Table 9 — cont'd

b) The 48 real observables in transversity quantization

$$\sigma_0, \quad t^{20}_{00}, t^{01}_{00}, t^{21}_{00}, \left.\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\} t^{20}_{20}, t^{21}_{20}, t^{21}_{11}, t^{21}_{1-1}$$

$$P_R = {}^zt^{00}_{00}, {}^zt^{20}_{00}, {}^zt^{01}_{00}, {}^zt^{21}_{00}, \left.\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\} {}^zt^{20}_{20}, {}^zt^{21}_{20}, {}^zt^{21}_{11}, {}^zt^{21}_{1-1}$$

$$\left.\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\} {}^xt^{20}_{10}, {}^xt^{01}_{01}, {}^xt^{21}_{10}, {}^xt^{21}_{01}, {}^xt^{21}_{01}, {}^xt^{21}_{2-1}$$

$$\left.\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\} {}^yt^{20}_{10}, {}^yt^{01}_{01}, {}^yt^{21}_{10}, {}^yt^{21}_{01}, {}^yt^{21}_{21}, {}^yt^{21}_{2-1}$$

($M + M' =$ even for $t^{LL'}_{MM'}$, ${}^zt^{LL'}_{MM'}$ and odd for ${}^xt^{LL'}_{MM'}$, ${}^yt^{LL'}_{MM'}$, by B-symmetry; $t^{LL'}_{-M-M'} = (-)^{M+M'}\, t^{LL'}_{MM'}$)

c) The 31 generalized spin rotation parameters and the 16 linear constraints

$$P_0 = \frac{1}{6}\left[1 - \sqrt{10}\, t^{20}_{00} + \sqrt{3}\, t^{01}_{00} - \sqrt{30}\, t^{21}_{00}\right] = \frac{1}{6}\left[P_R - \sqrt{10}\, {}^zt^{20}_{00} + \sqrt{3}\, {}^zt^{01}_{00} - \sqrt{30}\, {}^zt^{21}_{00}\right] = P_0$$

$$P_0' = \frac{1}{6}\left[1 - \sqrt{10}\, t^{20}_{00} - \sqrt{3}\, t^{01}_{00} + \sqrt{30}\, t^{21}_{00}\right] = \frac{-1}{6}\left[P_R - \sqrt{10}\, {}^zt^{20}_{00} - \sqrt{3}\, {}^zt^{01}_{00} + \sqrt{30}\, {}^zt^{21}_{00}\right] = P_0'$$

$$P = \frac{1}{6}\left[1 + \sqrt{\frac{5}{2}}\, t^{20}_{00} - \sqrt{3}\, t^{01}_{00} - \sqrt{\frac{15}{2}}\, t^{21}_{00}\right] = \frac{1}{6}\left[P_R + \sqrt{\frac{5}{2}}\, {}^zt^{20}_{00} - \sqrt{3}\, {}^zt^{01}_{00} - \sqrt{\frac{5}{2}}\, {}^zt^{21}_{00}\right] = P$$

$$P' = \frac{1}{6}\left[1 + \sqrt{\frac{5}{2}}\, t^{20}_{00} + \sqrt{3}\, t^{01}_{00} + \sqrt{\frac{15}{2}}\, t^{21}_{00}\right] = \frac{-1}{6}\left[P_R + \sqrt{\frac{5}{2}}\, {}^zt^{20}_{00} + \sqrt{3}\, {}^zt^{01}_{00} + \sqrt{\frac{5}{2}}\, {}^zt^{21}_{00}\right] = P'$$

$$P_0 + P_0' + 2(P + P') = 1 \qquad\qquad P_R = P_0 - P_0' + 2(P - P')$$

$$Q = \frac{1}{6}\left[\sqrt{15}\, \overline{t^{20}_{20}} - \sqrt{45}\, \overline{t^{21}_{20}}\right] = \frac{1}{6}\left[\sqrt{15}\, \overline{{}^zt^{20}_{20}} - \sqrt{45}\, \overline{{}^zt^{21}_{20}}\right] = Q$$

$$Q' = \frac{1}{6}\left[\sqrt{15}\, \overline{t^{20}_{20}} + \sqrt{45}\, \overline{t^{21}_{20}}\right] = \frac{-1}{6}\left[\sqrt{15}\, \overline{{}^zt^{20}_{20}} + \sqrt{45}\, \overline{{}^zt^{21}_{20}}\right] = Q'$$

$$S_1 = \frac{1}{6}\left[-\sqrt{45}\, \overline{t^{21}_{1-1}}\right]; \qquad \frac{1}{6}\left[-\sqrt{45}\, \overline{{}^zt^{21}_{1-1}}\right] = T_1$$

$$S_2 = \frac{1}{6}\left[\sqrt{45}\, \overline{t^{21}_{11}}\right]; \qquad \frac{-1}{6}\left[\sqrt{45}\, \overline{{}^zt^{21}_{11}}\right] = T_2$$

$$R_0 = \frac{i}{6}\left[-\sqrt{6}\, \overline{{}^yt^{01}_{01}} + \sqrt{60}\, \overline{{}^yt^{21}_{01}}\right] = \frac{1}{6}\left[-\sqrt{6}\, \overline{{}^xt^{01}_{01}} + \sqrt{60}\, \overline{{}^xt^{21}_{01}}\right] = R_0$$

Table 9 — cont'd

$$R_1 = \frac{i}{6}\left[\sqrt{90}\ \overline{{}^{y}t^{21}_{2-1}}\right] = \frac{1}{6}\left[\sqrt{90}\ \overline{{}^{x}t^{21}_{21}}\right] = R_1$$

$$R_2 = \frac{-i}{6}\left[-\sqrt{90}\ \overline{{}^{y}t^{21}_{21}}\right] = \frac{1}{6}\left[-\sqrt{90}\ \overline{{}^{x}t^{21}_{21}}\right] = R_2$$

$$R_3 = \frac{-i}{6}\left[-\sqrt{6}\ \overline{{}^{y}t^{01}_{01}} - \sqrt{15}\ \overline{{}^{y}t^{21}_{01}}\right] = \frac{1}{6}\left[-\sqrt{6}\ \overline{{}^{x}t^{01}_{01}} - \sqrt{15}\ \overline{{}^{x}t^{21}_{01}}\right] = R_3$$

$$A_1 = \frac{i}{6}\left[-\sqrt{\frac{15}{2}}\ \overline{{}^{y}t^{20}_{10}} + \sqrt{\frac{45}{2}}\ \overline{{}^{y}t^{21}_{10}}\right]; \qquad \frac{1}{6}\left[-\sqrt{\frac{15}{2}}\ \overline{{}^{x}t^{20}_{10}} + \sqrt{\frac{45}{2}}\ \overline{{}^{x}t^{21}_{10}}\right] = U_1$$

$$A_2 = \frac{-i}{6}\left[-\sqrt{\frac{15}{2}}\ \overline{{}^{y}t^{20}_{10}} - \sqrt{\frac{45}{2}}\ \overline{{}^{y}t^{21}_{10}}\right]; \qquad \frac{1}{6}\left[-\sqrt{\frac{15}{2}}\ \overline{{}^{x}t^{20}_{10}} - \sqrt{\frac{45}{2}}\ \overline{{}^{x}t^{21}_{10}}\right] = U_2$$

d) Transfer of Polarization

$\sigma\varrho^{00}_{++} = \sigma_0 P_0(1+z)$	$\sigma\varrho^{00}_{--} = \sigma_0 P_0'(1-z)$
$\sigma\varrho^{00e}_{--} = \sigma_0 P(1+z)$	$\sigma\varrho^{11e}_{++} = \sigma_0 P'(1-z)$
$\sigma\varrho^{1-1}_{--} = \sigma_0 Q(1+z)$	$\sigma\varrho^{1-1}_{++} = \sigma_0 Q'(1-z)$
$\sigma\varrho^{10e}_{-+} = \sigma_0(S_1 + T_1 z)$	$\sigma\varrho^{10e}_{+-} = \sigma_0(S_2 - T_2 z)$
$\sigma\varrho^{00}_{+-} = \sigma_0 R_0(x - iy)$	$\sigma\varrho^{11e}_{+-} = \sigma_0 R_3(x + iy)$
$\sigma\varrho^{1-1}_{-+} = \sigma_0 R_1(x - iy)$	$\sigma\varrho^{1-1}_{+-} = \sigma_0 R_2(x + iy)$
$\sigma\varrho^{10e}_{--} = \sigma_0(U_1 x - iA_1 y)$	$\sigma\varrho^{10e}_{++} = \sigma_0(U_2 x + iA_2 y)$

e) Transversity and helicity amplitudes as in Table 8e)

f) Expression of the observables in c) and d) as functions of the transversity amplitudes in Table 8e)

$2\sigma_0 P_0 = \lvert a\rvert^2$	$2\sigma_0 P_0' = \lvert a'\rvert^2$
$2\sigma_0 P = \frac{1}{2}(\lvert b\rvert^2 + \lvert c\rvert^2)$	$2\sigma_0 P' = \frac{1}{2}(\lvert b'\rvert^2 + \lvert c'\rvert^2$
$2\sigma_0 Q = b\bar{c}$	$2\sigma_0 Q' = c'\bar{b}'$
$2\sigma_0 S_1 = \frac{1}{2}(b\bar{a} - a'\bar{b}')$	$2\sigma_0 S_2 = \frac{1}{2}(c'\bar{a}' - a\bar{c})$
$2\sigma_0 T_1 = \frac{1}{2}(b\bar{a} + a'\bar{b}')$	$2\sigma_0 T_2 = \frac{1}{2}(c'\bar{a}' + a\bar{c})$
$2\sigma_0 R_0 = a\bar{a}'$	$2\sigma_0 R_3 = \frac{1}{2}(c'\bar{b} + b'\bar{c})$

Table 9 — cont'd

$2\sigma_0 R_1 = b\bar{b}'$	$2\sigma_0 R_2 = c'\bar{c}$
$2\sigma_0 U_1 = \frac{1}{2}(b\bar{a}' - a'\bar{c})$	$2\sigma_0 U_2 = \frac{1}{2}(c'\bar{a} - a\bar{b}')$
$2\sigma_0 A_1 = \frac{1}{2}(b\bar{a}' + a'\bar{c})$	$2\sigma_0 A_2 = \frac{1}{2}(c'\bar{a} + a\bar{b}')$

g) Complement to the algebraic reconstruction of amplitudes in Table 8g), for transversally polarized target

$$|b|^2 = 2\sigma_0|S_1 + T_1|^2/P_0 \qquad |b'|^2 = 2\sigma_0|S_1 - T_1|^2/P_0'$$

$$|c|^2 = 2\sigma_0|S_2 - T_2|^2/P_0 \qquad |c'|^2 = 2\sigma_0|S_2 + T_2|^2/P_0'$$

$$\phi_b - \phi_a = \mathrm{Arg}\,(S_1 + T_1) \qquad \phi_{c'} - \phi_{a'} = \mathrm{Arg}\,(S_2 + T_2)$$

Table 10

Amplitude reconstruction for reactions of type $\pi p \to \rho N$ ($0^-\ 1/2^+ \to 1^{-\varepsilon}\ 1/2^{+\varepsilon}$) with polarized target

a) Combined production and decay angular distribution and measurement of the polarization transfer by the method of moments.

$$I(\psi, \theta, \phi) = \frac{1}{2\pi} \sum_L C(L) \sum_M \left[\overline{t^L_M} + P_T \cos\psi\ {}^z\overline{t^L_M} + P_T \sin\psi\ {}^x\overline{t^L_M} + P_L \overline{{}^y t^L_M}\right] Y^L_M(\theta, \phi)$$

$$C(L)\,(t^L_M + P_L\,{}^y t^L_M) = \langle Y^L_M(\theta, \phi)\rangle$$

$$C(L)\,P_T\,{}^z t^L_M = \langle 2\cos\psi\, Y^L_M(\theta, \phi)\rangle$$

$$C(L)\,P_T\,{}^x t^L_M = \langle 2\sin\psi\ Y^L_M(\theta, \phi)\rangle$$

with

$$C(0) = 1/\sqrt{4\pi}, \qquad C(2) = -\sqrt{2\pi} \quad \text{(all other } C \text{ coefficients vanish)}$$

b) The 12 real observables in transversity quantization

$$\sigma_0, \qquad t_0{}^2, \qquad \mathrm{Re}\, t_2{}^2, \qquad \mathrm{Im}\, t_2{}^2$$

$$P_R = {}^z t_0{}^0, \quad {}^z t_0{}^2, \quad \mathrm{Re}\,{}^z t_2{}^2, \quad \mathrm{Im}\,{}^z t_2{}^2, \quad \mathrm{Re}\,{}^x t_1{}^2, \quad \mathrm{Im}\,{}^x t_1{}^2, \quad \mathrm{Re}\,{}^y t_1{}^2, \quad \mathrm{Im}\,{}^y t_1{}^2$$

(M = even for t^L_M and ${}^z t^L_M$ M = odd for ${}^x t^L_M$ and ${}^y t^L_M$, by B-symmetry; $t^L_{-M} = (-)^M\, t^L_M$).

Table 10 — cont'd

c) Generalized spin rotation parameters

$$P_0 = \frac{1}{6}\left[1 + P_R - \sqrt{10}\,(t_0^2 + {}^z t_0^2)\right]$$

$$P_0' = \frac{1}{6}\left[1 - P_R - \sqrt{10}\,(t_0^2 - {}^z t_0^2)\right]$$

$$P = \frac{1}{6}\left[1 + P_R + \sqrt{\frac{5}{2}}\,(t_0^2 + {}^z t_0^2)\right]$$

$$P' = \frac{1}{6}\left[1 - P_R + \sqrt{\frac{5}{2}}\,(t_0^2 - {}^z t_0^2)\right]$$

$$Q = \frac{1}{6}\sqrt{15}\left(\overline{t_2^2} + \overline{{}^z t_2^2}\right)$$

$$Q' = \frac{1}{6}\sqrt{15}\left(\overline{t_2^2} - \overline{{}^z t_2^2}\right)$$

$$U = \frac{-1}{6}\sqrt{30}\,\overline{{}^x t_1^2}$$

$$A = \frac{-i}{6}\sqrt{30}\,\overline{{}^y t_1^2}$$

d) Transfer of polarization

$$\sigma\varrho_{00} = \sigma_0[(P_0 + P_0') + (P_0 - P_0')\,z]$$

$$\sigma\varrho^e_{11} = \sigma_0[(P + P') + (P - P')\,z]$$

$$\sigma\varrho_{1-1} = \sigma_0[(Q + Q') + (Q - Q')\,z]$$

$$\sigma\varrho^e_{10} = \sigma_0[Ux - iAy]$$

e) Transversity and helicity amplitudes, as in Table 8e)

f) Expression of the observables in c) and d) as functions of the transversity amplitudes

$$2\sigma_0 P_0 = |a|^2 \qquad 2\sigma_0 P_0' = |a'|^2$$

$$2\sigma_0 P = \frac{1}{2}(|b|^2 + |c|^2) \qquad 2\sigma_0 P' = \frac{1}{2}(|b'|^2 + |c'|^2)$$

$$2\sigma_0 Q = b\bar{c} \qquad 2\sigma_0 Q' = c'\bar{b}'$$

$$2\sigma_0 U = \frac{1}{2}(b\bar{a}' - a\bar{b}' + c'\bar{a} - a'\bar{c})$$

$$2\sigma_0 A = \frac{1}{2}(b\bar{a}' - a\bar{b}' - c'\bar{a} + a'\bar{c})$$

4.1. Reactions of type $\pi p \to K\Lambda$

4.1.1. The Wolfenstein parameters

It is well known that reactions of the type $\pi p \to K\Lambda$ with polarized target and analysis of the final Λ polarization are completely described, at fixed energy and momentum transfer, by σ_0, the unpolarized differential cross section, and P, R, A, the 3 Wolfenstein parameters (cf. ref. [*21*]) which satisfy one quadratic constraint (cf. table 4). Indeed these 4 real numbers supply the whole phenomenological information, namely the differential cross section σ and the final polarization components (X, Y, Z) as functions of the initial ones (x, y, z). In the right part of table 4) we show these functions when the polarizations are quantized in s-transversity frames (cf. section 2.1.). The simple inspection of these functions shows that each one of the 4 parameters can be measured twice. That both experimental procedures must supply the same result constitutes a Wolfenstein theorem which will be proven below.

4.2.1. The complex spin rotation parameters

A complete measurement of the reaction $\pi p \to K\Lambda$ involves a measure of the differential cross section and an analysis of the combined angular distribution of the normal to the reaction plane and of the Λ decay products, as it was discussed in section 3.3. This angular distribution is given in table 4a) and its moment analysis yields the polarization transfer parameters t_M^L, ${}^zt_M^L$, ${}^xt_M^L$, ${}^yt_M^L$ as shown in the same table. There are 8 real, a priori non vanishing, parameters, given in table 4b) (in transversity quantization). They are not independent; they satisfy some linear and quadratic constraints. The method to derive systematically these constraints was given in section 3.3.2. In the present case we first write the B-symmetric matrices ϱ_0 and ϱ_z, and the B-antisymmetric ones ϱ_x and ϱ_y, which appear in the matrix W (eq. 3.9), in the form

$$\varrho_0 = \begin{array}{|c|c|}\hline P_0 & \\ \hline & P_0' \\ \hline\end{array}\,, \quad \varrho_z = \begin{array}{|c|c|}\hline {}^zP_0 & \\ \hline & {}^zP_0' \\ \hline\end{array}\,, \quad \varrho_x = \begin{array}{|c|c|}\hline & {}^xR_0 \\ \hline \overline{{}^xR_0} & \\ \hline\end{array}\,, \quad \varrho_y = \begin{array}{|c|c|}\hline & -i\,{}^yR_0 \\ \hline \overline{i\,{}^yR_0} & \\ \hline\end{array}$$

Then the polarization transfer matrix W reads

$$W = \sigma_0 \quad \begin{array}{cc|c|c|c|c|c} \lambda_p & \lambda_\Lambda & +1/2 & +1/2 & -1/2 & -1/2 & \lambda_p' \\ & & +1/2 & -1/2 & +1/2 & -1/2 & \lambda_\Lambda' \\ \hline +1/2 & +1/2 & P_0 + {}^zP_0 & & & {}^xR_0 + {}^yR_0 & \\ \hline +1/2 & +1/2 & & P_0' - {}^zP_0' & \overline{{}^xR_0} - \overline{{}^yR_0} & & \\ \hline -1/2 & +1/2 & & {}^xR_0 - {}^yR_0 & P_0 - {}^zP_0 & & \\ \hline -1/2 & -1/2 & \overline{{}^xR_0} + \overline{{}^yR_0} & & & P_0' + {}^zP_0' & \\ \hline \end{array}$$

It is the direct sum of the external matrix W_1 (with $\lambda_p - \lambda_\Lambda =$ even) and the internal one W_2 (with $\lambda_p - \lambda_\Lambda =$ odd). The rank of W must be **1** since the other two particles are spinless. This condition imposes either

$$W_2 = 0,\ \text{rank}\ W_1 = 1 \leftrightarrow P_0 = {}^zP_0,\quad P_0' = {}^zP_0',\quad {}^xR_0 = {}^yR_0,\quad |R_0|^2 = P_0P_0',$$

or

$$W_1 = 0,\ \text{rank}\ W_2 = 1 \leftrightarrow P_0 = -{}^zP_0,\quad P_0' = -{}^zP_0',\quad {}^xR_0 = -{}^yR_0,\quad |R_0|^2 = P_0P_0'.$$

In each case there are 4 linear constraints and one quadratic constraint. For our reaction, with relative parity $\eta = \eta_\pi\eta_p\eta_K\eta_\Lambda = +1$, the first alternative is realized, and the corresponding linear constraints are equivalent to the Wolfenstein theorem. The parameters P_0 and P_0' are real, the parameter $R_0 = {}^xR_0 = {}^yR_0$ is complex. They are also called "the spin rotation parameters" of the reaction. The definition of complex spin rotation parameters will be useful in the generalizations below. The relation of P_0, P_0', R_0 with the Wolfenstein's A, P, R is given in table 4c, where we also show their relations with the polarization transfer multipole parameters and the linear constraints between the latter. Finally, table 4d gives the quadratic constraint in terms of A, P, R and P_0, P_0', R_0 and table 4e shows the polarization transfer in terms of the density matrix elements in transversity quantization which are closer to the amplitudes we intend to reconstruct.

4.1.3. Reconstruction of amplitudes

Table 4f introduces the terminology for the helicity and transversity amplitudes (c.f. section 2.2.). Their relations with the spin rotation parameters are given in table 4g. Note that with unpolarized target, i.e. by measuring only σ_0 and P (or σ_0 and P_0) the moduli a and a' of the transversity amplitudes is determined and only their relative phase in ghost.
On the contrary, for the helicity amplitudes, the moduli are not determined with an unpolarized target.

4.2. Reactions of type $\pi p \to K\Sigma^*$ ($0^-\,1/2^+ \to 0^-\,3/2^+$)

In table 2a, 30 examples of such reactions are listed. For any of them, with unpolarized target but with analysis of the cascade decay of the final baryon, the transversity amplitudes can be measured, up to one ghost phase, by the procedure indicated in table 5. The determination of the ghost phase needs a polarized target and can be performed following the procedure described in table 6.

4.2.1. Reactions with unpolarized target

This section is a comment of table 5. Part a) gives the method for measuring the even multipole parameters by a moment analysis of the two body decay of the Σ^* (cf. sect. 2.4.1.). Part b) gives the method for measuring all the multipole parameters by a moment analysis of the cascade decay: $\Sigma^* \to \Lambda\pi$, $\Lambda \to p\pi$ (cf. section 2.4.3.). In fact, for L = even one has $L_1 = 0$ and the θ_1, ϕ_1 or ($\theta_1{}^h$, $\phi_1{}^h$) disappear; then by integrating on θ_1, ϕ_1 (or $\theta_1{}^h$, $\phi_1{}^h$), the same distribution as in Part a) is recovered, with $C(L, L_1 = 0, J = L) = C^h(L, L_1 = 0, M = 0) = C(L)/\sqrt{4\pi}$. For each quantization frame for the Λ, the parameters $t_M{}^1$ can be measured twice: with $L_1 = 1$, $J = 0{,}2$ for the canonical frame and with $L_1 = 1$, $M_1 = 0{,}1$ for the helicity frame. The compatibility of these two groups of measurements is a check of the experimental and theoretical assumptions (no biases, spin 3/2 for the Σ^* ...) and can be used to reduce the experimental errors.

The a priori non vanishing multipole parameters are listed in part c). Of course all other multipole moments of the cascade angular distribution can be measured too. Their vanishing is a check of parity conservation in the production and in the $\Sigma^* \to \Lambda\pi$ decay. From these values of the multipole parameters, the density matrix elements are easily obtained (cf. eq. (2.8)). In part d) we give explicitly the non-vanishing density matrix elements in transversity quantization. Remark that since the matrix elements are linear combinations of the multipole parameters, they could be obtained directly by the method of moments as mean values, of complicated linear expressions of spherical harmonics. This method can reduce the errors on density matrix elements, and should be applied when amplitude reconstruction is intended. Nevertheless one should perform the checks mentioned above.
The positivity and rank 2 conditions of the 4×4 density matrix (cf. section 2.3.) impose to its elements the constraints written in part e). Part f) introduces some simple terminology for the transversity and helicity amplitudes which satisfy the B-symmetry conditions (cf. eq. (2.6)). We give also the relation between these amplitudes when the conventions of section 2.1. are used. Finally part g) shows the very simple connection of the transversity amplitudes with the measurable transversity density matrix elements. Remark that the argument of Q and Q' give the relative phases between the amplitudes a and b and between a' and b'. But the relative phase between these two groups of amplitudes is ghost. Therefore the moduli of the helicity amplitudes are also ghosts.

4.2.2. Reactions with polarized target

The determination of the ghost phase requires an experiment with a polarized target. Table 6 shows the method for measuring all the observables of the reaction and gives the corresponding generalized spin rotation parameters (for comparison, see section 4.1.). Part a) shows the combined production and cascade decay angular distribution for an arbitrary target polarization (cf. section 3.3.4.). Its moment analysis yields the polarization transfer multipole parameters. In part b) we list those which are not a priori vanishing. Of course the other moments of the combined distribution could be measured and should be found compatible with zero.
Part c) shows the linear constraints on the observed transfer multipole parameters (cf. section 3.2.2.) and introduces the linearly independent generalized spin rotation parameters. The real P's and the complex Q's can be measured with an unpolarized target by the left side equations. The complex R's can be measured with a longitudinally polarized target by the left side equations. Besides, all parameters can be measured with a transversally polarized target by the right side equations. Therefore, the experiments with only transverse or only longitudinal target polarization are equivalent: both supply the whole physical information. But the first experiment allows the check of 8 linear constraints; the other 8 constraints can be checked only when both kinds of experiments are performed.
The generalized spin rotation parameters must still satisfy some quadratic constraints and some positivity conditions (cf. section 3.2.2.) given explicitly in part d). These parameters are the coefficients of the polarization transfer from the target polarization to the final particle density matrix as shown in part e).
At last part g) shows the observables in terms of the transversity amplitudes defined in table 5f). The ghost phase between the amplitudes a, b and the amplitudes a', b' is contained in the arguments of the parameters R, R', R_1 and R_2. As emphasized above, it can be measured either with transverse or longitudinal target polarization.

4.3. Reactions of type $\pi p \to \pi\Delta$ ($0^- \, 1/2^+ \to 0^- \, 3/2^{+e}$)

In table 3a, 34 examples of this type of reactions have been listed. Their amplitudes can be completely reconstructed with a transversally polarized target. Table 7 that we now comment gives the recipes for the reconstruction by measuring some generalized spin rotation parameters (cf. section 4.1.).

Part a) shows the combined angular distribution of the normal to the reaction plane and of the Δ decay products (cf. section 3.3.2.) It allows the measurement of the polarization transfer multipole parameters by a moment analysis as indicated in the same part a). The list of those parameters which are not a priori vanishing is given in part b). Remark that ${}^{y}t_1{}^2$ can be measured only with a longitudinally polarized target. Of course all other moments of the combined angular distribution could be measured and should be found compatible with zero, as a check of parity conservation in the reaction and in the Δ decay.

Part c) introduces the generalized spin rotation parameters which are linear combinations of the transfer multipole parameters and as them could be directly measured by the moment method. The last line of this part c) shows that a longitudinally polarized target provides no new information, only a complex linear constraint can be checked. This part c) uses the same terminology as part c) of table 6 (reaction type $\pi p \to K\Sigma^*$). But for the Δ we can only measure the even multipole parameters, i.e., expressions of the type $P_1 + P_1'$ given by the left side equations and $P_1 - P_1'$ given by the right side equations; R_- is simply $R - R'$ and can be obtained from the left side and the right side equations, whence the linear constraint.

The spin rotation parameters must satisfy the non linear rank constraints and the positivity conditions written in part d).

Part e) shows the polarization transfer from the initial polarization to the final density matrix.

Part f) introduces some simple terminology for the transversity and helicity amplitudes which must be B-symmetric (cf. eq. (2.6)), and gives their linear relations for the conventions of section 2.1.

Finally part g) exhibits the relations between observables and amplitudes. All moduli and relative phases of the transversity amplitudes are easily obtained, even the relative phase between the amplitudes a, b and the amplitudes a', b' can be directly obtained for instance from the expression in brackets.

4.4. Reactions of type $\pi p \to K^*\Lambda$ ($0^- \, 1/2^+ \to 1^{-e} \, 1/2^+$)

Forty examples of such reactions are listed in table 2b. For each of them, with unpolarized target but with analysis of the joint angular distribution of the final decay, the transversity amplitudes can be reconstructed, up to one ghost phase, following the procedure described in table 8. The determination of the ghost phase needs a polarized target and can be performed following the procedure described in table 9.

4.4.1. Experiment with unpolarized target

This section is a comment of table 8. Part a) gives the method for measuring the double multipole parameters by a moment analysis of the joint two body decays of K^* and Λ (cf. sect. 2.4.2.). Unprimed indices and arguments correspond to K^* polarization and decay while the primed ones correspond to those of Λ. The a priori non vanishing multipole parameters are listed in part b). Of course all other multipole moments of the joint

angular distribution can be measured too. Their vanishing is a check of parity conservation in the production and in the K* decay.
From these values of the double multipole parameters, the joint density matrix is easily obtained (cf. eq. (2.9.)). In part c) we give explicitly the non vanishing elements of the measurable joint density matrix, in transversity quantization. Upper indices refer to the K* transversities and the index e labels elements of the even density matrix; lower indices are twice the Λ transversities. Remark that since the density matrix elements are linear expressions of the multipole parameters, they could be obtained directly, by the method of moments, as mean values of similar linear expressions of spherical harmonics. This method reduces the errors on density matrix elements and should be applied when amplitude reconstruction is intended. Nevertheless one should first perform the parity checks mentioned above.
The positivity and rank 2 conditions of the total 6×6 density matrix (the measured part plus the ghost part) imposes to its measured elements the constraints written in part d). The two equalities are the rank constraints. They are rather cumbersome but they constitute a new check and furthermore they have a diacrital function. Indeed they contain the square root of a complex number Δ (function of 4th degree in density matrix elements); the constraints decide which of the two possible roots must be chosen, since they will be satisfied for one of the roots and not for the other. This choice eliminates any discrete ambiguity in the reconstruction of the transversity amplitudes. Of course the check of these rank constraints and the possibility of discriminating the two roots require accurate experimental results and hence high statistics.

Part e) introduces some simple terminology for the transversity and helicity amplitudes which satisfy the B-symmetry conditions (cf. eq. (2.6.)). We give also the relations between these amplitudes when the conventions of section 2.1. are used. Finally we give the relation between our transversity amplitudes and those introduced by Byers and Yang, who use a cartesian basis for the spin 1 particle and a transversity quantization axis which violates the Basel convention.

Part f) shows the expressions of the measured observables as functions of the defined transversity amplitudes, and part g) gives the inverse expressions which allow an algebric reconstruction of the amplitudes. Of course the relative phase between the two sets of amplitudes a, b, c and a', b', c' is ghost and therefore the moduli of the helicity amplitudes cannot be determined with unpolarized target. Remark also that the determination of $|b|$ and $|c|$ from P and Q, and similarly for the primed quantities, contains a discrete ambiguity indicated by the sign ε or ε'. These signs can nevertheless be fixed by the last inequalities of part g), when the choice of the complex square root of Δ can be done as discussed above.
Another method for amplitude reconstruction is to fit the expressions in part f), imposing for instance that a and a' be real. A mixed method would be to obtain directly by the method of moments the moduli $|a|^2$, $|b \pm c|^2$, $|a'|^2$, $|b' + c'|^2$ and to fit afterwards the relative phase between these sets of amplitudes by using the values of $\operatorname{Im} Q$, $\operatorname{Im} Q'$, S_1 and S_2.

4.4.2. Experiment with polarized target

The determination of the ghost phase requires a polarized target. Table 9 shows the method for measuring all the observables and introduces the corresponding generalized spin rotation parameters (for comparison see section 4.1.).

Part a) shows the combined production and joint decay angular distribution for an arbitrary target polarization (cf. section 3.3.2.). Its moment analysis yields the polarization transfer joint multipole parameters. In part b) we list those which are not a priori vanishing. They are 48 and are set according to the way they are measured: the first line (12) can be measured with unpolarized target, the two following lines (24) with transverse target polarization, the last line (12) with longitudinal one. Of course, all other (48) parameters can be measured and should be found compatible with zero.

Part c) shows that among those 48 transfer multipole parameters there exist 16 linear constraints (cf. section 3.2.2.) so that, besides the unpolarized differential cross section σ_0, 31 generalized spin rotation parameters can be defined. The real P's and the complex Q's and S's can be measured with unpolarized target. In addition to these, a transverse polarized target allows the measurement of 16 more parameters the T's, R's and U's, and a longitudinal polarized target yields the R's and A's, i.e. 12 more than with unpolarized target and 4 more than with transversal polarization.
These parameters are the coefficients of the polarization transfer to the final joint density matrix as shown in part d).

Part f) gives the expression of the observables in terms of the transversity amplitudes defined in table 8e). The ghost phase between the two sets of amplitudes a, b, c and a', b', c' is contained in the R's, U's and A's parameters and can be measured either by longitudinal or by transverse polarization of the target. In this last case, the measurement of the parameters T_1 and T_2 allows a more direct reconstruction of the transversity amplitudes as given in part g).

4.5. Reactions of type $\pi p \to \rho N$ ($0^- \, 1/2^+ \to 1^{-e} \, 1/2^{+e}$)

In Table 3b) 22 examples of this type of reactions are listed. Their transversity amplitudes can be reconstructed with a transversally polarized target up to one ghost phase and some discrete ambiguities. The experiment with longitudinally polarized target supplies two more observables which allow to check two non linear constraints and to eliminate the discrete ambiguities. But the ghost phase could only be obtained from the polarization of the final nucleon[9]).
Table 10, that we now comment, gives the practical recipes for the amplitude reconstruction, by measuring some generalized spin rotation parameters (cf. Section 4.1.).

Part a) shows the combined angular distribution of the normal to the reaction plane and the ρ decay products (cf. Section 3.3.2.). It allows the measurement of the polarization transfer multipole parameters by a moment analysis as indicated in the same part a). The list of these parameters which are not a priori vanishing is given in Part b). Remark that ${}^{y}t_1{}^2$ can be measured only with a longitudinally polarized target. Of course all other moments of the combined angular distribution could be measured and should be found compatible with zero, as a check of parity conservation in the reaction and in the ρ decay.

Part c) introduces the generalized spin rotation parameters, which are linear combinations of the transfer multipole parameters, and as them could be directly measured by the moment method. This Part c) uses the same terminology as Part c) of Table 9 (reaction type $\pi p \to K^*\Lambda$). But in the present case we can only measure the polarization of the first particle, i.e. the transfer double multipole parameters with $L' = M' = 0$. They are given by expressions of the type $P_0 + P_0'$ written in the left side equations and of the type $P_0 - P_0'$ written in the right side equations in Table 9c). The parameters U and A correspond to $U_1 + U_2$ and $A_1 - A_2$. Part d) of Table 10 shows the polarization transfer from the target polarization to the final density matrix.

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[9]) That is a simple application of the Simonius theorem (cf. ref. [9] and Appendix).

The transition amplitudes in this type of reactions are the same as in the reaction type $\pi p \to K^*\Lambda$. Thus we refer to the simple terminology introduced in Table 8e) for the transversity and helicity amplitudes which must be B-symmetric (cf. Eq. (2.6)), and for their linear relations, when the conventions of Section 2.1. are adopted.

Part f) exhibits the relations between the observable spin rotation parameters and the transversity amplitudes. Remark that amplitudes corresponding to opposite polarizations of the final nucleon, i.e. $(ab'c')$ and $(a'bc)$, are never mixed. Therefore the relative phase between those sets of amplitudes is ghost. The situation is equivalent to that of the reaction type $\pi p \to K^*\Lambda$ with unpolarized initial state. From the polarization point of view both reaction types are related by crossing of the baryons. Indeed Table 10f) is obtained from Table 8f) by means of the substitutions $b \leftrightarrow c'$, $c \leftrightarrow b'$, $P \leftrightarrow P'$, $Q \leftrightarrow Q'$, $S_1 \to 1/2(U - A)$, $S_2 \to 1/2(U + A)$. Therefore in our present case, when all the spin rotation parameters are observed (experiment with transverse and longitudinal target polarization), all the moduli and relative phases (up to ghost one) can be unambiguously reconstructed, and two non linear constraints can be checked. For this purpose the expressions in Table 8d) and 8g can be used with the substitutions mentioned above. When the experiment is only performed with transversally polarized target, the parameter A and the last expression in Table 10f) are ignored, and the diacritical constraints are not available. Then, from the Table 10f), the six moduli can be obtained up to two discrete ambiguities for the moduli of b, c and the moduli of b', c'. The two relative phases between these couples of amplitudes are unambiguously measurable. The two relative phases between a and b', c', and between a' and b, c can be determined from the expression of U, up to at most a 2^4-uple discrete ambiguity.

4.6. Reactions of type $\pi p \to K^{**}\Lambda(0^- 1/2^+ \to 2^{+e} 1/2^+)$

Forty examples of this type of reactions can be obtained from Table 2b). Their transversity amplitudes can be reconstructed (up to one ghost phase) with unpolarized target, but with analysis of the joint angular distribution of the final decays. Table 11 gives the practical recipes for this amplitude reconstruction. It is a simple extension of Table 8, which gives the amplitude reconstruction for reaction type $\pi p \to K^*\Lambda$ and has been commented in Section 4.4.1. We refer to these comments, which can be easily applied to Table 11, although we have omitted here the explicit expressions of the 12 non linear constraints and the algebraic expressions for reconstructing the amplitudes. They are very cumbersome and can be obtained from the equations in Table 11c) by elementary algebra. Anyway the simplest method to reconstruct the amplitudes will be a best fit of these expressions as was commented in Section 4.4.1.
The determination of the ghost phase requires a polarized target. The experiments with transverse target polarization and with longitudinal one supplies 72 and 30 new observables including the ghost phase and new constraints. We have not tabulated all these generalized spin rotation parameters. The corresponding Table would be an extension of Table 9. The measurement of only one final polarization is enough to fix the ghost phase (cf. ref. [9]). If only the polarization of K** is measured, Table 12 could be used.

4.7. Reactions of type $\pi p \to A_2 N$ $(0^- 1/2^+ \to 2^{+e} 1/2^{+e})$

Twenty two examples of this type of reactions can be obtained from Table 3b). Their transversity amplitudes can be reconstructed (up to one ghost phase) with polarized target and measurement of the A_2 polarization. Table 12 gives the practical recipes for this

Table 11

Amplitude reconstruction for reactions of type $\pi p \to K^{**}\Lambda(0^- \, 1/2^+ \to 2^{+\varepsilon} \, 1/2^+)$ with unpolarized target

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a) Joint angular distribution of the K^{**} and Λ decays, and measurement of the double multipole parameters by the method of moments

$$I(\theta,\phi;\theta',\phi') = \sum_{L,L'} C_{K^{**}}(L)\, C_\Lambda(L') \sum_{M_1M'} \overline{t^{LL'}_{MM'}}\, Y^L_M(\theta,\phi)\, Y^{L'}_{M'}(\theta',\phi')$$

$$C_{K^{**}}(L)\, C_\Lambda(L')\, t^{LL'}_{MM'} = \langle Y^L_M(\theta,\phi)\, Y^{L'}_{M'}(\theta',\phi')\rangle$$

with: $C_{K^{**}}(0) = C_\Lambda(0) = 1/\sqrt{4\pi}, \qquad C_\Lambda(1) = \alpha_\Lambda/\sqrt{4\pi}$

$C_{K^{**}}(2) = -\sqrt{5/14\pi}, \qquad C_{K^{**}}(4) = \sqrt{9/14\pi}$, for $2 \to 00$ decay

$C_{K^{**}}(2) = -\sqrt{5/56\pi}, \qquad C_{K^{**}}(4) = -\sqrt{2/7\pi}$, for $2^+ \to 1^-0^-$ decay

(all other C coefficients vanish)

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b) The 30 real observables in transversity quantization

$\sigma_0,\ t^{20}_{00},\ t^{40}_{00},\ t^{01}_{00},\ t^{21}_{00},\ t^{41}_{00},$

$$\left.\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}\left\{\begin{matrix} t^{20}_{20},\ t^{40}_{20},\ t^{40}_{40},\ t^{21}_{20},\ t^{41}_{20},\ t^{41}_{40},\\ t^{21}_{11},\ t^{21}_{1-1},\ t^{41}_{11},\ t^{41}_{1-1},\ t^{41}_{31},\ t^{41}_{3-1},\end{matrix}\right.$$

(L = even, $M + M'$ = even by B-symmetry, $t^{LL'}_{-M-M'} = (-1)^{M+M'}\, \overline{t^{LL'}_{MM'}}$

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c) Observable elements of this joint density matrix in transversity quantization

$$\left.\begin{matrix} P_0 = \varrho^{00}_{++} \\ P_0' = \varrho^{00}_{--}\end{matrix}\right\} = \frac{1}{10}\left[1 - \sqrt{\frac{50}{7}}\, t^{20}_{00} + \sqrt{\frac{162}{7}}\, t^{40}_{00} \pm \left(\sqrt{3}\, t^{01}_{00} - \sqrt{\frac{150}{7}}\, t^{21}_{00} + \sqrt{\frac{486}{7}}\, t^{41}_{00}\right)\right]$$

$$\left.\begin{matrix} P_1 = \varrho^{11e}_{--} \\ P_0' = \varrho^{11e}_{++}\end{matrix}\right\} = \frac{1}{10}\left[1 - \sqrt{\frac{25}{14}}\, t^{20}_{00} - \sqrt{\frac{72}{7}}\, t^{40}_{00} \mp \left(\sqrt{3}\, t^{01}_{00} - \sqrt{\frac{75}{14}}\, t^{21}_{00} - \sqrt{\frac{216}{7}}\, t^{41}_{00}\right)\right]$$

$$\left.\begin{matrix} P_2 = \varrho^{22e}_{++} \\ P_2' = \varrho^{22e}_{--}\end{matrix}\right\} = \frac{1}{10}\left[1 + \sqrt{\frac{50}{7}}\, t^{20}_{00} + \sqrt{\frac{9}{14}}\, t^{40}_{00} \pm \left(\sqrt{3}\, t^{01}_{00} + \sqrt{\frac{150}{7}}\, t^{21}_{00} + \sqrt{\frac{27}{14}}\, t^{41}_{00}\right)\right]$$

$$P_0 + P_0' + 2(P_1 + P_1' + P_2 + P_2') = 1$$

$$\left.\begin{matrix} Q_1 = \varrho^{1-1}_{--} \\ Q_1' = \varrho^{1-1}_{++}\end{matrix}\right\} = \frac{1}{10}\left[\sqrt{\frac{75}{7}}\, \overline{t^{20}_{20}} - \sqrt{\frac{180}{7}}\, \overline{t^{40}_{20}} \mp \left(\sqrt{\frac{225}{7}}\, \overline{t^{21}_{20}} - \sqrt{\frac{540}{7}}\, \overline{t^{41}_{20}}\right)\right]$$

$$\left.\begin{matrix} Q_3 = \varrho^{20e}_{++} \\ Q_3' = \varrho^{20e}_{--}\end{matrix}\right\} = \frac{1}{10}\left[\sqrt{\frac{50}{7}}\, \overline{t^{20}_{20}} + \sqrt{\frac{135}{14}}\, \overline{t^{40}_{20}} \pm \left(\sqrt{\frac{150}{7}}\, \overline{t^{21}_{20}} + \sqrt{\frac{405}{14}}\, \overline{t^{41}_{20}}\right)\right]$$

$$\left.\begin{matrix} Q_2 = \varrho^{2-2}_{++} \\ Q_2' = \varrho^{2-2}_{--}\end{matrix}\right\} = \frac{1}{10}\left[\sqrt{45}\, \overline{t^{40}_{40}} \pm \sqrt{135}\, \overline{t^{41}_{40}}\right]$$

Table 11 — cont'd

$$S_1 = \varrho^{10e}_{-+} = \frac{1}{10}\left[-\sqrt{75/7}\,\overline{t^{21}_{1-1}} + \sqrt{810/7}\,\overline{t^{41}_{1-1}}\right]$$

$$S_2 = \varrho^{21e}_{-+} = \frac{1}{10}\left[-\sqrt{450/7}\,\overline{t^{21}_{1-1}} - \sqrt{135/7}\,\overline{t^{41}_{1-1}}\right]$$

$$S_3 = \varrho^{10e}_{+-} = \frac{1}{10}\left[\sqrt{75/7}\,\overline{t^{21}_{11}} - \sqrt{810/7}\,\overline{t^{41}_{11}}\right]$$

$$S_4 = \varrho^{21e}_{+-} = \frac{1}{10}\left[\sqrt{450/7}\,\overline{t^{21}_{11}} + \sqrt{135/7}\,\overline{t^{41}_{11}}\right]$$

$$S_5 = \varrho^{2-1e}_{+-} = \frac{1}{10}\left[\sqrt{135}\,\overline{t^{41}_{31}}\right]$$

$$S_6 = \varrho^{2-1e}_{-+} = \frac{1}{10}\left[-\sqrt{135}\,\overline{t^{41}_{3-1}}\right]$$

d) Transversity and helicity amplitudes, and their relations

λ_Λ	$\lambda_{K^{**}}$	$\lambda_p = +\frac{1}{2}$	$\lambda_p = -\frac{1}{2}$			
$+\frac{1}{2}$	2		d		$-D'$	D
	1	c'			C	C'
	0		a		$-A'$	A
	-1	b'			B	B'
	-2		e		$-E'$	E
$-\frac{1}{2}$	2	e'			E	E'
	1		b		$-B'$	B
	0	a'			A	A'
	-1		c		$-C'$	C
	-2	d'			D	D'

(left block) $= T^{\lambda_{K^{**}}}_{2\lambda_\Lambda, 2\lambda_p}$; (right block) $= H^{\lambda_{K^{**}}}_{2\lambda_\Lambda, 2\lambda_p}$

$$4(A - iA') = -2a - \sqrt{6}\,(d + e), \qquad 4(A + iA') = -2a' - \sqrt{6}\,(d' + e')$$

$$4(B - iB') = -2(b + c) - 2(d - e), \qquad 4(B + iB') = -2(b' + c') - 2(d' - e')$$

$$4(C - iC') = -2(b + c) + 2(d - e), \qquad 4(C + iC') = -2(b' + c') + 2(d' - e')$$

$$4(D - iD') = -\sqrt{6}\,a - 2(b - c) + (d + e), \qquad 4(D + iD') = -\sqrt{6}\,a' - 2(b' - c') + (d' + e')$$

$$4(E - iE') = -\sqrt{6}\,a + 2(b - c) + (d + e), \qquad 4(E + iE') = -\sqrt{6}\,a' + 2(b' - c') + (d' + e')$$

d') Collinearity constraints

$$\left.\begin{aligned} &A = B = C = D = E = 0 \\ &C' = D' = E' = 0 \end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned} &a = -a',\ d = -d' = \sqrt{\frac{3}{2}}\,a + b \\ &b = -b' = c = -c',\ e = -e' = \sqrt{\frac{3}{2}}\,a - b \end{aligned}\right.$$

Table 11 — cont'd

e) Expression of the observables in c) and d) as functions of the transversity amplitudes

$2\sigma P_0 = \|a\|^2$	$2\sigma P_0' = \|a'\|^2$
$2\sigma P_1 = \frac{1}{2}(\|b\|^2 + \|c\|^2)$	$2\sigma P_1' = \frac{1}{2}(\|b'\|^2 + \|c'\|^2)$
$2\sigma P_2 = \frac{1}{2}(\|d\|^2 + \|e\|^2)$	$2\sigma P_2' = \frac{1}{2}(\|d'\|^2 + \|e'\|^2)$
$2\sigma Q_1 = b\bar{c}$	$2\sigma Q_1' = c'\bar{b}'$
$2\sigma Q_2 = d\bar{e}$	$2\sigma Q_2' = e'\bar{d}'$
$2\sigma Q_3 = \frac{1}{2}(d\bar{a} + a\bar{e})$	$2\sigma Q_3' = \frac{1}{2}(e'\bar{a}' + a'\bar{d}')$
$2\sigma S_1 = \frac{1}{2}(b\bar{a} - a'\bar{b}')$	$2\sigma S_2 = \frac{1}{2}(e'\bar{c}' - c\bar{e})$
$2\sigma S_3 = \frac{1}{2}(c'\bar{a}' - a\bar{c})$	$2\sigma S_4 = \frac{1}{2}(d\bar{b} - b'\bar{d}')$
$2\sigma S_5 = \frac{1}{2}(d\bar{c} - c'\bar{d}')$	$2\sigma S_6 = \frac{1}{2}(e'\bar{b}' - b\bar{e})$

Table 12

Amplitude reconstruction for reactions of type $\pi p \to A_2 N(0^- 1/2^+ \to 2^{+\epsilon} 1/2^{+\epsilon})$ with polarized target

a) Combined production and decay angular distribution and measurement of the polarization transfer by the method of moments

$$I(\psi, \theta, \phi) = \frac{1}{2\pi} \sum_L C(L) \sum_M \left[\overline{t^L_M} + P_T\left(\cos\psi \overline{{}^z t^L_M} + \sin\psi \overline{{}^x t^L_M}\right) + P_L \overline{{}^y t^L_M}\right] Y^L_M(\theta, \phi)$$

$$C(L)\,(t^L_M + P_L {}^y t^L_M) = \langle Y^L_M(\theta,\phi)\rangle$$

$$C(L)\, P_T {}^z t^L_M = \langle 2\cos\psi\, Y^L_M(\theta,\phi)\rangle$$

$$C(L)\, P_T {}^x t^L_M = \langle 2\sin\psi\, Y^L_M(\theta,\phi)\rangle$$

with

$C(2) = -\sqrt{5/14\pi}$, $C(4) = \sqrt{9/14\pi}$ for $2 \to 00$ decay

$C(2) = -\sqrt{5/56\pi}$, $C(4) = -\sqrt{2/7\pi}$ for $2^+ \to 1^- 0^-$ decay

$C(0) = 1/\sqrt{4\pi}$ all other C coefficients vanish)

Table 12 — cont'd

b) The 30 real observables in transversity quantization

$$\sigma_0,\ t_0^2,\ t_0^4,\ \left.\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}\ t_2^2,\ t_2^4,\ t_4^4$$

$$P_R = {}^zt_0^0,\ {}^zt_0^2,\ {}^zt_0^4,\ \left.\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}\ {}^zt_2^2\ {}^zt_2^4,\ {}^zt_4^4,\ {}^xt_1^2,\ {}^xt_1^4,\ {}^xt_3^4$$

$$\left.\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\}\ {}^yt_1^2,\ {}^yt_1^4,\ {}^yt_3^4$$

$\left(M = \text{even for } t_M^L \text{ and } {}^zt_M^L,\ M = \text{odd for } {}^xt_M^L \text{ and } {}^yt_M^L, \text{ by B-symmetry; } t_M^L = (-1)^M\, \overline{t_M^L}\right)$

c) The 29 generalized spin rotation parameters

$$\left.\begin{matrix}P_0\\ P_0'\end{matrix}\right\} = \frac{1}{10}\left[(1 \pm P_R) - \sqrt{\frac{50}{7}}\,(t_0^2 \pm {}^zt_0^2) + \sqrt{\frac{162}{7}}\,(t_0^4 \pm {}^zt_0^4)\right]$$

$$\left.\begin{matrix}P_1\\ P_1'\end{matrix}\right\} = \frac{1}{10}\left[(1 \pm P_R) - \sqrt{\frac{25}{14}}\,(t_0^2 \pm {}^zt_0^2) - \sqrt{\frac{72}{7}}\,(t_0^4 \pm {}^zt_0^4)\right]$$

$$\left.\begin{matrix}P_2\\ P_2'\end{matrix}\right\} = \frac{1}{10}\left[(1 \pm P_R) + \sqrt{\frac{50}{7}}\,(t_0^2 \pm {}^zt_0^2) + \sqrt{\frac{9}{14}}\,(t_0^4 \pm {}^zt_0^4)\right]$$

$$P_0 + P_0' + 2(P_1 + P_1' + P_2 + P_2') = 1$$

$$P_0 - P_0' + 2(P_1 - P_1' + P_2 - P_2') = P_R$$

$$\left.\begin{matrix}Q_1\\ Q_1'\end{matrix}\right\} = \frac{1}{10}\left[\sqrt{\frac{75}{7}}\left(\overline{t_2^2} \pm \overline{{}^zt_2^2}\right) - \sqrt{\frac{180}{7}}\left(\overline{t_2^4} \pm \overline{{}^zt_2^4}\right)\right]$$

$$\left.\begin{matrix}Q_3\\ Q_3\end{matrix}\right\} = \frac{1}{10}\left[\sqrt{\frac{50}{7}}\left(\overline{t_2^2} \pm \overline{{}^zt_2^2}\right) + \sqrt{\frac{135}{14}}\left(\overline{t_2^4} \pm \overline{{}^zt_2^4}\right)\right]$$

$$\left.\begin{matrix}Q_2\\ Q_2'\end{matrix}\right\} = \frac{1}{10}\sqrt{45}\left(\overline{t_4^4} \pm \overline{{}^zt_4^4}\right)$$

$$U_1 = \frac{1}{10}\left[-\sqrt{\frac{50}{7}}\,\overline{{}^xt_1^2} + \sqrt{\frac{540}{7}}\,\overline{{}^xt_1^4}\right]$$

$$U_2 = \frac{1}{10}\left[-\sqrt{\frac{300}{7}}\,\overline{{}^xt_1^2} - \sqrt{\frac{90}{7}}\,\overline{{}^xt_1^4}\right]$$

$$U_3 = \frac{1}{10}\left[-\sqrt{90}\,\overline{{}^xt_3^4}\right]$$

$$A_1 = \frac{i}{10}\left[-\sqrt{\frac{50}{7}}\,\overline{{}^yt_1^2} + \sqrt{\frac{540}{7}}\,\overline{{}^yt_1^4}\right]$$

$$A_2 = \frac{i}{10}\left[-\sqrt{\frac{300}{7}}\,\overline{{}^yt_1^2} - \sqrt{\frac{90}{7}}\,\overline{{}^yt_1^4}\right]$$

$$A_3 = \frac{i}{10}\left[-\sqrt{90}\,\overline{{}^yt_3^4}\right]$$

Table 12 — cont'd

d) Transfer of polarization

$$\sigma\varrho_{00} = \sigma_0[(P_0 + P_0') + (P_0 - P_0')\,z]$$

$$\sigma\varrho^e_{11} = \sigma_0[(P_1 + P_1') + (P_1 - P_1')\,z]$$

$$\sigma\varrho^e_{22} = \sigma_0[(P_2 + P_2') + (P_2 - P_2')\,z]$$

$$\sigma\varrho_{1-1} = \sigma_0[(Q_1 + Q_1') + (Q_1 - Q_1')\,z]$$

$$\sigma\varrho_{2-2} = \sigma_0[(Q_2 + Q_2') + (Q_2 - Q_2')\,z]$$

$$\sigma\varrho_{20} = \sigma_0[(Q_3 + Q_3') + (Q_3 - Q_3')\,z]$$

$$\sigma\varrho_{10} = \sigma_0[U_1 x - iA_1 y]$$

$$\sigma\varrho_{21} = \sigma_0[U_2 x - iA_2 y]$$

$$\sigma\varrho_{2-1} = \sigma_0[U_3 x - iA_3 y]$$

e) Transversity and helicity amplitudes as in Table 11 d)

f) Expression of the observables in c) and d) as functions of the transversity amplitudes

$$2\sigma_0 P_0 = |a|^2 \qquad 2\sigma_0 P_0' = |a'|^2$$

$$2\sigma_0 P_1 = \frac{1}{2}(|b|^2 + |c|^2) \qquad 2\sigma_0 P_1' = \frac{1}{2}(|b'|^2 + |c'|^2)$$

$$2\sigma_0 P_2 = \frac{1}{2}(|d|^2 + |e|^2) \qquad 2\sigma_0 P_2' = \frac{1}{2}(|d'|^2 + |e'|^2)$$

$$2\sigma_0 Q_1 = b\bar{c} \qquad 2\sigma_0 Q_1' = c'\bar{b}'$$

$$2\sigma_0 Q_2 = d\bar{e} \qquad 2\sigma_0 Q_2' = e'\bar{d}'$$

$$2\sigma_0 Q_3 = \frac{1}{2}(d\bar{a} + a\bar{e}) \qquad 2\sigma_0 Q_3' = \frac{1}{2}(a'\bar{d}' + e'\bar{a}')$$

$$2\sigma_0 U_1 = \frac{1}{2}(b\bar{a}' - a\bar{b}' + c'\bar{a} - a'\bar{c}) \qquad 2\sigma_0 A_1 = \frac{1}{2}(b\bar{a}' - a\bar{b}' - c'\bar{a} + a'\bar{c})$$

$$2\sigma_0 U_2 = \frac{1}{2}(d\bar{c}' - c\bar{d}' + e'\bar{b} - b'\bar{e}) \qquad 2\sigma_0 A_2 = \frac{1}{2}(d\bar{c}' - c\bar{d}' - e'\bar{b} + b'\bar{e})$$

$$2\sigma_0 U_3 = \frac{1}{2}(d\bar{b}' - b\bar{d}' + e'\bar{c} - c'\bar{e}) \qquad 2\sigma_0 A_3 = \frac{1}{2}(d\bar{b}' - b\bar{d}' - e'\bar{c} + e'\bar{e})$$

amplitude reconstruction. It is a simple extension of Table 10, which gives the amplitude reconstruction for reaction type $\pi p \to \rho N$ and has been commented in Section 4.5. We refer to these comments, which can be easily applied to Table 12, although we have omitted here the explicit expressions of the non linear constraints and the algebraic expressions for reconstructing the amplitudes. All this can be obtained from Table 12f) by elementary algebra. Anyway the simplest method to reconstruct the amplitudes will be a best fit of these expressions by fixing two arbitrary phases (e.g. a and a' real).
The ghost phase between the sets of amplitudes (a, b, e, b', c'), (a', d', e', b, c) could only be measured by analysis of the final nucleon polarization. The situation is equivalent to that of the reaction $\pi p \to K^{**}\Lambda$ with unpolarized initial state. Indeed Table 12f) can be obtained from Table 11e) by the substitutions $b \leftrightarrow c'$, $c \leftrightarrow b'$, $P_1 \leftrightarrow P_1'$, $Q_1 \leftrightarrow Q_1'$, $S_2 \pm S_1 \to U_1, A_1$, $S_5 \pm S_6 \to U_2, A_2$, $S_4 \pm S_3 \to U_3, A_3$.

4.8. Reaction of type $\pi p \to K^*\Sigma^*$ ($0^-\ 1/2^+ \to 1^{-e}\ 3/2^+$)

Thirty examples of this type of reactions can be obtained from Table 2 e). Their transversity amplitudes can be reconstructed (up to one ghost phase) with unpolarized target, but with analysis of the joint angular distribution of the K^* decay and the Σ^* cascade decay. Table 13 gives the practical recipes for this amplitude reconstruction. It is a simple extension of Table 8, which gives the amplitude reconstruction for reaction type $\pi p \to K^*\Lambda$ and has been commented in Section 4.4.1. We refer to these comments, which can be easily applied to Table 13, although we have omitted here the explicit expressions of the non linear constraints and the algebraic expressions for reconstructing the amplitudes. They can all be obtained from equations in Table 13 f) by elementary algebra. Anyway the simplest method to reconstruct the amplitudes will be a best fit of these expressions as was commented in Section 4.4.1.
The determination of the ghost phase requires a polarized target. The experiments with transverse target polarization and with longitudinal one supplies 122 and 48 new observables, including the ghost phase and new constraints. We have not tabulated all these generalized spin rotation parameters. The corresponding table would be an extension of Table 9.

Table 13

Amplitude reconstruction for reactions of type $\pi p \to K^*\Sigma^*(0^+\ 1/2^+ \to 1^{+e}\ 3/2^+)$
with unpolarized target

a) Joint angular distribution of the $K^*\Sigma^*$ decays, and measurement of the L and L' even multipole parameters by the method of moments

$$I(\theta, \phi; \theta', \phi') = \sum_{LL'} C_{K^*}(L)\, C_{\Sigma^*}(L') \sum_{M,M'} \overline{t^{LL'}_{MM'}}\, Y^L_M(\theta, \phi)\, Y^{L'}_{M'}(\theta', \phi')$$

$$C_{K^*}(L)\, C_{\Sigma^*}(L')\, t^{LL'}_{MM'} = \langle Y^L_M(\theta, \phi)\, Y^{L'}_{M'}(\theta', \phi')\rangle$$

with

$$C_{K^*}(0) = C_{\Sigma^*}(0) = 1/\sqrt{4\pi}, \qquad C_{K^*}(2) = -1/\sqrt{2\pi}, \qquad C_{\Sigma^*}(2) = -1/\sqrt{4\pi}$$

(all other C coefficients vanish)

Table 13 — cont'd

b) Joint angular distribution of the K* decay and Σ^* cascade decay and measurement of the L even double multipole parameters by the method of moments

b 1) Canonical frame for the Λ

$$I(\theta, \phi, \theta', \phi'; \theta_1, \phi_1) = \sum_{LL'JL_1} C_{K^*}(L)\, C(L'L_1J) \sum_{MM'N} \langle JL_1\, NM_1 \mid L'M'\rangle$$

$$\overline{t^{LL'}_{MM'}}\, Y^L_M(\theta, \phi)\, Y^J_M(\theta', \phi')\, Y^{L_1}_{M_1}(\theta_1, \phi_1)$$

$$C_{K^*}(L)\, C(L'L_1J)\, t^{LL'}_{MM'} = \sum_{M_1} \langle JL_1NM_1 \mid L'M'\rangle \langle Y^L_M(\theta, \phi)\, Y^J_M(\theta', \phi')\, Y^{L_1}_{M_1}(\theta_1, \phi_1)\rangle$$

b 2) Helicity frame for the Λ

$$I(\theta, \phi, \theta', \phi', \theta_1{}^h, \phi_1{}^h) = \sum_{LL'L_1M_1} C_{K^*}(L)\, C^h(L'L_1M_1) \sum_{MM'} \overline{t^{LL'}_{MM'}}\, Y^L_M(\theta, \phi)$$

$$\sqrt{\frac{2L'+1}{4\pi}}\, \overline{D^{L'}(\phi'\ \theta'\ 0)^{M'}_{M_1}}\, Y^{L_1}_{M_1}(\theta_1{}^h \phi_1{}^h)$$

$$C_{K^*}(L)\, C^h(L'L_1M_1)\, t^{LL'}_{MM'} = \sqrt{\frac{2L'+1}{4\pi}} \langle Y^L_M(\theta, \varphi)\, \overline{D^{L'}(\phi'\theta'0)^{M'}_{M_1}}\, Y^{L'}_{M'}(\theta_1{}^h, \phi_1{}^h)\rangle$$

with $C_{K^*}(L)$ as in a) and $C(L'L_1J)$ and $C^h(L'L_1M_1)$ given in Table 5 b3)

c) The 48 observables in transversity quantization

$\sigma, t^{20}_{00}, t^{01}_{00}, t^{02}_{00}, t^{03}_{00}, t^{21}_{00}, t^{22}_{00}, t^{23}_{00}$

$$\left.\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\right\} \begin{matrix} t^{20}_{20}, t^{02}_{02}, t^{03}_{02}, t^{21}_{20}, t^{22}_{20}, t^{23}_{20}, t^{23}_{02}, t^{22}_{22}, t^{23}_{22}, \\ t^{22}_{2-2}, t^{23}_{2-2}, t^{21}_{11}, t^{22}_{11}, t^{23}_{11}, t^{21}_{1-1}, t^{22}_{1-1}, t^{23}_{13}, t^{23}_{1-3}, \end{matrix}$$

(L = even, $M + M'$ = even by B-symmetry, $t^{LL'}_{-M-M'} = (-1)^{M+M'}\, t^{LL'}_{MM'}$)

d) Observable elements of the joint density matrix in transversity quantization

$$\left.\begin{matrix} P_0 = \varrho^{00}_{11} \\ P_0{}' = \varrho^{00}_{-1-1} \end{matrix}\right\} = \frac{1}{12}\left[1 - \sqrt{10}\, t^{20}_{00} - \sqrt{5}\, t^{02}_{00} + \sqrt{50}\, t^{22}_{00} \pm \left(\sqrt{\frac{3}{5}}\, t^{01}_{00} - \sqrt{\frac{63}{5}}\, t^{03}_{00} - \sqrt{6}\, t^{21}_{00} + \sqrt{126}\, t^{23}_{00}\right)\right]$$

$$\left.\begin{matrix} P_1 = \varrho^{11e}_{-1-1} \\ P_1{}' = \varrho^{11e}_{11} \end{matrix}\right\} = \frac{1}{12}\left[1 + \sqrt{\frac{5}{2}}\, t^{20}_{00} - \sqrt{5}\, t^{02}_{00} - \sqrt{\frac{25}{2}}\, t^{22}_{00} \mp \left(\sqrt{\frac{3}{5}}\, t^{01}_{00} - \sqrt{\frac{63}{5}}\, t^{03}_{00}\right.\right.$$

$$\left.\left. + \sqrt{\frac{3}{2}}\, t^{21}_{00} - \sqrt{\frac{63}{2}}\, t^{23}_{00}\right)\right]$$

$$\left.\begin{matrix} P_2 = \varrho^{11e}_{33} \\ P_2{}' = \varrho^{11e}_{-3-3} \end{matrix}\right\} = \frac{1}{12}\left[1 + \sqrt{\frac{5}{2}}\, t^{20}_{00} + \sqrt{5}\, t^{02}_{00} + \sqrt{\frac{25}{2}}\, t^{22}_{00} \pm \left(\sqrt{\frac{27}{5}}\, t^{01}_{00} + \sqrt{\frac{7}{5}}\, t^{03}_{00}\right.\right.$$

$$\left.\left. + \sqrt{\frac{27}{2}}\, t^{21}_{00} + \sqrt{\frac{7}{2}}\, t^{23}_{00}\right)\right]$$

Table 13 — cont'd

$$\left.\begin{array}{l} P_3 = \varrho^{00}_{-3-3} \\ P_3{}' = \varrho^{00}_{33} \end{array}\right\} = \frac{1}{12}\left[1 - \sqrt{10}\, t^{20}_{00} + \sqrt{5}\, t^{02}_{00} - \sqrt{50}\, t^{22}_{00} \mp \left(\sqrt{\frac{27}{5}}\, t^{01}_{00} + \sqrt{\frac{7}{5}}\, t^{03}_{00} - \sqrt{54}\, t^{21}_{00} - \sqrt{14}\, t^{23}_{00}\right)\right]$$

$$P_0 + P_0{}' + 2(P_1 + P_1{}' + P_2 + P_2{}') + P_3 + P_3{}' = 1$$

$$\left.\begin{array}{l} Q_1 = \varrho^{1-1}_{-1-1} \\ Q_1{}' = \varrho^{1-1}_{11} \end{array}\right\} = \frac{1}{12}\left[\sqrt{15}\, \overline{t^{20}_{20}} - \sqrt{75}\, \overline{t^{22}_{20}} \mp \left(3\overline{t^{21}_{20}} - \sqrt{189}\, \overline{t^{23}_{20}}\right)\right]$$

$$\left.\begin{array}{l} Q_2 = \varrho^{1-1}_{33} \\ Q_2{}' = \varrho^{1-1}_{-3-3} \end{array}\right\} = \frac{1}{12}\left[\sqrt{15}\, \overline{t^{20}_{20}} + \sqrt{75}\, \overline{t^{22}_{20}} \pm \left(9\overline{t^{21}_{20}} + \sqrt{21}\, \overline{t^{23}_{20}}\right)\right]$$

$$\left.\begin{array}{l} Q_3 = \varrho^{00}_{1-3} \\ Q_3{}' = \varrho^{00}_{3-1} \end{array}\right\} = \frac{1}{12}\left[\sqrt{10}\, \overline{t^{02}_{02}} - 10\, \overline{t^{22}_{02}} \mp \left(\sqrt{14}\, \overline{t^{03}_{02}} - \sqrt{140}\, \overline{t^{23}_{02}}\right)\right]$$

$$\left.\begin{array}{l} Q_4 = \varrho^{11\epsilon}_{3-1} \\ Q_4{}' = \varrho^{11\epsilon}_{1-3} \end{array}\right\} = \frac{1}{12}\left[\sqrt{10}\, \overline{t^{02}_{02}} + 5\, \overline{t^{22}_{02}} \pm \left(\sqrt{14}\, \overline{t^{03}_{02}} + \sqrt{35}\, \overline{t^{23}_{02}}\right)\right]$$

$$\left.\begin{array}{l} Q_5 = \varrho^{1-1}_{-13} \\ Q_5{}' = \varrho^{1-1}_{-31} \end{array}\right\} = \frac{1}{12}\left[\sqrt{150}\, \overline{t^{22}_{2-2}} \pm \sqrt{210}\, \overline{t^{23}_{2-2}}\right]$$

$$\left.\begin{array}{l} Q_6 = \varrho^{1-1}_{3-1} \\ Q_6{}' = \varrho^{1-1}_{1-3} \end{array}\right\} = \frac{1}{12}\left[\sqrt{150}\, \overline{t^{22}_{22}} \pm \sqrt{210}\, \overline{t^{23}_{22}}\right]$$

$$S_1 = \varrho^{10\epsilon}_{-11} = \frac{-1}{12}\left[6\, \overline{t^{21}_{1-1}} - \sqrt{126}\, \overline{t^{23}_{1-1}}\right]$$

$$\left.\begin{array}{l} S_3 = \varrho^{10\epsilon}_{13} \\ S_5 = \varrho^{10\epsilon}_{-3-1} \end{array}\right\} = \frac{-1}{12}\left[\sqrt{27}\, \overline{t^{21}_{1-1}} + \sqrt{42}\, \overline{t^{23}_{1-1}} \pm \sqrt{75}\, \overline{t^{22}_{1-1}}\right]$$

$$S_2 = \varrho^{10\epsilon}_{1-1} = \frac{1}{12}\left[6\, \overline{t^{21}_{11}} - \sqrt{126}\, \overline{t^{23}_{11}}\right]$$

$$\left.\begin{array}{l} S_4 = \varrho^{10\epsilon}_{31} \\ S_6 = \varrho^{10\epsilon}_{-1-3} \end{array}\right\} = \frac{1}{12}\left[\sqrt{27}\, \overline{t^{21}_{11}} + \sqrt{42}\, \overline{t^{23}_{11}} \pm \sqrt{75}\, \overline{t^{22}_{11}}\right]$$

$$S_7 = \varrho^{10\epsilon}_{-33} = \frac{-1}{12}\left[\sqrt{210}\, \overline{t^{23}_{1-3}}\right]$$

$$S_8 = \varrho^{10\epsilon}_{3-3} = \frac{1}{12}\left[\sqrt{210}\, \overline{t^{23}_{13}}\right]$$

Table 13 — cont'd

e) Transversity and helicity amplitudes

λ_{Σ^*}	λ_{K^*}	$\lambda_p = +\frac{1}{2}$	$\lambda_p = -\frac{1}{2}$
$+\frac{3}{2}$	1	e	
	0		d'
	-1		
$+\frac{1}{2}$	1		c'
	0	a	
	-1		b'
$-\frac{1}{2}$	1	b	
	0		a'
	-1	c	
$-\frac{3}{2}$	1		f'
	0	d	
	-1		e'

$= T^{\lambda_{K^*}}_{2\lambda_{\Sigma^*}, 2\lambda_p}$

E	$-E'$
D'	D
F	$-F'$
C'	C
A	$-A'$
B'	B
B	$-B'$
A'	A
C	$-C'$
F'	F
D	$-D'$
E'	E

$= H^{\lambda_{K^*}}_{2\lambda_{\Sigma^*}, 2\lambda_p}$

$$4(A - iA') = -\sqrt{2}\,(b + c) - \sqrt{6}\,(e + f) \qquad 4(A + iA') = -\sqrt{2}\,(b' + c') - \sqrt{6}\,(e' + f'),$$

$$4(B - iB') = -\sqrt{2}\,a - (b - c) - \sqrt{6}\,d - \sqrt{3}\,(e - f)$$

$$4(B + iB') = -\sqrt{2}\,a' - (b' - c') - \sqrt{6}\,d' - \sqrt{3}\,(e' - f')$$

$$4(C - iC') = -\sqrt{2}\,a + (b - c) - \sqrt{6}\,d + \sqrt{3}\,(e - f)$$

$$4(C + iC') = -\sqrt{2}\,a' + (b' - c') - \sqrt{6}\,d + \sqrt{3}\,(e' - f')$$

$$4(D - iD') = -\sqrt{6}\,(b + c) + \sqrt{2}\,(e + f) \qquad 4(D + iD') = -\sqrt{6}\,(b' + c') + \sqrt{2}\,(e' + f')$$

$$4(E - iE') = -\sqrt{6}\,a - \sqrt{3}\,(b - c) + \sqrt{2}\,d + (e - f)$$

$$4(E + iE') = -\sqrt{6}\,a' - \sqrt{3}\,(b' - c') + \sqrt{2}\,d' + (e' - f')$$

$$4(F - iF') = -\sqrt{6}\,a + \sqrt{3}\,(b - c) + \sqrt{2}\,d - (e - f)$$

$$4(F + iF') = -\sqrt{6}\,a' + \sqrt{3}\,(b' - c') + \sqrt{2}\,d' - (e' - f')$$

e') Collinearity constraints

$$\left.\begin{matrix} A' = B' = C' = D' = E' = F' = 0 \\ C = D = E = 0 \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} d = d', & a = a' = \sqrt{2}\,(c - b) + \sqrt{3}\,d \\ b = b', & e = e' = \sqrt{3}\,c + \sqrt{2}\,d \\ c = c', & f = f' = \sqrt{3}\,b - \sqrt{2}\,d \end{matrix}\right.$$

Table 13 — cont'd

f) Expression of the observables in d) as functions of the transversity amplitudes

$2\sigma P_0 = \|a\|^2$	$2\sigma P_0' = \|a'\|^2$
$2\sigma P_1 = \frac{1}{2}(\|b\|^2 + \|c\|^2)$	$2\sigma P_1' = \frac{1}{2}(\|b'\|^2 + \|c'\|^2)$
$2\sigma P_2 = \frac{1}{2}(\|e\|^2 + \|f\|^2)$	$2\sigma P_2' = \frac{1}{2}(\|e'\|^2 + \|f'\|^2)$
$2\sigma P_3 = \|d\|^2$	$2\sigma P_3' = \|d'\|^2$
$2\sigma Q_1 = b\bar{c}$	$2\sigma Q_1' = c'\bar{b}'$
$2\sigma Q_2 = e\bar{f}$	$2\sigma Q_2' = f'\bar{e}'$
$2\sigma Q_3 = a\bar{d}$	$2\sigma Q_3' = d'\bar{a}'$
$2\sigma Q_4 = \frac{1}{2}(e\bar{b} + f\bar{c})$	$2\sigma Q_4' = \frac{1}{2}(b'\bar{c}' + e'\bar{f}')$
$2\sigma Q_5 = b\bar{f}$	$2\sigma Q_5' = f'\bar{b}'$
$2\sigma Q_6 = e\bar{c}$	$2\sigma Q_6' = c'\bar{e}'$
$2\sigma S_1 = \frac{1}{2}(b\bar{a} - a'\bar{b}')$	$2\sigma S_2 = \frac{1}{2}(c'\bar{a}' - a\bar{c})$
$2\sigma S_3 = \frac{1}{2}(c'\bar{d}' - a\bar{f})$	$2\sigma S_5 = \frac{1}{2}(f'\bar{a}' - d\bar{c})$
$2\sigma S_4 = \frac{1}{2}(e\bar{a} - d'\bar{b}')$	$2\sigma S_6 = \frac{1}{2}(b\bar{d} - a'\bar{e}')$
$2\sigma S_7 = \frac{1}{2}(f'\bar{d}' - d\bar{f})$	$2\sigma S_8 = \frac{1}{2}(e\bar{d} - d'\bar{e}')$

4.9. Reactions of type $\pi p \to \rho\Delta$ ($0^- 1/2^+ \to 1^{-e} 3/2^{+e}$)

Thirty four examples of this type of reactions can be obtained from Table 3a). Their amplitudes can be completely reconstructed with transversally polarized target and measurement of the joint angular distribution of the final decays. Table 14 gives the practical recipes for this amplitude reconstruction. It is a simple extension of Table 10, which gives the amplitude reconstruction for reaction type $\pi p \to \rho N$ and has been commented in Section 4.5. We refer to these comments, which can be easily applied to Table 14, although we have omitted here the explicit expressions of the non linear constraints and the algebraic expressions for reconstructing the amplitudes. All them can be obtained from Table 14 f) by elementary algebra. Anyway the simplest method to reconstruct the amplitudes will be a best fit of these expressions by fixing one arbitrary phase (e.g. a = real). Remark that the amplitudes are here obtained without any ghost phase, up to the overall one.

Table 14

Amplitude reconstruction for reactions of type $\pi p \to \rho\Delta$ with polarized target

---

a) Combined production and joint decay angular distribution and measurement of the polarization transfer by the method of moments

$$I(\psi; \theta, \phi, \theta', \phi') = \frac{1}{2\pi} \sum_{L,L} C_\rho(L)\, C_\Delta(L')$$

$$\times \sum_{M,M'} \left[\overline{t^{LL'}_{MM'}} + P_T\left(\cos\psi \overline{{}^z t^{LL'}_{MM'}} + \sin\psi \overline{{}^x t^{LL'}_{MM'}}\right) + P_L \overline{{}^y t^{LL'}_{MM'}}\right] Y^L_M(\theta, \phi)\, Y^{L'}_{M'}(\theta', \phi')$$

$$C_\rho(L)\, C_\Delta(L')\, (t^{LL'}_{MM'} + P_L t^{LL'}_{MM'}) = \langle Y^L_M(\theta, \phi)\, Y^{L'}_{M'}(\theta, \phi') \rangle$$

$$C_\rho(L)\, C_\Delta(L')\, P_T {}^z t^{LL'}_{MM'} = \langle 2 \cos\psi\, Y^L_M(\theta, \phi)\, Y^{L'}_{M'}(\theta', \phi') \rangle$$

$$C_\rho(L)\, C_\Delta(L')\, P_T {}^x t^{LL'}_{MM'} = \langle 2 \sin\psi\, Y^L_M(\theta, \phi)\, Y^{L'}_{M'}(\theta', \phi') \rangle$$

with

$$C_\rho(0) = C_\Delta(0) = 1/\sqrt{4\pi}, \qquad C_\rho(2) = -1/\sqrt{2\pi}, \qquad C_\Delta(2) = -1/\sqrt{4\pi}$$

(all other C coefficients vanish)

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b) The 72 real observables in transversity quantization

$$\sigma_0,\ t^{20}_{00},\ t^{02}_{00},\ t^{22}_{00},\ \begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\Big\}\ t^{20}_{20},\ t^{02}_{02},\ t^{22}_{20},\ t^{22}_{02},\ t^{22}_{22},\ t^{22}_{2-2},\ t^{22}_{11},\ t^{22}_{1-1}$$

$$P_R,\ {}^z t^{20}_{00},\ {}^z t^{02}_{00},\ {}^z t^{22}_{00},\ \begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\Big\}\ {}^z t^{20}_{20},\ {}^z t^{02}_{02},\ {}^z t^{22}_{20},\ {}^z t^{22}_{02},\ {}^z t^{22}_{22},\ {}^z t^{22}_{2-2},\ {}^z t^{22}_{11},\ {}^z t^{22}_{1-1}$$

$$\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\Big\}\ {}^x t^{20}_{10},\ {}^x t^{02}_{01},\ {}^x t^{22}_{10},\ {}^x t^{22}_{01},\ {}^x t^{22}_{21},\ {}^x t^{22}_{12},\ {}^x t^{22}_{2-1},\ {}^x t^{22}_{1-2}$$

$$\begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\Big\}\ {}^y t^{20}_{10},\ {}^y t^{02}_{01},\ {}^y t^{22}_{10},\ {}^y t^{22}_{01},\ {}^y t^{22}_{21},\ {}^y t^{22}_{12},\ {}^y t^{22}_{2-1},\ {}^y t^{22}_{1-2}$$

---

c) The 63 generalized spin rotation parameters and the 8 linear constraints

$$\begin{matrix}P_0\\ P_0'\end{matrix}\Big\} = \frac{1}{12}\left[(1 \pm P_R) - \sqrt{10}\,(t^{20}_{00} \pm {}^z t^{20}_{00}) - \sqrt{5}\,(t^{02}_{00} \pm {}^z t^{02}_{00}) + \sqrt{50}\,(t^{22}_{00} + {}^z t^{22}_{00})\right]$$

$$\begin{matrix}P_1\\ P_1'\end{matrix}\Big\} = \frac{1}{12}\left[(1 \pm P_R) + \sqrt{\frac{5}{2}}\,(t^{20}_{00} \pm {}^z t^{20}_{00}) - \sqrt{5}\,(t^{02}_{00} \pm {}^z t^{20}_{00}) - \sqrt{\frac{25}{2}}\,(t^{22}_{00} \pm {}^z t^{22}_{00})\right]$$

$$\begin{matrix}P_2\\ P_2'\end{matrix}\Big\} = \frac{1}{12}\left[(1 \pm P_R) + \sqrt{\frac{5}{2}}\,(t^{20}_{00} \pm {}^z t^{20}_{00}) + \sqrt{5}\,(t^{02}_{00} \pm {}^z t^{02}_{00}) + \sqrt{\frac{25}{2}}\,(t^{22}_{00} \pm {}^z t^{22}_{00})\right]$$

$$\begin{matrix}P_3\\ P_3'\end{matrix}\Big\} = \frac{1}{12}\left[(1 \pm P_R) - \sqrt{10}\,(t^{20}_{00} \pm {}^z t^{20}_{00}) + \sqrt{5}\,(t^{02}_{00} \pm {}^z t^{02}_{00}) - \sqrt{50}\,(t^{22}_{00} \pm {}^z t^{25}_{00})\right]$$

Table 14 — cont'd

$$P_0 + P_0' + 2(P_1 + P_1' + P_2 + P_2') + P_3 + P_3' = 1$$

$$P_0 - P_0' + 2(P_1 - P_1' + P_2 - P_2') + P_3 - P_3' = P_R$$

$$\left.\begin{matrix} Q_1 \\ Q_1' \end{matrix}\right\} = \frac{1}{12}\left[\sqrt{15}\left(\overline{t^{20}_{20}} \pm \overline{{}^{z}t^{20}_{20}}\right) - \sqrt{75}\left(\overline{t^{22}_{20}} \pm \overline{{}^{z}t^{22}_{20}}\right)\right]$$

$$\left.\begin{matrix} Q_2 \\ Q_2' \end{matrix}\right\} = \frac{1}{12}\left[\sqrt{15}\left(\overline{t^{20}_{20}} \pm \overline{{}^{z}t^{20}_{20}}\right) + \sqrt{75}\left(\overline{t^{22}_{20}} \pm \overline{{}^{z}t^{22}_{20}}\right)\right]$$

$$\left.\begin{matrix} Q_3 \\ Q_3' \end{matrix}\right\} = \frac{1}{12}\left[\sqrt{10}\left(\overline{t^{02}_{02}} \pm \overline{{}^{z}t^{02}_{02}}\right) - 10\left(\overline{t^{22}_{02}} \pm \overline{{}^{z}t^{22}_{02}}\right)\right]$$

$$\left.\begin{matrix} Q_4 \\ Q_4' \end{matrix}\right\} = \frac{1}{12}\left[\sqrt{10}\left(\overline{t^{02}_{02}} \pm \overline{{}^{z}t^{02}_{02}}\right) + 5\left(\overline{t^{22}_{02}} \pm \overline{{}^{z}t^{22}_{02}}\right)\right]$$

$$\left.\begin{matrix} Q_5 \\ Q_5' \end{matrix}\right\} = \frac{1}{12}\sqrt{150}\left(\overline{t^{22}_{2-2}} \pm \overline{{}^{z}t^{22}_{2-2}}\right)$$

$$\left.\begin{matrix} Q_6 \\ Q_6' \end{matrix}\right\} = \frac{1}{12}\sqrt{150}\left(\overline{t^{22}_{22}} \pm \overline{{}^{z}t^{22}_{2-2}}\right)$$

$$\left.\begin{matrix} T_1 \\ T_1' \end{matrix}\right\} = \frac{-1}{12}\sqrt{75}\left(\overline{t^{22}_{1-1}} \pm \overline{{}^{z}t^{22}_{1-1}}\right)$$

$$\left.\begin{matrix} T_2 \\ T_2' \end{matrix}\right\} = \frac{1}{12}\sqrt{75}\left(\overline{t^{22}_{11}} \pm \overline{{}^{z}t^{22}_{11}}\right)$$

c) Continued

$$R_1 = \frac{1}{12}\left[-\sqrt{10}\,\overline{{}^{x}t^{02}_{01}} + 10\,\overline{{}^{x}t^{22}_{01}}\right] = \frac{-i}{12}\left[-\sqrt{10}\,\overline{{}^{y}t^{02}_{01}} + 10\,\overline{{}^{y}t^{22}_{01}}\right] = R_1$$

$$R_2 = \frac{1}{12}\left[-\sqrt{10}\,\overline{{}^{x}t^{02}_{01}} - 5\,\overline{{}^{x}t^{22}_{01}}\right] = \frac{i}{12}\left[-\sqrt{10}\,\overline{{}^{y}t^{02}_{01}} - 5\,\overline{{}^{y}t^{22}_{01}}\right] = R_2$$

$$R_3 = \frac{1}{12}\left[\sqrt{150}\,\overline{{}^{x}t^{22}_{2-1}}\right] = \frac{i}{12}\left[\sqrt{150}\,\overline{{}^{y}t^{22}_{2-1}}\right] = R_3$$

$$R_4 = \frac{1}{12}\left[-\sqrt{150}\,\overline{{}^{x}t^{22}_{21}}\right] = \frac{i}{12}\left[-\sqrt{150}\,\overline{{}^{y}t^{22}_{21}}\right] = R_4$$

$$U_1 = \frac{1}{12}\left[-\sqrt{\frac{15}{2}}\,\overline{{}^{x}t^{20}_{10}} + \sqrt{\frac{75}{2}}\,\overline{{}^{x}t^{22}_{20}}\right]; \qquad \frac{i}{12}\left[-\sqrt{\frac{15}{2}}\,\overline{{}^{y}t^{20}_{10}} + \sqrt{\frac{72}{2}}\,\overline{{}^{y}t^{22}_{10}}\right] = A_1$$

$$U_2 = \frac{1}{12}\left[-\sqrt{\frac{15}{2}}\,\overline{{}^{x}t^{20}_{10}} - \sqrt{\frac{75}{2}}\,\overline{{}^{x}t^{22}_{10}}\right]; \qquad \frac{i}{12}\left[-\sqrt{\frac{15}{2}}\,\overline{{}^{y}t^{20}_{10}} - \sqrt{\frac{75}{2}}\,\overline{{}^{y}t^{22}_{10}}\right] = A_2$$

$$U_3 = \frac{1}{12}\left[-\sqrt{75}\,\overline{{}^{x}t^{22}_{1-2}}\right]; \qquad \frac{i}{12}\left[-\sqrt{75}\,\overline{{}^{y}t^{22}_{1-2}}\right] = A_3$$

$$U_4 = \frac{1}{12}\left[-\sqrt{75}\,\overline{{}^{x}t^{22}_{12}}\right]; \qquad \frac{i}{12}\left[-\sqrt{75}\,\overline{{}^{y}t^{22}_{12}}\right] = A_4$$

Table 14 — cont'd

d) Transfer of polarization

$$\sigma\varrho^{00}_{11e} = \sigma_0 \frac{1}{2} [(P_0 + P_0') + (P_0 - P_0') z],$$

$$\sigma\varrho^{11e}_{11e} = \sigma_0 \frac{1}{2} [(P_1 + P_1') + (P_1 - P_1') z],$$

$$\sigma\varrho^{11e}_{33e} = \sigma_0 \frac{1}{2} [(P_2 + P_2') + (P_2 - P_2') z],$$

$$\sigma\varrho^{00}_{33e} = \sigma_0 \frac{1}{2} [(P_3 + P_3') + (P_3 - P_3') z],$$

$$\sigma\varrho^{1-1}_{11e} = \sigma_0 \frac{1}{2} [(Q_1 + Q_1') + (Q_1 - Q_1') z],$$

$$\sigma\varrho^{1-1}_{33e} = \sigma_0 \frac{1}{2} [(Q_2 + Q_2') + (Q_2 - Q_2') z],$$

$$\sigma\varrho^{00}_{3-1e} = \sigma_0 \frac{1}{2} [(Q_3 + Q_3') + (Q_3 - Q_3') z],$$

$$\sigma\varrho^{11e}_{3-1e} = \sigma_0 \frac{1}{2} [(Q_4 + Q_4') + (Q_4 - Q_4') z],$$

$$\sigma\varrho^{1-1}_{-13e} = \sigma_0 \frac{1}{2} [(Q_5 + Q_5') + (Q_5 - Q_5') z],$$

$$\sigma\varrho^{1-1}_{3-1e} = \sigma_0 \frac{1}{2} [(Q_6 + Q_6') + (Q_6 - Q_6') z],$$

$$\sigma\varrho^{10e}_{13e} = \sigma_0 \frac{1}{2} [(T_1 + T_1') + (T_1 - T_1') z],$$

$$\sigma\varrho^{10e}_{31e} = \sigma_0 \frac{1}{2} [(T_2 + T_2') + (T_2 - T_2') z],$$

$$\sigma\varrho^{00}_{31e} = \sigma_0 R_1 (x + iy),$$

$$\sigma\varrho^{11e}_{31e} = \sigma_0 R_2 (x - iy),$$

$$\sigma\varrho^{1-1}_{13e} = \sigma_0 R_3 (x + iy),$$

$$\sigma\varrho^{1-1}_{31e} = \sigma_0 R_4 (x - iy),$$

$$\sigma\varrho^{10e}_{11e} = \sigma_0 (U_1 x - iA_1 y),$$

$$\sigma\varrho^{10e}_{33e} = \sigma_0 (U_2 x - iA_2 y)$$

$$\sigma\varrho^{10e}_{-13e} = \sigma_0 (U_3 x - iA_3 y)$$

$$\sigma\varrho^{10e}_{3-1e} = \sigma_0 (U_4 x - iA_4 y)$$

e) Transversity and helicity amplitudes as in Table 13e)

f) Expression of the observables in c) and d) as functions of the transversity amplitudes

$$2\sigma_0 P_0 = |a|^2 \qquad 2\sigma_0 P_0' = |a'|^2$$

$$2\sigma_0 P_1 = \frac{1}{2} (|b|^2 + |c|^2) \qquad 2\sigma_0 P_1' = \frac{1}{2} (|b'|^2 + |c'|^2)$$

$$2\sigma_0 P_1 = \frac{1}{2} (|e|^2 + |f|^2) \qquad 2\sigma_0 P_2' = \frac{1}{2} (|e'|^2 + |f'|^2)$$

$$2\sigma_0 P_2 = |d|^2 \qquad 2\sigma_0 P_3' = |d'|^2$$

Table 14 — cont'd

$2\sigma_0 Q_1 = b\bar{c}$	$2\sigma_0 Q_1' = c'\bar{b}'$
$2\sigma_0 Q_2 = e\bar{f}$	$2\sigma_0 Q_2' = f'\bar{e}'$
$2\sigma_0 Q_3 = a\bar{d}$	$2\sigma_0 Q_3' = d'\bar{a}'$
$2\sigma_0 Q_4 = \frac{1}{2}(e\bar{b} + f\bar{e})$	$2\sigma_0 Q_4' = \frac{1}{2}(b'\bar{e}' + c'\bar{f}')$
$2\sigma_0 Q_5 = b\bar{f}$	$2\sigma_0 Q_5' = f'\bar{b}'$
$2\sigma_0 Q_6 = e\bar{c}$	$2\sigma_0 Q_6' = c'\bar{e}'$
$2\sigma_0 T_1 = \frac{1}{2}(d\bar{c} + a\bar{f})$	$2\sigma_0 T_1' = \frac{1}{2}(f'\bar{a}' + c'\bar{d}')$
$2\sigma_0 T_2 = \frac{1}{2}(e\bar{a} - b\bar{d})$	$2\sigma_0 T_2' = \frac{1}{2}(a'\bar{e}' + d'\bar{b}')$
$2\sigma_0 R_1 = \frac{1}{2}(d'\bar{a} - a'\bar{d}')$	$2\sigma_0 R_3 = \frac{1}{2}(c'\bar{f} - f'\bar{c})$
$2\sigma_0 R_2 = \frac{1}{4}(e\bar{c}' - c\bar{e}' + b\bar{f}' - f\bar{b}')$	$2\sigma_0 R_4 = \frac{1}{2}(e\bar{b}' - b\bar{e}')$
$2\sigma_0 U_1 = \frac{1}{4}(c'\bar{a} - a'\bar{c} + b\bar{a}' - a\bar{b}')$	$2\sigma_0 U_3 = \frac{1}{4}(f'\bar{a} - a'\bar{f} + b\bar{d}' - d\bar{b}')$
$2\sigma_0 U_2 = \frac{1}{4}(f'\bar{d} - d'\bar{f} + e\bar{d}' - d\bar{e}')$	$2\sigma_0 U_4 = \frac{1}{4}(c'\bar{d} - d'\bar{c} + e\bar{a}' - a\bar{e}')$
$2\sigma A_1 = \frac{i}{4}(e\bar{c}' - c\bar{e}' - b\bar{f}' + f\bar{b}')$	$2\sigma_0 A_3 = \frac{i}{4}(f'\bar{a} - a'\bar{f} - b\bar{d}' + d\bar{b}')$
$2\sigma_0 A_2 = \frac{i}{4}(f'\bar{d} - d'\bar{f} - e\bar{d}' + d\bar{e}')$	$2\sigma_0 A_4 = \frac{i}{4}(c'\bar{d} - d'\bar{c} - e\bar{a}' + a\bar{e}')$

Appendix

1. The matrix W of polarization transfer

We consider the linear map w from the initial polarization space of density operators on $\mathscr{H}_e$ to the final polarization space of density operators on $\mathscr{H}_f$:

$$\varrho_e \to w(\varrho_e) \equiv \sigma\varrho_f = T\varrho_e T^\dagger . \tag{A 1}$$

There is a complete mathematical similarity between this polarization transfer and the polarization correlation for a system of two particles. It is therefore very convenient and more elegant to describe polarization transfer by a matrix W analogous to the joint density matrix [*20*]. For an initial polarization density matrix ϱ_e and a final analyser of polarization A_f, the transition rate is

$$w(\varrho_e, A_f) = \operatorname{tr} A_f T\varrho_e T^\dagger \tag{A 2}$$

or, writing down the indices (upper = lines, lower = columns),

$$w(\varrho_e, A_f) = \sum_{\substack{\mu\mu' \\ \lambda\lambda'}} (A_f)^{\mu'}_{\mu}\, T_{\lambda}{}^{\mu}(\varrho_e)^{\lambda}_{\lambda'}\, (T^\dagger)^{\lambda'}_{\mu'} = \sum_{\substack{\mu\mu' \\ \lambda\lambda'}} (\tilde{\varrho}_e)_{\lambda}{}^{\lambda'}\, (A_f)_{\mu}{}^{\mu'}\, (\tilde{T})^{\lambda\mu}\, (\tilde{T}^\dagger)_{\lambda'\mu'}$$

where the symbol $\sim$ means transposition in the initial space. The last expression can be written

$$w(\varrho_e, A_f) = \operatorname{tr}(\tilde{\varrho}_e \otimes A_f)\, W\,, \tag{A 3}$$

by using the polarization transfer matrix W, whose elements are

$$W^{\lambda\mu}_{\lambda'\mu'} = (\tilde{T})^{\lambda\mu}(\tilde{T}^\dagger)_{\lambda'\mu'}\,. \tag{A 4}$$

This matrix represents an observable, rank one, positive, Hermitian operator acting on the spin space $\mathscr{H}_e\, {}^*\!\otimes \mathscr{H}_f$. The final state density matrix $\sigma\varrho_f$ is obtained by taking the partial trace in the inital spin space

$$\sigma\varrho_f = \operatorname{tr}_e(\tilde{\varrho}_e \otimes \mathbf{1}_f)\, W\,. \tag{A 5}$$

Thus the knowledge of W is equivalent to that of T (or $\tilde{T}$) up to an overall phase. Indeed given any rank one, positive, Hermitian operator, H, it is easy to find a vector $|x\rangle$ such that

$$H = |x\rangle\,\langle x|\,. \tag{A 6}$$

For any well defined ordering of indices in H, let λ^0 be the first index for which $H^{\lambda^0}_{\lambda^0} \neq 0$. Then a possible vector $|x\rangle$ is defined by components

$$x^\lambda = H^{\lambda}_{\lambda^0}/\sqrt{H^{\lambda^0}_{\lambda^0}}\,. \tag{A 7}$$

(Remark that $x^\lambda = 0$ for $\lambda < \lambda^0$, since $H_\lambda{}^\lambda = 0$ implies $H^\lambda_{\lambda'} = 0 = H_\lambda{}^{\lambda'}$ for any λ'). We call this procedure a "conventional amplitude reconstruction", and denote symbolically

$$|x\rangle = \mathrm{CAR}\,(H)\,. \tag{A 7'}$$

Since Eq. (A 4) is of the type (A 6) with some double indicing, a particular solution of (A 4) is the vector $(\tilde{T})^{\lambda\mu}$

$$|\tilde{T}\rangle = \mathrm{CAR}\,(W)\,. \tag{A 8}$$

The general solution is obtained by multiplication of this particular one by an arbitrary phase.

Let us study now the structure of the T and W matrices describing two body reactions in which parity is conserved. For this purpose it is convenient to adopt transversity quantizations, for which half of the transition amplitudes vanish (we consider the most frequent case of reactions in which some fermions are present). It is also convenient to introduce a "separation order" for lines and columns of the transition matrix (cf. [*13*]), which segregates the vanishing from the non necessarily vanishing amplitudes.

For one particle with spin j and parity η, such an ordering classifies the magnetic quantum numbers μ in two sets S_e and S_0:

$$\text{for } j - \mu = \begin{cases} \text{even}, & \mu \in S_e\,, \\ \text{odd}, & \mu \in S_0\,. \end{cases} \tag{A 9}$$

Keeping this ordering, the B-symmetry operator (cf. Section 2.2.) for this particle can be written in block form

$$B = \eta e^{-i\pi j} \begin{array}{c} \quad S_e \qquad S_0 \\ \left(\begin{array}{c|c} \mathbf{1}_{j+1/2} & 0 \\ \hline 0 & -\mathbf{1}_{j1/2} \end{array}\right)\begin{array}{l} S_e \\ S_0 \end{array} \end{array} \text{ for } j = \text{half odd integer} \tag{A 10a}$$

$$B = \eta e^{-i\pi j} \left(\begin{array}{c|c} \mathbf{1}_{j+1} & 0 \\ \hline 0 & -\mathbf{1}_j \end{array}\right) \text{ for } j = \text{integer}. \tag{A 10b}$$

For a system of two particles with spins j, j' and parities η, η' the separation order classifies similarly the couples (μ, μ') of magnetic quantum numbers in two sets (cf. [*13*b]):

$$\text{for } j - \mu + j' - \mu' = \begin{cases} \text{even}, & (\mu, \mu') \in S_e \\ \text{odd}, & (\mu, \mu') \in S_0. \end{cases} \tag{A 11}$$

The corresponding B-symmetry operator reads

$$B = \eta\eta' e^{-i\pi(j+j')} \begin{array}{c} \quad S_e \quad S_0 \\ \left(\begin{array}{c|c} \mathbf{1}_n & 0 \\ \hline 0 & -\mathbf{1}_n \end{array}\right)\begin{array}{l} S_e \\ S_0 \end{array} \end{array} \tag{A 12}$$

where $n = 1/2(2j + 1)\,(2j' + 1)$ was supposed to be integer. For our problem we need two such operators: B_e acting on $\mathscr{H}_e$, the initial spin space of particles 1 and 2, and B_f acting on $\mathscr{H}_f$, the final spin space of particles 3, 4.
Parity conservation in the two body reaction implies

$$B_f T B_e^{\dagger} = T, \tag{A 13}$$

and imposes to the transition matrix T, written in this separation order, one of the two following block structures,

$$T = \begin{array}{c} \quad S_e \quad S_0 \\ \left(\begin{array}{c|c} T_+ & 0 \\ \hline 0 & T_- \end{array}\right)\begin{array}{l} S_e \\ S_0 \end{array} \end{array}, \quad \text{for } \varepsilon = +1; \qquad T = \begin{array}{c} \quad S_e \quad S_0 \\ \left(\begin{array}{c|c} 0 & T_+ \\ \hline T_- & 0 \end{array}\right)\begin{array}{l} S_e \\ S_0 \end{array} \end{array}, \quad \text{for } \varepsilon = -1 \tag{A 14}$$

where

$$\varepsilon = \bar{\eta}_1 \bar{\eta}_2 \eta_3 \eta_4 e^{i\pi(j_1 + j_2 - j_3 - j_4)}. \tag{A 15}$$

By transposition of initial indices, from $T_{\pm}$ we obtain $\tilde{T}_{\pm}$, and from T we obtain T, which can be written for both values of ε:

$$\tilde{T} = \begin{vmatrix} \dfrac{1+\varepsilon}{2} & \tilde{T}_+ \\ \dfrac{1-\varepsilon}{2} & \tilde{T}_- \\ \dfrac{1-\varepsilon}{2} & \tilde{T}_+ \\ \dfrac{1+\varepsilon}{2} & \tilde{T}_- \end{vmatrix} \begin{array}{l} (S_e, S_e) \\ (S_e, S_0) \\ (S_0, S_e) \\ (S_0, S_0). \end{array} \tag{A 16}$$

Thus, in this separation order for initial and final indices, the matrix W has the block form

$$W = \tilde{T}\tilde{T}^\dagger = \left(\begin{array}{c|c|c|c} \frac{1+\varepsilon}{2}\tilde{T}_+\tilde{T}_+{}^\dagger & 0 & 0 & \frac{1+\varepsilon}{2}\tilde{T}_+\tilde{T}_-{}^\dagger \\ \hline 0 & \frac{1-\varepsilon}{2}\tilde{T}_+\tilde{T}_-{}^\dagger & \frac{1-\varepsilon}{2}\tilde{T}_+\tilde{T}_+{}^\dagger & 0 \\ \hline 0 & \frac{1-\varepsilon}{2}\tilde{T}_+\tilde{T}_-{}^\dagger & \frac{1-\varepsilon}{2}\tilde{T}_+\tilde{T}_+{}^\dagger & 0 \\ \hline \frac{1+\varepsilon}{2}\tilde{T}_-\tilde{T}_+{}^\dagger & 0 & 0 & \frac{1+\varepsilon}{2}\tilde{T}_-\tilde{T}_-{}^\dagger \end{array}\right) \qquad \text{(A 17)}$$

By further reordering, it could be written as a direct sum of two blocks, one of which necessarily vanishes:

$$W = \frac{1+\varepsilon}{2} W_0 \oplus \frac{1-\varepsilon}{2} W_0, \qquad W_0 = \begin{pmatrix} \tilde{T}_+\tilde{T}_+{}^\dagger & \tilde{T}_+\tilde{T}_-{}^\dagger \\ \tilde{T}_-\tilde{T}_+{}^\dagger & \tilde{T}_-\tilde{T}_-{}^\dagger \end{pmatrix}. \qquad \text{(A 18)}$$

2. Number of ghost amplitudes in experiments with unpolarized or polarized spin 1/2 initial particle[10])

We consider now an initial state composed of particle 1 with spin j_1, which will be assumed to be unpolarized, and particle 2 with spin $j_2 = 1/2$, whose polarization will be considered. In the main text $j_1 = 0$, and 1 is the beam, 2 the target. This section is more general and can be applied to the case $j_1 = 1/2$ (nucleon scattering), or to the case where 1 is a higher spin nucleus target and 2 a polarized beam. We also assume that the dimension of the final spin space $\mathscr{H}_f$ is not smaller than $2(2j_1 + 1)$, the dimension of the initial one $\mathscr{H}_e$, and that the final density matrix $\sigma\varrho_f$ is completely observed.
According to Eq. (A 12) the B-symmetry operator of the initial state, B_e, decomposes in two blocks for a separation order of indices:

$$e^{i\pi(j_1+1/2)}\eta_1\eta_2 B_e = \mathbf{1}_{2j_1+1} \oplus (-\mathbf{1}_{2j_1+1}). \qquad \text{(A 19)}$$

The number N_U of real ghost amplitudes, for unpolarized initial state, is the number of parameters of the set of matrices U which transforms T into TU but leaves the observables unchanged, i.e., $TU(TU)^\dagger = TT^\dagger$. This transformation must preserve the B-symmetric structure of T; since furthermore we disregard the overall phase of T, the matrices U must satisfy the conditions:

$$UU^\dagger = \mathbf{1}, \qquad B_e U B_e{}^\dagger = U, \qquad \det U = +1. \qquad \text{(A 20)}$$

Because of the structure of B_e, Eq. (A 19), these conditions imply that U belongs to the group $S[U(2j_1 + 1) \otimes U(2j_1 + 1)]$. The number N_U of ghost amplitudes is the dimension of this group

$$N_U = 2(2j_1 + 1)^2 - 1. \qquad \text{(A 21)}$$

In the case of an experiment with polarized spin 1/2 initial particle, in addition to Eq. (A 20), U must satisfy the new condition

$$[U, \mathbf{1}_{2j_1+1} \otimes \varrho_2] = 0. \qquad \text{(A 22)}$$

10) This section summarizes published and unpublished work of SIMONIUS, cf. [9].

for the density matrix ϱ_2 of particle 2. Condition (A 20) and condition (A 22) for arbitrary ϱ_2 (we assume that the experiment with longitudinal *and* transverse polarization is performed) are equivalent to

$$U = U_1 \otimes \mathbf{1}_2, \qquad U_1 U_1^\dagger = \mathbf{1}, \qquad \det U_1 = 1, \qquad B_1 U_1 B_1^\dagger = U_1. \qquad \text{(A 23)}$$

Because of the structure of B_1, Eq. (A 10), these conditions imply that U belongs to the group $S[U(j_1 + 1) \otimes U(j_1)]$ for j_1 integer, or to the group $S[U(j_1 + 1/2) \otimes U(j_1 + 1/2)]$ for j_1 half odd integer. The number N_P of real ghost amplitudes in these cases is given by the dimension of the groups

$$N_P = \begin{cases} (j_1 + 1)^2 + j_1^2 - 1 = 2j_1(j_1 + 1) & \text{for integer } j_1, \\ 2\left(j_1 + \dfrac{1}{2}\right)^2 - 1 & \text{for half odd integer } j_1. \end{cases} \qquad \text{(A 24)}$$

Table A 1 gives the value of N_U, N_P for the low values of j_1. It also gives $N_R = N_U - N_P$, the number of additional amplitudes which can be reached by using a polarized spin 1/2 initial particle.

3. The matrices ϱ_α of polarization transfer from spin 1/2 initial particle

We consider further the case of an initial state composed of an unpolarized spin j_1 particle and a spin $j_2 = 1/2$ particle whose polarization is described by the density matrix ϱ_2. With the usual expansion for ϱ_2

$$\varrho_2 = \frac{1}{2}(\mathbf{1} + x\tau_x + y\tau_y + z\tau_z), \qquad \text{(A 25)}$$

the final density matrix can be written

$$\sigma\varrho_f = \sigma_0(\varrho_0 + x\varrho_x + y\varrho_y + z\varrho_z), \qquad \text{(A 26)}$$

where σ_0 and ϱ_0 are the differential cross section and the final polarization for an unpolarized initial state, and the matrices $\varrho_i (i = x, y, z)$ add the information on the polarization transfer.

The polarization domain of ϱ_2 is the Poincaré sphere $x^2 + y^2 + z^2 \leqq 1$; the linear map w transforms this sphere into an ellipsoid centered at $\sigma_0\varrho_0$. The principal axes of this ellipsoid are the new observables which can be measured when the initial state is polarized. They are not arbitrary but must satisfy some conditions, e.g. when rank $\varrho_2 = 1$ (total polarization) rank $\varrho_f \leqq 2j_1 + 1$, and hence the density matrix $\sigma\varrho_f$ (is on the surface of the cone C of positive matrices acting on $\mathscr{H}_f$ (we assume dim $\mathscr{H}_f > 2j_1 + 1$).

For parity conserving two body reactions it is easy to prove that the joint density matrices ϱ_0 and ϱ_z are B-symmetric, while ϱ_x and ϱ_y are B-antisymmetric. Indeed the B-symmetry operator for particle 2 [cf. (A 1)],

$$B_2 = -i\eta_2\tau_3, \qquad \text{(A 27)}$$

decomposes the matrix ϱ_2 of (A 24) into

$$\varrho_2 = \varrho_{2+} + \varrho_{2-}, \qquad B_2\varrho_{2\pm}B_2^\dagger = \pm\varrho_{2\pm}, \qquad \text{(A 28)}$$

with

$$\varrho_{2+} = \frac{1}{2}(\mathbf{1} + z\tau_z), \qquad \varrho_{2-} = \frac{1}{2}(x\tau_x + y\tau_y). \qquad \text{(A 29)}$$

Since the density matrix $\varrho_1 = \mathbf{1}_{2j_1+L}/(2j_1+1)$ and the transition matrix T are B-symmetric, if we call

$$\sigma\varrho_{f\pm} = T(\varrho_1 \otimes \varrho_{2\pm})\, T^\dagger, \tag{A 30}$$

we obtain

$$B_f \sigma\varrho_{f\pm} B_f^\dagger = \pm\sigma\varrho_{f\pm} \tag{A 31}$$

and

$$\sigma\varrho_{f+} = \sigma_0(\varrho_0 + z\varrho_z), \qquad \sigma\varrho_{f-} = \sigma_0(x\varrho_x + y\varrho_y). \tag{A 32}$$

4. Amplitude reconstruction for reactions with spinless beam and spin 1/2 target when the final polarization is completely observed

We suppose for the beam spin $j_1 = 0$. Then the initial state density matrix is that of the target, $\varrho_e = \varrho_z$, with dimension, $\dim \mathscr{H}_e = 2$. The most general form for the operator W on $\mathscr{H}_e^* \otimes \mathscr{H}_f$ is

$$W = \Sigma_\alpha \tilde{\tau}_\alpha \otimes X_\alpha, \tag{A 33}$$

since $\tilde{\tau}_\alpha = \mathbf{1}, \tau_x, \tilde{\tau}_y, \tau_z$ form an orthonormal basis on $\mathscr{H}_e^*$.
Substitution in Eq. (A 5) of the expansion (A 33) of W and (A 25) of ϱ_e, use of the identity $\mathrm{tr}\, \tau_\alpha\tau_\beta = 2\delta_{\alpha\beta}$, and comparison with the definition (A 26) of ϱ_α, yield $X_\alpha = \sigma_0\varrho_\alpha$, i.e.,

$$W = \sigma_0 \Sigma_\alpha \tilde{\tau}_\alpha \otimes \varrho_\alpha, \tag{A 34}$$

which is Eq. (3.9) of the main text. Using the ordinary representation of the Pauli matrices, this equation reads

$$W = \sigma_0 \left(\begin{array}{c|c} \varrho_0 + \varrho_z & \varrho_x + i\varrho_y \\ \hline \varrho_x - i\varrho_y & \varrho_0 - \varrho_z \end{array}\right), \tag{A 34'}$$

The separation order is superfluous for the initial spin space, since there are only two indices: $\mu_2 = 1/2 \in S_e$ and $\mu_2 = -1/2 \in S_0$. The matrices $\tilde{T}_\pm$ are then identical to the matrices $T_\pm$. If we introduce the separation order for the final spin space, we may identify the two W expressions (A 32) and (A 34') and we obtain the following block expressions for the polarization transfer matrices:

$$\varrho_0 = \left(\begin{array}{c|c} A & 0 \\ \hline 0 & B \end{array}\right), \quad \varrho_z = \varepsilon \left(\begin{array}{c|c} A & 0 \\ \hline 0 & -B \end{array}\right), \quad \varrho_x = \left(\begin{array}{c|c} & C \\ \hline C^\dagger & \end{array}\right), \quad \varrho_y = \varepsilon \left(\begin{array}{c|c} & -iC \\ \hline iC^\dagger & \end{array}\right) \tag{A 35}$$

with

$$2\sigma_0 A = T_+T_+^\dagger, \qquad 2\sigma_0 B = T_-T_-^\dagger, \qquad 2\sigma_0 C = T_+T_-^\dagger. \tag{A 36}$$

The sign ε is the function of parities and spins given in (A 15). Remark that A, B, C are rank one matrices, and A, B are Hermitian and positive. The so called "reaction polarization" is

$$P_R \equiv \mathrm{tr}\, \varrho_z = \varepsilon(\mathrm{tr}\, A - \mathrm{tr}\, B). \tag{A 37}$$

Let us study the amplitude reconstruction in the cases in which the final joint polarization can be completely measured, i.e., when one observes $\sigma_0\varrho_0$ in the experiment with unpolarized target, $\sigma_0\varrho_z$ and $\sigma_0\varrho_x$ in the experiment with transversally polarized target, and $\sigma_0\varrho_y$ in the experiment with longitudinally polarized target. In this case, with unpolarized target, all the transversity amplitudes T_+ and T_- can be reconstructed up to the overall phase and one ghost phase, as proved by Simonius (cf. Section 2, and Table A1 for $j_1 = 0$). Indeed, by the "conventional amplitude reconstruction" (cf. Section A1), from the observed matrices $\sigma_0 A$ and $\sigma_0 B$ we can obtain

$$|a\rangle = \mathrm{CAR}\,(2\sigma_0 A)\,, \qquad |b\rangle = \mathrm{CAR}\,(2\sigma_0 B)\,, \tag{A 38}$$

which are T_+ and T_- up to arbitrary phases. The Simonius ghost phase φ, is the relative phase between T_+ and T_-. It can only be obtained from C, which is observable in experiments with either transverse or longitudinal target polarization. From Eqs. (A 36) and (A 38), we see that the observed matrix $\sigma_0 C$ must satisfy

$$s\sigma_0 C = |a\rangle\,\langle b|\,e^{i\varphi}\,, \tag{A 39}$$

and that the amplitude vectors $|a\rangle$ and $e^{-i\varphi}|b\rangle$ can only differ from T_+ and T_- by an overall phase, which we disregard.

The number of amplitudes is given by the dimension of $T_\pm$. The number of observables and of their linear and non linear constraints in experiments with different target polarizations are easily obtained from the dimension, Hermiticity and rank properties of the matrices A, B, C. The dimension of the one column matrices $T_\pm$ and the square matrices A, B, C is

$$n = (2l + 1)\,(j + 1/2)\,, \tag{A 40}$$

where l and j are the integer and half odd integer final spins. We recall that a $n \times n$ Hermitian matrix depends on n^2 real parameters. If the matrix has rank k, these parameters satisfy $(n - k)^2$ constraints of degree $k + 1$. The results are summarized in Table A 2. For $l = 0$ equivalent numbers are presented in Table 1. For $l = 0$ and $j = 3/2$, the explicit amplitude reconstruction is presented in Section 4.2. of the main text and in Tables 5—6.

Table A1

Number of ghost amplitudes in reactions with an unpolarized spin j particle and an (unpolarized or polarized) spin 1/2 particle as initial state

j	0	1/2	1	3/2	2	integer	half odd integer
N_U	1	7	17	31	49	$8j(j+1)+1$	$8(j+1/2)^2 - 1$
N_P	0	1	4	7	12	$2j(j+1)$	$2(j+1/2)^2 - 1$
N_R	1	6	13	24	37	$6j(j+1)+1$	$6(j+1/2)^2$

The tabulated numbers of real amplitudes are:

N_U = ghost amplitudes for unpolarized initial state
N_P = ghost amplitudes for polarized spin 1/2 initial particle
$N_R = N_U - N_P$ = amplitudes reached by initial polarization.

Note that identity of particles, internal symmetry (isospin charge conjugation) and, for elastic reactions, time reversal may decrease N_U.

Table A2

Number of amplitudes, independent observables, and observable constraints in reactions of type $0\ 1/2 \to lj$ (l = integer, j = half odd integer) for different initial polarizations and complete measurement of the final polarization

$(2l+1)(j+1/2)$	A	U_I	U_N	T_I	T_N	T_L	L_I	L_N
1	3	2	0	1	1	2	1	1
2	7	6	2	1	7	8	1	7
3	11	10	8	1	17	18	1	17
4	15	14	18	1	31	32	1	31
5	19	18	32	1	49	50	1	49
n	$4n-1$	$4n-2$	$2(n-1)^2$	1	$2n^2-1$	$2n^2$	1	$2n^2-1$

Terminology:

A = number of real amplitudes (up to the overall phase)
U = number of observables for unpolarized target (or beam)
T = number of new observables reached with transversally polarized target (or beam)
L = idem with longitudinally polarized target (or beam)

The subindices classify these observables into

I = independent observables
N = non linear observable constraints
L = linear observable constraints

(Linear constraints coming from B-symmetry have not been counted, otherwise the total number of observables is multiplied by 2).

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