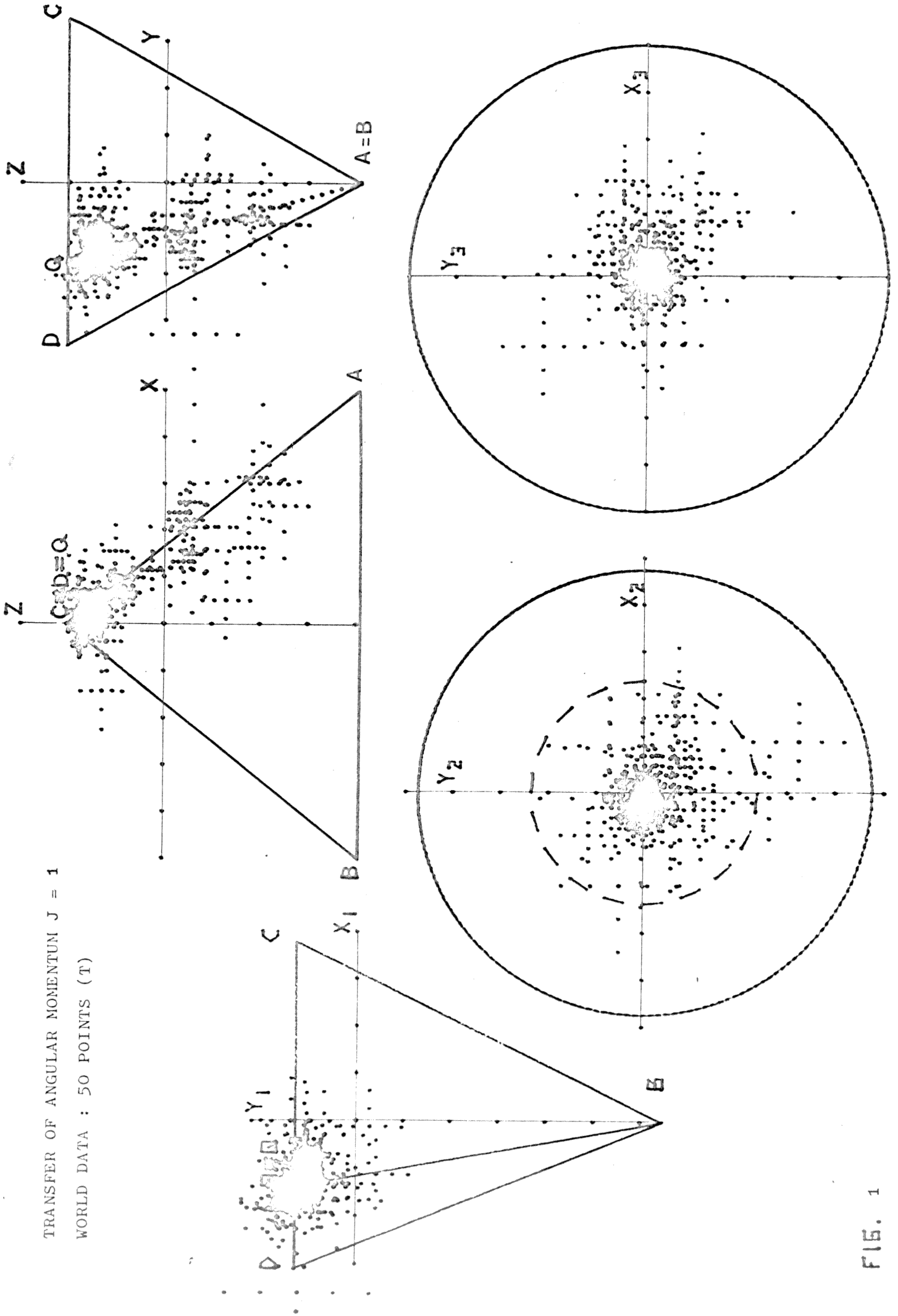
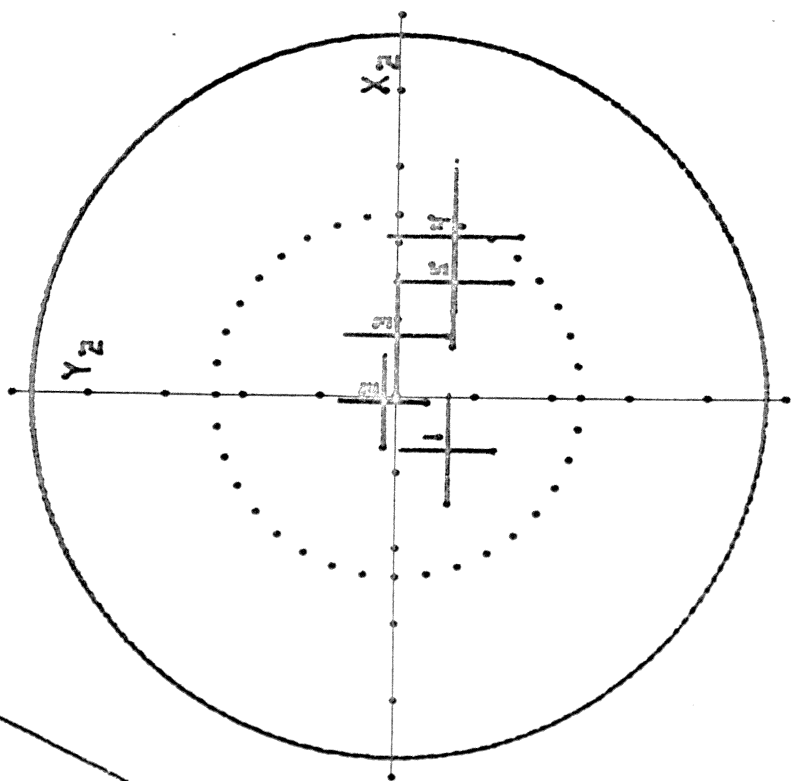
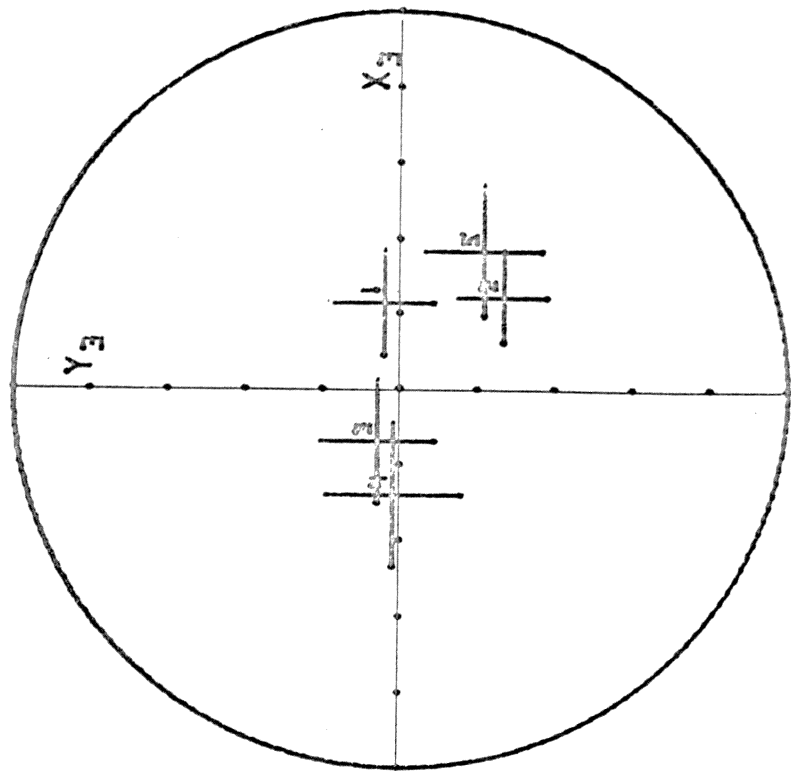
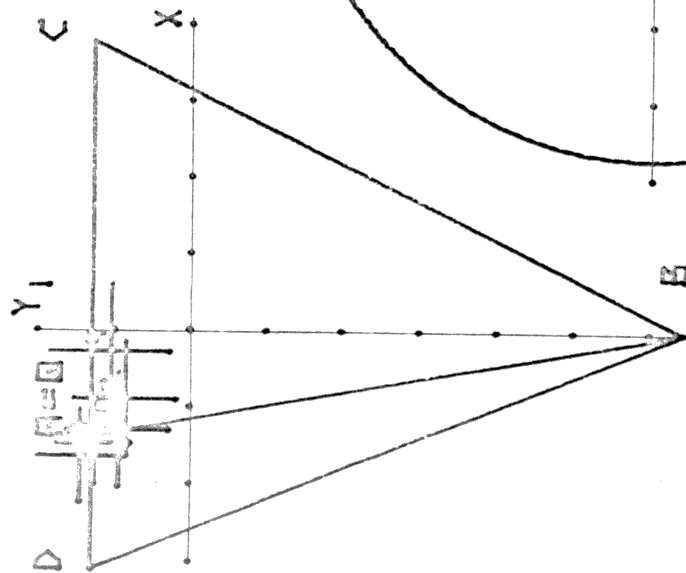
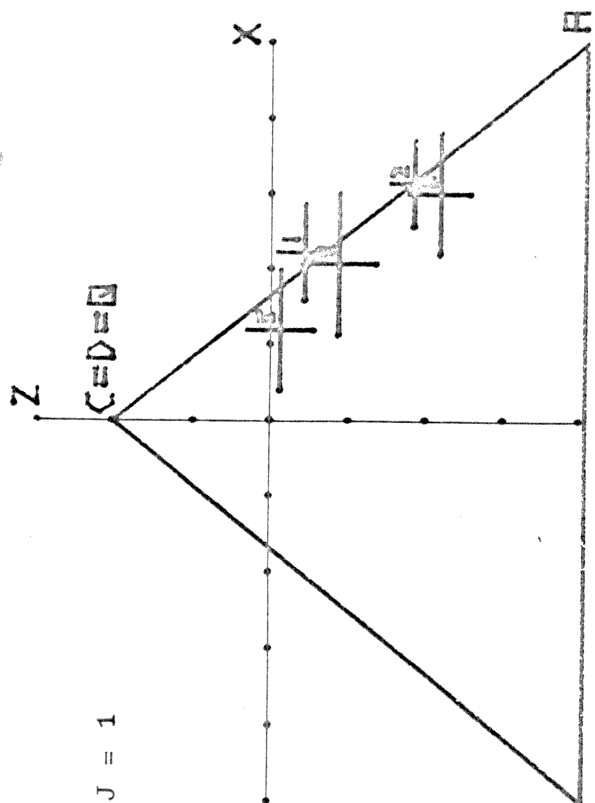
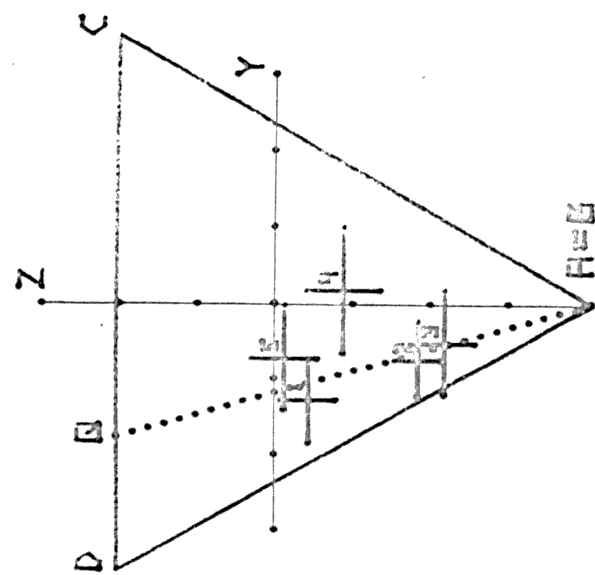


FIG. 1

TRANSFER OF ANGULAR MOMENTUM  $J = 1$



$\omega\Delta^{++}$  13 GeV (T)

Ref. 19

FIG. 2