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THE POLAR ANGLE DISTRIBUTION IN JOINT
DECAY OF SPIN 1 AND 3/2 RESONANCES

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ABSTRACT

Plots for the presentation of experimental results on joint polar angle distributions are described. They allow easy checks of the positivity of the density matrix and of the angular distribution, and they visualize the comparison of the experimental results with the class A predictions of the quark model.

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1. DIAGONAL, EVEN POLARIZATION DOMAIN

A recent preprint [1] proposes a partial test of the quark model [2] for associated production of spin 1 and 3/2 resonances of the type

$$\pi p \rightarrow \omega \Delta \tag{1}$$

It seems to us that (complete or partial) plots of the polarization domain^x are still the best tools for such a test.

The final state of reactions of type (1) is described by a 12 x 12 density matrix $\rho(\omega, \Delta)$. Parity conservation in the production reaction leaves 71, a priori non vanishing, joint multipole parameters $T_{MM'}^{LL'}(\omega, \Delta)$ (those with $L = 0, 1, 2$, $L' = 0, 1, 2, 3$ and e.g. in transversity quantization $M + M' = \text{even}$). The analysis of the joint angular distribution of the decays $\omega \rightarrow 3\pi$, $\Delta \rightarrow N\pi$ allows only the measurement of 19 multipole parameters (those with L and $L' = 0$ or 2).

It should be known that the observed density matrix built with these 19 parameters (with the unobservable parameters set equal to zero) needs not be positive. This is easily understood in geometrical terms. The complete density matrix is represented by a point ρ in a 71-dimensional Euclidean space, the point ρ^E which represents the measured matrix is the projection of ρ in the 19-dimensional plane E of measurable parameters. The point ρ belongs to the positivity domain D , but ρ^E belongs to the projection of D on E , which is generally larger than the intersection of D by E . Hence ρ^E does not necessarily belong to this intersection and the measured density matrix can be non positive.

However it is possible to find out of the 19 measurable parameters a set of parameters spanning a plane E' such that projection and intersection are equal (such a plane is called an equatorial plane of D). This is the

x For a general study of the polarization domain see ref. [3,4,5].

case for the 3 diagonal parameters T_{00}^{20} , T_{00}^{02} and T_{00}^{22*} . In the following we restrict ourselves to these parameters and for notational convenience we omit their lower indices. The intersection of D by E' is the tetrahedron ABCD drawn in Fig. 1. It is defined by the inequalities**

$$1 - \sqrt{24} T^{20} - \sqrt{12} T^{02} + \sqrt{24} T^{22} \geq 0 \quad (2a)$$

$$1 - \sqrt{24} T^{20} + \sqrt{12} T^{02} - \sqrt{24} T^{22} \geq 0 \quad (2b)$$

$$1 + \sqrt{6} T^{20} - \sqrt{12} T^{02} - \sqrt{6} T^{22} \geq 0 \quad (2c)$$

$$1 + \sqrt{6} T^{20} + \sqrt{12} T^{02} + \sqrt{6} T^{22} \geq 0 \quad (2d)$$

which express the positivity of the diagonal elements.

This tetrahedron is self-conjugated by polar transform^x with respect to the sphere of radius $r = \sqrt{-1/12}$. Hence its opposite edges are orthogonal. It is convenient to define new orthogonal axis in the directions

AB and CD. The change of coordinates reads

$$Z = T^{20}, \quad (3a)$$

$$X = -\sqrt{1/3} T^{02} + \sqrt{2/3} T^{22}, \quad (3b)$$

$$Y = -\sqrt{2/3} T^{02} - \sqrt{1/3} T^{22}. \quad (3c)$$

* That these parameters span an equatorial plane is easily proved. The diagonal elements form an equatorial plane [3,4,5] and the $L + L' = \text{even}$ parameters form a symmetry plane, therefore their intersection is an equatorial plane E'' spanned by T_{00}^{20} , T_{00}^{02} , T_{00}^{22} , T_{00}^{11} and T_{00}^{31} . Furthermore, the L and $L' = \text{even}$ parameters form a symmetry plane E' of $D \cap E''$, and hence it is an equatorial plane of D.

** We normalize the 3 diagonal parameters in such a way that for two density matrices ρ_A and ρ_B one has

$$\text{tr } \rho_A \rho_B = \frac{1}{12} + T_A^{20} T_B^{20} + T_A^{02} T_B^{02} + T_A^{22} T_B^{22}$$

This is the case for the parameters defined in ref. [6].

x The positivity domain D is self conjugated by polar transform [3,5], hence the intersection of D by the equatorial plane E' has the same property. Concretely one verifies that the line issued from each summit through the center is perpendicular to the opposite facet and that the product of the algebraic distances from the center to the summits and to the opposite facets is $-1/12$.

The projections of the tetrahedron on the planes (X,Z) and (Y,Z) are the triangles ABC and ADC drawn in Fig. 2. An experimental point is inside the tetrahedron if and only if its projections on these planes are inside these triangles. Note that if one considers independently the "physical bounds" [7] of each parameter, the corresponding domain is an orthohedron, the volume of which is 4 times bigger than that of the polarization domain.

2. JOINT POLAR ANGLE DISTRIBUTION

The joint polar angle distribution of the resonances ω and Δ involves only these 3 parameters T^{20} , T^{02} , T^{22} . It can be written

$$I(\theta, \theta') = \frac{1}{4} \left[1 - \sqrt{6} T^{20} (3 \cos^2 \theta - 1) - \sqrt{3} T^{02} (3 \cos^2 \theta' - 1) + \sqrt{\frac{3}{2}} T^{22} (3 \cos^2 \theta - 1) (3 \cos^2 \theta' - 1) \right], \quad (4)$$

where θ and θ' are the angles of the normal to the ω decay plane and the direction of the Δ decay, with respect to their quantization axis. The positivity of $I(\theta, \theta')$, for arbitrary θ and θ' , imposes constraints on the multipole parameters; they are

$$1 - \sqrt{24} T^{20} - \sqrt{12} T^{02} + \sqrt{24} T^{22} \geq 0, \quad (5a)$$

$$1 - \sqrt{24} T^{20} + \sqrt{3} T^{02} - \sqrt{6} T^{22} \geq 0, \quad (5b)$$

$$1 + \sqrt{6} T^{20} - \sqrt{12} T^{02} - \sqrt{6} T^{22} \geq 0, \quad (5c)$$

$$1 + \sqrt{6} T^{20} + \sqrt{3} T^{02} + \sqrt{\frac{3}{2}} T^{22} \geq 0. \quad (5d)$$

These conditions yield the tetrahedron A'BC'D (drawn in Fig. 1) which have 2 facets and 3 edges in common with the tetrahedron ABCD. The projections of this tetrahedron on the planes (X,Z) and (Y,Z) are the triangles A'BC and ADC' drawn in Fig. 2. In any case the projections of the experimental results must fall inside these triangles, otherwise they would correspond to a non positive angular distribution.

Recently another parametrization of the angular distribution has been proposed [1], namely

$$I(\theta, \theta') = \frac{9}{4} \left[\frac{r}{3+a} (1 + a \cos^2 \theta') \cos^2 \theta + \frac{1-r}{3+b} (1+b \cos^2 \theta') \frac{1}{2} \sin^2 \theta \right] \quad (6)$$

The relations of the a,b,r parameters with the X,Y,Z parameters are

$$r = \frac{1}{3} (1 - \sqrt{24} z), \quad (7a)$$

$$a = 3 \left[\frac{1 - \sqrt{24}z}{3X} - 1 \right]^{-1}, \quad b = 3 \left[\frac{\sqrt{2} + \sqrt{12}z}{3Y} - 1 \right]^{-1}. \quad (7b)$$

The lines of constant r (or a or b) are indicated in Fig. 2, for some characteristic values of the parameters.

We see two major drawbacks to this parametrization :

- i) the parameters a and b cannot be obtained by a linear moment analysis of the angular distribution,
- ii) the parameters are not orthogonal and hence the errors on the experimental values obtained by maximum likelihood are strongly correlated.

3. QUARK MODEL PREDICTIONS

Up to now we have not at all precised which quantization frames was to be used for the ω and the Δ . Indeed the positivity conditions for the density matrix and for the polar angle distribution must be satisfied whatever the quantization frames (any transversity and helicity frames and even frames with z-axis neither in the reaction plane nor along the normal to the reaction plane). Since, in transversity quantization, the diagonal parameters are invariant by rotations around the normal, the position of an experimental point in the tetrahedron is independent of the choice of transversity frame (e.g. s,t or u transversity). This would not be the case for the different helicity frames, and the positivity

conditions must be satisfied for each of them separately.

The quark model gives relations between the multipole parameters of the ω and the Δ . For the diagonal parameters, in transversity quantization, the class A prediction of the model [2] is the line AQ drawn in Figs. 1 and 2 (the point Q is in the edge CD, and therefore the line AQ in the facet ACD). Its equations, in the different parametrizations discussed above, are

$$-\frac{1}{\sqrt{6}} \leq T^{20} = \sqrt{2} T^{02} = \frac{1}{\sqrt{6}} - 2 T^{22} \leq \frac{1}{\sqrt{24}} \quad (8a)$$

$$-\frac{1}{\sqrt{6}} \leq Z = -\sqrt{\frac{2}{3}} (X + \sqrt{2} Y) = \frac{1}{\sqrt{6}} - \frac{2}{\sqrt{3}} (\sqrt{2} X - Y) \leq \frac{1}{\sqrt{24}} \quad (8b)$$

$$a = 3, \quad b = -\frac{3}{5}, \quad 0 \leq r \leq 1. \quad (8c)$$

We suggest that the 2 plots of Fig. 2 be used for the presentation of the experimental results on joint polar angle distribution of spin 1 and $\frac{3}{2}$ resonances. They allow a very clean cut check of the positivity of the density matrix and of the joint polar angle distribution, for any quantization frame. Besides, the experimental results being plotted with their statistical errors, the relative size of the errors and of the polarization domain is visualized on the plots. Finally we find it much easier to compare the experimental results to the class A predictions of the quark model by a glance at the plots, rather than by checking the equality of the two sides of some equations.

As an example, we have plotted in Fig. 2, the experimental results reported in refs. [8,9]. They fall along and not far from the predicted line AQ.

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FIGURE CAPTIONS

Fig.1 Positivity domains of joint polarization and of decay angular distributions for particles of spin 1 and 3/2. The tetrahedron ABCD is the positivity domain for the three measurable diagonal parameters of the joint density matrix. The tetrahedron A'BC'D is the positivity domain for the corresponding parameters of the polar angle distribution in the joint decay. The class A prediction of the quark model is the segment AQ on the facet ACD of the first tetrahedron. The axes are drawn with length 1. The coordinates (T^{02}, T^{22}, T^{20}) of the significant points are: $A = (-1, \sqrt{2}, -\sqrt{2})/\sqrt{12}$, $B = (1, -\sqrt{2}, -\sqrt{2})/\sqrt{12}$, $C = (-\sqrt{2}, -1, 1)/\sqrt{24}$, $D = (\sqrt{2}, 1, 1)/\sqrt{24}$, $A' = (-\sqrt{2}, 2, -1)/\sqrt{6}$, $C' = (-\sqrt{8}, -2, 1)/\sqrt{24}$, $Q = (\sqrt{2}, 1, 2)/\sqrt{96}$.

Fig.2 Two-dimensional plots for the domains in Fig.1. The triangles are projections of the tetrahedra on two orthogonal planes defined by the axis $T^{20} = Z$ and the two orthogonal edges BAA' (part a) and DCC' (part b). Four experimental points have been plotted; they correspond to the reactions $\pi^+ p \rightarrow \rho^0 \Delta^{++}$ (points 1,2) and $\pi^+ p \rightarrow \omega^0 \Delta^{++}$ (points 3,4) at 5 GeV/c (points 1,3) and 8 GeV/c (points 2,4), as analysed in refs [8],[9]. Point 2 satisfies scarcely the positivity of the density matrix (to fall inside the triangles ABC and ACD), and even the positivity of the angular distributions (to fall inside the triangles A'BC and AC'D). The four points fall along and not far off the line AQ, which is the class A prediction of the quark model.

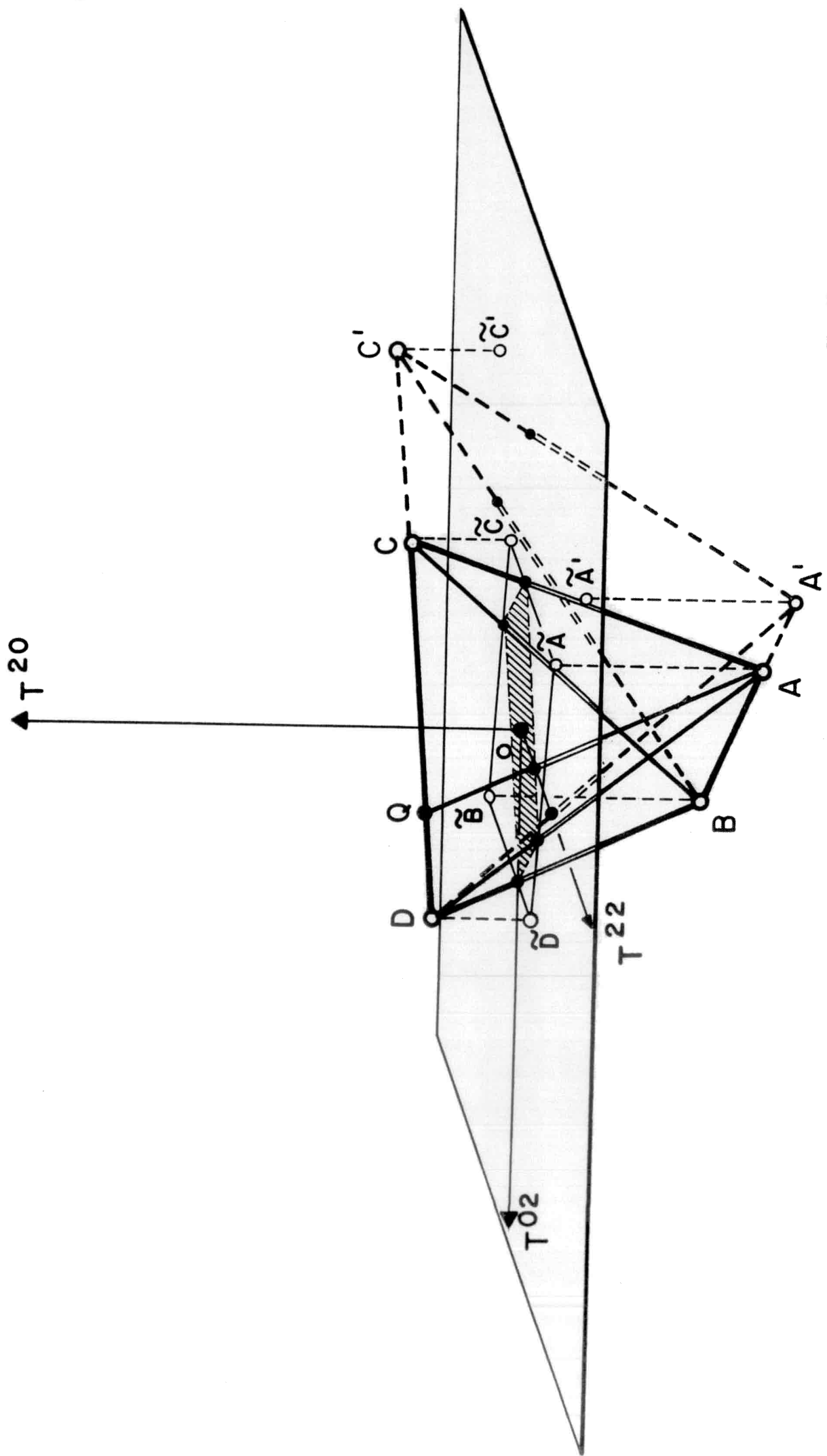


FIG. 1

