4. Odd polarization of the spin 3/2 particle.

4.1. Measurement of the odd polarization of the spin 3/2 particle.

The angular distribution of the strong two-body decay of a spin 3/2 particle supplies information on its even polarization only (cf. 3.1). But the odd polarization of this spin 3/2 particle (that we call A) can be measured whenever the polarization of the spin 1/2 decay product (that we call B) is observed. This is the case if, as indicated in Table 4.1 (a) , B undergoes a weak decay (case of the cascade of Σ^* and Ξ^* particles). Let us call C the spinless decay product of B . Then for measuring the odd polarization of particle A it is sufficient to consider the degree of longitudinal polarization of particle B in the rest frame of A . The angular distribution of this longitudinal polarization , $I(\theta,\varphi).P_L(\theta,\varphi)$, is related to the odd polarization parameters $t^{(1)}_M$ and $t^{(3)}_M$ of particle A by expression in Table 4.1(b). Recall that this longitudinal polarization, P_L , is defined by the mean value (c) of the same table, as function of α , the asymmetry parameter of the parity violating second decay (cf. Table 1.1(c) and (d_1)). The angle θ_1 is simply defined in the rest system of B as the angle between the three-momenta of A and C . The inverse expressions, which supply a method of independent measurement for each one of these odd polarization parameters are also given in Table 4.1(d) and (e). The brackets $<\ldots P_L \ldots>$ in (d) properly indicate a double experimental mean value : (a) the mean value P_L of the B decay angle θ_1 , for each sub-ensemble of events with the same direction, θ,φ, for the decay of A, and (b) the mean value of inclosed expressions on θ,φ for the different directions of this first decay. In fact it is enough to measure the three angles, θ,φ,θ_1 , for each event and to obtain a single mean value of the expressions for the whole ensemble (cf. part I, A5).

In the case of B-symmetry for the production process of the spin 3/2 particle, 6 of these 10 odd polarization parameters have to be zero. They are the $r_M^{(L)}$ parameters with M odd in the case of transversity quantization, and these with M positive or nule in the case of helicity quantization, as indicated in Table 4.1(f) and (g).

TABLE [illegible] . Measurement of the odd polarization of spin $\frac{3}{2}$ particle

(a) Cascade decay

$$\frac{3}{2} \rightarrow \frac{1}{2} + 0 \qquad \text{(strong interaction)}$$

$$\searrow\!\longrightarrow \frac{1}{2} + 0 \qquad \text{(weak interaction)}$$

Angular distribution of the longitudinal polarization

(b) $$I(\theta,\varphi)\, P_L(\theta,\varphi) = \sqrt{\frac{1}{20\pi}} \left[\sum_{M=-1}^{+1} \bar{t}^{(1)}_{M}\, Y^{(1)}_{M}(\theta,\varphi) - 3 \sum_{M=-3}^{+3} \bar{t}^{(3)}_{M}\, Y^{(3)}_{M}(\theta,\varphi) \right]$$

(c) $$P_L(\theta,\varphi) = \frac{3}{\alpha} < \cos\theta_1 >_{\theta,\varphi}$$

Multipole parameters

(d) $$t^{(1)}_{M} = \sqrt{20\pi} < Y^{(1)}_{M}(\theta,\varphi)\, P_L(\theta,\varphi) > \qquad M = -1,\ 0,\ +1$$

$$t^{(3)}_{M} = -\sqrt{\frac{20\pi}{9}} < Y^{(3)}_{M}(\theta,\varphi)\, P_L(\theta,\varphi) > \qquad M = -3,\ -2, \ldots +3$$

(e) $$r^{(L)}_{0} = \sqrt{\frac{2L+1}{3}}\, t^{(L)}_{0} \qquad L = 1,\ 3$$

$$r^{(L)}_{M} = (-1)^{M} \sqrt{\frac{2(2L+1)}{3}}\ \mathrm{Re}\ t^{(L)}_{M} \qquad M = 1,\ 2,\ \ldots\ L$$

$$r^{(L)}_{-M} = (-1)^{M} \sqrt{\frac{2(2L+1)}{3}}\ \mathrm{Im}\ t^{(L)}_{M}$$

Condition of B-symmetry in the production process

(f) For transversity quantization

$$T_{r\,1}^{\,(1)} = T_{r\,-1}^{\,(1)} = T_{r\,1}^{\,(3)} = T_{r\,-1}^{\,(3)} = T_{r\,3}^{\,(3)} = T_{r\,-3}^{\,(3)} = 0$$

(g) For helicity quantization

$$H_{r\,1}^{\,(1)} = H_{r\,0}^{\,(1)} = H_{r\,3}^{\,(3)} = H_{r\,2}^{\,(3)} = H_{r\,1}^{\,(3)} = H_{r\,0}^{\,(3)} = 0$$

4.2. Relation between different odd polarization parameters of the spin 3/2 particle.

In any reference frame, the odd part of the density matrix of the spin 3/2 particle has the form indicated in Table 4.2(a) (cf. Part I, A6). Remark that it is traceless and has to be added to the trace one, even part given in Table 3.2(a) in order to obtain a meaningful density matrix. These matrix elements of the odd part are related to the multipole parameters of Table 4.1 by the relations given in Table 4.2(b).

The relations between the multipole parameters for associated transversity and helicity quantizations are given in Table 4.2(c). And the relation between the multipole parameters for two transversity quantizations are given in Table 4.2(d), as function of the rotation angle arount the normal ψ_{ba} .

The six parameters that must be zero in the case of B-symmetric production are conveniently indicated in the second column of Table 4.2(c) and in the last lines or column of Table 4.2(b). Remark there that for transversity quantization the odd part of the density matrix in Table 4.2(a) has "checker-board pattern" and is antisymmetric through the second diagonal (the even part had the same pattern and was symmetric, cf. 3.2). And in the case of helicity quantization this B-symmetric odd part is pure imaginary (the even part was real, cf. 3.2).

TABLE 4.2 - Relation between different odd polarization parameters of spin $\frac{3}{2}$ particl

(a) Odd part of the density matrix for spin $\frac{3}{2}$ particle

$$\rho^{o} = \begin{vmatrix} \rho^{o}_{33} & \rho^{o}_{31} & \rho^{o}_{3-1} & \rho^{o}_{3-3} \\ \bar{\rho}^{o}_{31} & \rho^{o}_{11} & \rho^{o}_{1-1} & -\rho^{o}_{3-1} \\ \bar{\rho}^{o}_{3-1} & \bar{\rho}^{o}_{1-1} & -\rho^{o}_{11} & \rho^{o}_{31} \\ \bar{\rho}^{o}_{3-3} & -\bar{\rho}^{o}_{3-1} & \bar{\rho}^{o}_{31} & -\rho^{o}_{33} \end{vmatrix}$$

(b) Relation between the matrix elements in (a) and the multipole parameters in Table 3.1b

		(BH)
		$r^{(1)}_{0} = \sqrt{\frac{12}{5}}\ \rho^{o}_{33} + \sqrt{\frac{4}{15}}\ \rho^{o}_{11}$
		$r^{(3)}_{0} = \sqrt{\frac{4}{15}}\ \rho^{o}_{33} - \sqrt{\frac{12}{5}}\ \rho^{o}_{11}$
	$r^{(3)}_{-2} = -\sqrt{\frac{16}{3}}\ \mathrm{Im}\ \rho^{o}_{3-1}$	$r^{(3)}_{2} = \sqrt{\frac{16}{3}}\ \mathrm{Re}\ \rho^{o}_{3-1}$
	$r^{(1)}_{-2} = -\sqrt{\frac{16}{5}}\ \mathrm{Im}\ \rho^{o}_{31} - \sqrt{\frac{16}{15}}\ \mathrm{Im}\ \rho^{o}_{1-1}$	$r^{(1)}_{1} = \sqrt{\frac{16}{5}}\ \mathrm{Re}\ \rho^{o}_{31} + \sqrt{\frac{16}{15}}\ \mathrm{Re}\ \rho^{o}_{1-1}$
(BT)	$r^{(3)}_{-1} = -\sqrt{\frac{32}{15}}\ \mathrm{Im}\ \rho^{o}_{31} + \sqrt{\frac{8}{5}}\ \mathrm{Im}\ \rho^{o}_{1-1}$	$r^{(3)}_{1} = \sqrt{\frac{32}{15}}\ \mathrm{Re}\ \rho^{o}_{31} - \sqrt{\frac{8}{5}}\ \mathrm{Re}\ \rho^{o}_{1-1}$
	$r^{(3)}_{-3} = -\sqrt{\frac{8}{3}}\ \mathrm{Im}\ \rho^{o}_{3-3}$	$r^{(3)}_{3} = \sqrt{\frac{8}{3}}\ \mathrm{Re}\ \rho^{o}_{3-3}$

(BH) For B-symmetry and helicity quantization, the parameters in this column are zero

(BT) For B-symmetry and transversity quantization, the parameters in these three lines are zero.

(See continuation)

(Continuation of Table 4.2)

(c) Relation between the multipole parameters for transversity and helicity quantizations

	(B)
	$H_r{}^{(1)}_{0} = -T_r{}^{(1)}_{-1}$
$H_r{}^{(1)}_{-1} = T_r{}^{(1)}_{0}$	$H_r{}^{(1)}_{1} = T_r{}^{(1)}_{1}$
	$H_r{}^{(3)}_{0} = \sqrt{\frac{3}{8}}\, T_r{}^{(3)}_{-1} + \sqrt{\frac{5}{8}}\, T_r{}^{(3)}_{-3}$
$H_r{}^{(3)}_{-2} = -T_r{}^{(3)}_{-2}$	$H_r{}^{(3)}_{2} = \sqrt{\frac{5}{8}}\, T_r{}^{(3)}_{-1} - \sqrt{\frac{3}{8}}\, T_r{}^{(3)}_{-3}$
$H_r{}^{(3)}_{-1} = -\sqrt{\frac{5}{8}}\, T_r{}^{(3)}_{2} - \sqrt{\frac{3}{8}}\, T_r{}^{(3)}_{0}$	$H_r{}^{(3)}_{1} = -\sqrt{\frac{15}{16}}\, T_r{}^{(3)}_{3} - \sqrt{\frac{1}{16}}\, T_r{}^{(3)}_{1}$
$H_r{}^{(3)}_{-3} = \sqrt{\frac{3}{8}}\, T_r{}^{(3)}_{2} - \sqrt{\frac{5}{8}}\, T_r{}^{(3)}_{0}$	$H_r{}^{(3)}_{3} = \sqrt{\frac{1}{16}}\, T_r{}^{(3)}_{3} - \sqrt{\frac{15}{16}}\, T_r{}^{(3)}_{1}$

(d) Relation between the multipole parameters for two different transversity quantizations (rotation on the normal of angle ϕ_{ba})

$${}_bT_r{}^{(L)}_{0} = {}_aT_r{}^{(L)}_{0} \qquad L = 1,\ 3$$

$${}_bT_r{}^{(L)}_{M} = \cos(M\phi_{ba})\ {}_aT_r{}^{(L)}_{M} - \sin(M\phi_{ba})\ {}_aT_r{}^{(L)}_{-M} \qquad M = 1,\ 2, \ldots L$$

(B2)

$${}_bT_r{}^{(L)}_{-M} = \sin(M\phi_{ba})\ {}_aT_r{}^{(L)}_{M} + \cos(M\phi_{ba})\ {}_aT_r{}^{(L)}_{-M}$$

(B) For B-symmetry, the parameters in this column are zero.

(B2) For B-symmetry, $M = 2$.

4.3. Polarization domain for the B-symmetric spin 3/2 particle.

The positivity of the full (even plus odd) density matrix defines a seven-dimensional polarization domain for the B-symmetric spin 3/2 particle. This domain can be visualized by projections and sections of dimension three or less. We have studied in 3.3 its projection on the three-dimensional space of even polarization : a sphere of radius $\sqrt{1/3}$. Its projection on the orthogonal four-dimensional space of odd polarization is a truncated revolution hypercylinder with spheres of radius $\sqrt{1/3}$ as cross sections and height $2\sqrt{1/3}$. Fig. 4.1 represents the sphere (P) of even polarization, and the projections on the basis (Q) and on the height (R) of the hypercylinder of odd polarization. The unpolarized state is represented there by its projections P_o,Q_o, and R_o . This "isotropy center" can be used as origin of a vector space. Any polarization state can be represented by two three-dimensional vectors $\vec{P}$ and $\vec{Q}$ inside the spheres (P) and (Q) , and a unidimensional vector R inside the segment (R) . The coordinates of these different projections are given in Fig. 4.1 and Table 4.3 as function of the transversity multipole parameters ${}^{T}r^{(L)}_{M}$. This allows to fix the relative orientation of these two spheres in a natural way. Remark that, according to Tables 3.2(d) and 4.2(d), for a rotation of the frame by ψ_{ab} around the normal, the spheres (P) and (Q) rotate by twice this angle around their vertical diameters.

But these three projections do not completely describe the positivity domain, i.e. , for $\vec{P}$, $\vec{Q}$ and R inside these projections, the representative point can be outside the seven-dimensional positivity domain. The supplementary condition indicated in Table 4.3 is that the vector $\vec{R}$ has to be inside a shorter segment R_1R_2 . This segment is obtained for each value of $\vec{P}$ and $\vec{Q}$ by reducing the length of (R) by the lengths

$$d_{\pm} = |\vec{P} \pm \vec{Q}|$$

as indicated in Fig. 4.1(R). Of course this supplementary condition implies that

$$d_+ + d_- \leq 2\sqrt{1/3} \ .$$

So, for a fixed $\vec{P}$, $\vec{Q}$ cannot be any vector inside the sphere (Q) . It has to be inside a revolution cigar-like ellipsoid, which has as a major axis, the diameter of (Q) parallel to $\vec{P}$ and as foci the points $\pm\vec{P}$, as visualized in Fig. 4.1(Q).

The polarization degree is given by the distance of the representative point to the isotropy center in the seven-dimensional polarization space. It is easily obtained by the Pythagoric addition of its three orthogonal projections $|\vec{P}|$, $|\vec{Q}|$, and R . Thus pure states have

$$|\vec{P}| = |\vec{Q}| = |R| = \sqrt{1/3} \ .$$

The last equation imposes d_- or d_+ to be zero, and therefore $\vec{P}$ and $\vec{Q}$ equal or opposite. Thus a pure state must satisfy

$$|\vec{P}| = \sqrt{1/3} \ , \ \vec{Q} = \pm\vec{P} \ , \ R = \pm\sqrt{1/3} \ .$$

It is fixed by the direction of $\vec{P}$ and the sign of R , $\varepsilon = \pm 1$. The direction of $\vec{P}$ can be fixed by the polar and azimuthal angles θ,Φ defined in the standard way by the axis $T_r{}_2^{(2)}$, $T_r{}_{-2}^{(2)}$ and $T_r{}_o^{(2)}$ as axis X, Y, Z . In this transversity quantization, and labelling pure states by their magnetic quantum number we have the expression for the general pure state (up to an arbitrary phase)

$$|\theta,\Phi,\varepsilon > = \cos\frac{\theta}{2}|\varepsilon\,\frac{3}{2} > + \sin\frac{\theta}{2}\,e^{i\varepsilon\Phi}|-\varepsilon\tfrac{1}{2} > \ .$$

These pure states form two connected sets, corresponding to $\varepsilon = +1$ and $\varepsilon = -1$ which are orthogonal. In each set, to be orthogonal, the pure states must have opposite directions of $\vec{P}$, i.e.

$$\theta' = \pi - \theta \ , \ \Phi' = \Phi \pm \pi \ .$$

Any B-symmetric polarization state can be decomposed in four orthogonal B-symmetric pure states, defined by the sign of R , $\varepsilon = \pm 1$, and the direction of their $\vec{P}$ vector:

$$\vec{\pi}(\epsilon,\eta) = \eta(\vec{P} + \epsilon\vec{Q}) \ ,$$

with η an independent sign $\eta = \pm 1$. The four corresponding probabilities are given by

$$\lambda(\epsilon,\eta) = \frac{1+\eta\sqrt{3}\,|\vec{P} + \epsilon\vec{Q}| + \epsilon\sqrt{3}\ R}{4}$$

This decomposition is unique if $|\vec{P}| \neq 0$, $|\vec{Q}| \neq 0$ and $R \neq 0$.

Fig. 4.2 proposes a convenient set of two dimensional diagrams, to plot a point in the seven-dimensional polarization domain. The positivity condition and the polarization degree are visualized in diagrams (P_1), (Q_1) and (R). These diagrams are invariant under rotations of the quantization tetrad around the normal to the reaction plane. On the other hand, diagrams (P_2) and (Q_2) giving the angles ${}^{T}r_{\theta 2}^{(2)}$ and ${}^{T}r_{\theta 2}^{(3)}$ depend completely on the chosen parametrization. It would be therefore worthwhile to measure and to draw these diagrams for s, t and u frames of quantization.

TABLE 4.3. - Positivity conditions for the odd polarization parameters of B-symmetric spin $\frac{3}{2}$ particle

(a) Positivity conditions for transversity parametrization

$$[{}^{T}r^{(3)}_{2}]^2 + [{}^{T}r^{(3)}_{-2}]^2 + [r'^{(1)}_{0}]^2 \leq \frac{1}{3}$$

$$-\sqrt{\frac{1}{3}} + d_{+} \leq r'^{(3)}_{0} \leq \sqrt{\frac{1}{3}} - d_{-}$$

(b) Terminology

$$r'^{(1)}_{0} = \sqrt{\frac{4}{5}}\,{}^{T}r^{(1)}_{0} - \sqrt{\frac{1}{5}}\,{}^{T}r^{(3)}_{0}$$

$$r'^{(3)}_{0} = \sqrt{\frac{1}{5}}\,{}^{T}r^{(1)}_{0} + \sqrt{\frac{4}{5}}\,{}^{T}r^{(3)}_{0}$$

$$d_{\pm} = \sqrt{[{}^{T}r^{(2)}_{2} \pm {}^{T}r^{(3)}_{2}]^2 + [{}^{T}r^{(2)}_{-2} \pm {}^{T}r^{(3)}_{-2}]^2 + [{}^{T}r^{(2)}_{0} \pm r'^{(1)}_{0}]^2}$$

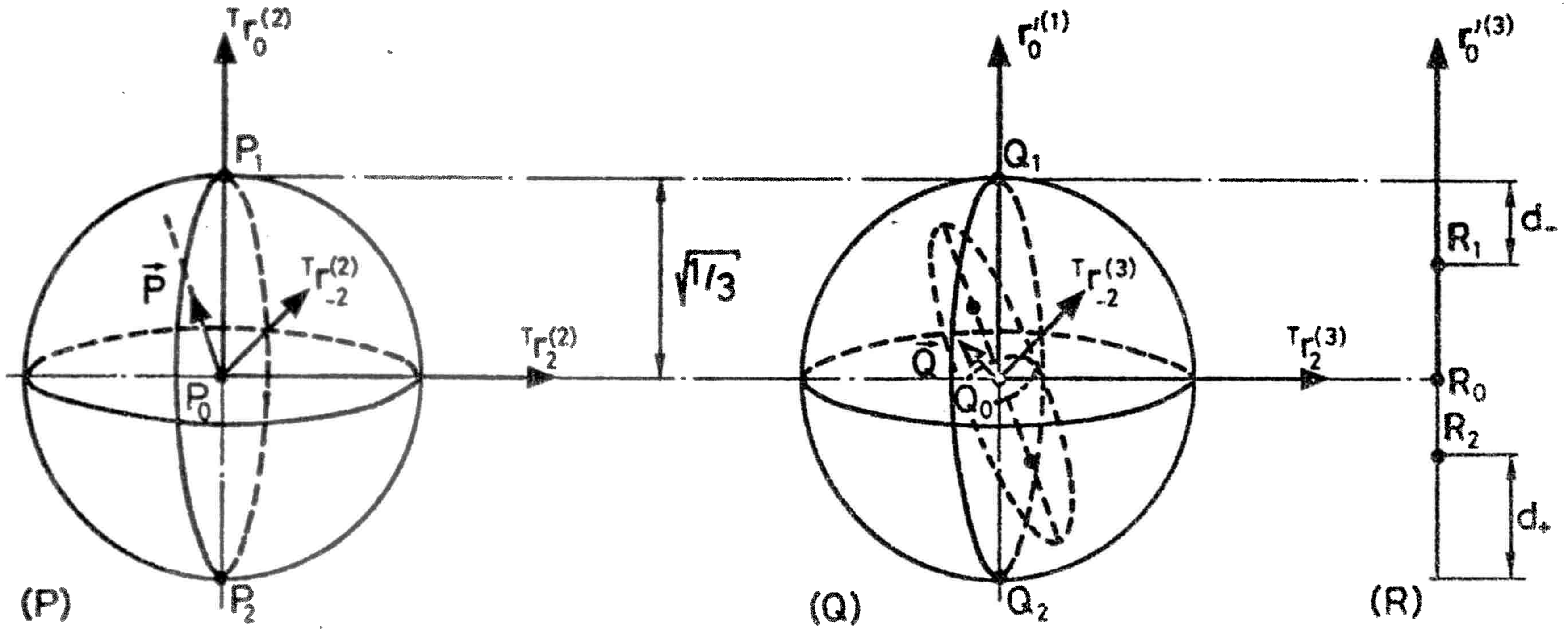

FIG.4.1

Domain of the B-symmetric polarization of spin $\frac{3}{2}$ particle. The sphere (P) is the projection on the space of even polarization. The projection on the space of odd polarization is a hypercylinder of spherical cross section (Q) and height (R). When the even polarization $\vec{P}$ is known, the allowed intersection in (Q) is the ellipsoide with foci $\pm\vec{P}$. For P and Q known the allowed intersection in (R) is the segment R_1R_2 defined by $d_\pm = |\vec{P} \pm \vec{Q}|$. The parameters $r_0'^{(1)}$ and $r_0'^{(3)}$ are defined in Table 4.3.(b).

Diagrams for plotting experimental points inside the polarization domain in FIG.4.1. (P_1) and (Q_1) are meridian section of the spheres (P) and (Q). The points plotted in (P_1), (Q_1) and (R) are invariant under frame rotations around the normal. (P_2) and (Q_2) are vertical projections of the same spheres. The azimuth of the points plotted in (P_2) and (Q_2) are different for s, t, and u frames of quantization.

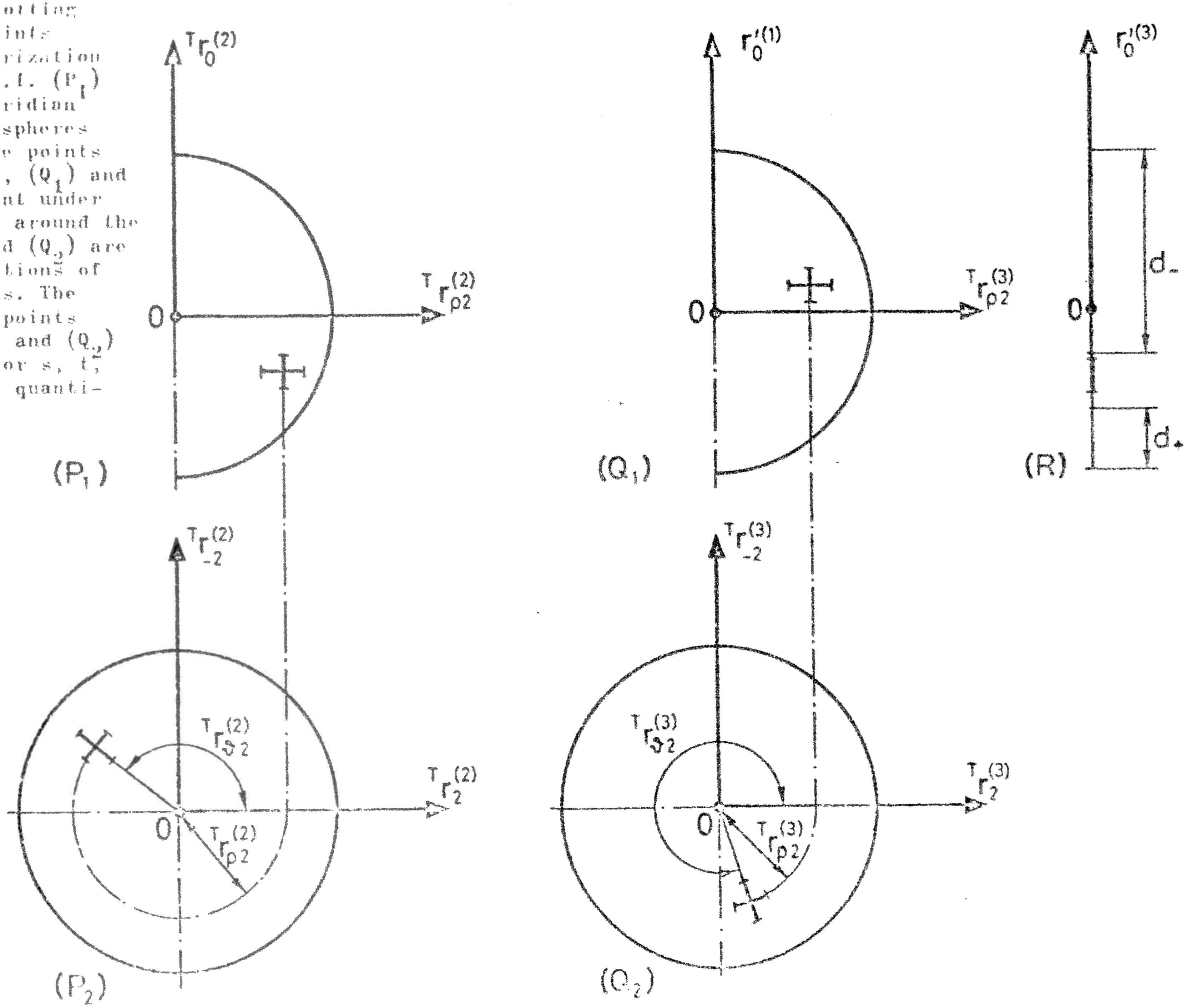

FIG. 4.2

4.4. Rank condition and model predictions on the odd polarization of the spin 3/2 particle.

For reactions of the type

$$0^- \frac{1}{2}^+ \longrightarrow 0^- \frac{3}{2}^+ , \qquad (1)$$

the density matrix of the initial state being of rank 2 , the density matrix of the spin 3/2 particle must be also of rank 2 (cf. Part I, A3). This reduces by two dimensions the seven-dimensional positivity domain. For a given even polarization $\vec{P}$ (cf. Fig. 4.1) the odd polarization $\vec{Q}$ must be in the surface of the ellipsoid on (Q), and R is

$$R = \sqrt{1/3} - |\vec{P}-\vec{Q}| = |\vec{P}+\vec{Q}| - \sqrt{1/3} .$$

Several models predict a null odd polarization for the spin 3/2 particles produced in two-body reactions. This is the case for the Stodolsky-Sakurai model for reactions of type (1), for the models of single particle or trajectory exchange in the t-channel (helicity amplitudes relatively real), and for models imposing the higher symmetry $SU(6)_W$ (with real Clebsch Gordan coefficients) to the t-channel vertex. Then $\vec{Q} = 0$, $R = 0$, and in the case of reaction (1), $|\vec{P}| = \sqrt{1/3}$. The representative point is then $(\vec{P},\vec{Q}_o,R_o)$ with $\vec{P}$ on the surface of (P). It describes a polarization state with a rank 2 density matrix, as expected from I A.1.6 and 7.

5. Polarization of the spin 2 particle.

5.1. Measurement of the even polarization of the spin 2 particle.

The most simple decay of spin 2 particles is the two-body decay into spinless particles, as indicated in Table 5.1(a_1). The other common decay mode of spin 2^+ particles is into one vector and one pseudoscalar particles, as indicated in Table 5.1(a_2). In both cases only one decay amplitude, corresponding to the d-wave, is allowed by angular momentum or angular momentum and parity conservation The decay angular distribution depends only on the even multipole parameters $t^{(2)}_M$ and $t^{(4)}_M$, and has the form given in Table 5.1(b_1) and (b_2) for the decay modes (a_1) and (a_2) respectively (cf. Part I, A4). The inverse expressions, which supply a method of independent measurement for each multipole parameter, are given also in Table 5.1(c_1) and (c_2) for the decay modes (a_1) and (a_2) respectively. The angular brackets < ... > indicate experimental mean values of the enclosed expression for the ensemble of events. $Y^{(2)}_M$ and $Y^{(4)}_M$ are the usual spherical harmonics, given in A4. Their arguments θ,φ are the polar and azimuthal angles, fixing for each event the direction of any of the decay products, in any frame in which the spin 2 particle is at rest. The relation between the real, orthonormalized multipole parameters $r^{(L)}_M$, and the standard ones $t^{(L)}_M$, are explicitely given in Table 5.1(d).

In the case of B-symmetry for the production process of the spin 2 particle (i.e., production in a parity conserving reaction with unpolarized target and beam, cf. Part I, A3), 6 of these 14 even polarization parameters must be zero (see Table 0.1(d)). For transversity quantization (i.e., quantization along the normal to the production plane) the 6 multipole parameters $r^{(L)}_M$ with M odd must be zero, while for helicity quantization (i.e., quantization along any direction inside the production plane) the 6 multipole parameters $r^{(L)}_M$ with M negative must be zero, as indicated in Table 5.1(e) and (f)

We recall that in these decay modes of the spin 2 particle the 10 odd polarization parameters, $r_M^{(1)}$ and $r_M^{(3)}$, cannot be measured since they do not appear in the expression of the angular distributions in Table 5.1(b_1) and (b_2). They can neither be measured in the case of the decay mode (a_2), by observing the angular distribution of the even polarization of the vector particle, the only polarization that can be normally observed. In the case of B-symmetric production 6 of these 10 parameters must be zero (see Table 0.1(d)), but the other 4 are not necessarily zero and cannot be measured. They have to be practically considered as "ghost parameters". Only rare cascade decay modes (e.g., $A_2 \rightarrow \rho^o \pi$ with $\rho^o \rightarrow \mu^+ \mu^-$) or accurate correlation measurements in some two body reactions (e.g., $K^- p \rightarrow f^o \Lambda$) at fixed scattering angle, could supply information about this odd polarization of spin 2 particles.

TABLE 5.1. - Measurement of the even polarization of spin 2 particle

Decays

(a$_1$) $2 \rightarrow 0 + 0$

(a$_2$) $2^+ \rightarrow 1^- + 0^-$ (strong interaction)

Angular distributions

(b$_1$) $$I(\theta,\varphi) = \frac{1}{4\pi} - \sqrt{\frac{1}{14\pi}}\left[\sqrt{5}\sum_{M=-2}^{+2} \bar{t}^{(2)}_M Y^{(2)}_M - 3\sum_{M=-4}^{+4} \bar{t}^{(4)}_M Y^{(4)}_M\right]$$

(b$_2$) $$I(\theta,\varphi) = \frac{1}{4\pi} - \sqrt{\frac{1}{56\pi}}\left[\sqrt{5}\sum_{M=-2}^{+2} \bar{t}^{(2)}_M Y^{(2)}_M + 4\sum_{M=-4}^{+4} \bar{t}^{(4)}_M Y^{(4)}_M\right]$$

Multipole parameters

(c$_1$) $t^{(2)}_M = -\sqrt{\frac{14\pi}{5}} < Y^{(2)}_M >$ $\qquad t^{(4)}_M = \sqrt{\frac{14\pi}{9}} < Y^{(4)}_M >$

(c$_2$) $t^{(2)}_M = -\sqrt{\frac{56\pi}{5}} < Y^{(2)}_M >$ $\qquad t^{(4)}_M = -\sqrt{\frac{7\pi}{2}} < Y^{(4)}_M >$

$M = -2, \ldots +2$ $\qquad M = -4, \ldots +4$

(d) $r^{(L)}_0 = \sqrt{\frac{2L+1}{4}}\; t^{(L)}_0$ $\qquad L = 2,\ 4$

$r^{(L)}_M = (-1)^M \sqrt{\frac{2L+1}{2}}\; \mathrm{Re}\; t^{(L)}_M$ $\qquad M = 1,\ 2 \ldots L$

$r^{(L)}_{-M} = (-1)^M \sqrt{\frac{2L+1}{2}}\; \mathrm{Im}\; t^{(L)}_M$

Condition of B-symmetry in the production process

(e) For transversity quantization

$$T_r{}^{(2)}_1 = T_r{}^{(2)}_{-1} = T_r{}^{(4)}_1 = T_r{}^{(4)}_{-1} = T_r{}^{(4)}_3 = T_r{}^{(4)}_{-3} = 0$$

(f) For helicity quantization

$$H_r{}^{(2)}_{-1} = H_r{}^{(2)}_{-2} = H_r{}^{(4)}_{-1} = H_r{}^{(4)}_{-2} = H_r{}^{(4)}_{-3} = H_r{}^{(4)}_{-4} = 0$$

5.2. Relation between different even polarization parameters of the spin 2 particle.

For any one reference frame, the even part of the density matrix of the spin 2 particle has the form indicated in Table 5.2(a) (cf. Part I, A6). The very elements of this even density matrix can be used as polarization parameters. They are related to the measurable multipole parameters of Table 5.1 by the relations given in Table 5.2(b).

For each particle, several pairs of reference frames for transversity and helicity quantization can be intrinsically defined. Each pair of frames is fixed by the normal to the production three-plane and some space or time-like direction (like the four-momentum transfer, which is associated with the s , t , or u channel of a two-body reaction, cf. Part I, A1). Associated transversity and helicity frames are simply related in the standard conventions by a rotation of $\frac{\pi}{2}$ radians around the x-axis. The corresponding linear transformation on the multipole parameters is given in Table 5.2(c), where the left T and H superscripts refer to transversity and helicity quantizations.

Two different pairs of transversity-helicity frames, say a and b , are related through a rotation around the normal by an angle ψ_{ba} (which usually will be a complicated function of the kinematical invariants s , t , u of a two-body reaction, cf. Part I, A1). The corresponding linear transformation on the multipole parameters is very simple for transversity quantization. It is also given in Table 5.2(d), where the left subscripts, a , b , label any transversity frames.

As has been mentioned, in the case of B-symmetric production the transversity multipole parameters with M odd or helicity multipole parameters with M negative must be zero. They are the parameters written in the last lines of Table 5.2(c), and in the last column or three last lines of Table 5.2(b). Note that in this case for transversity quantization the matrix elements ρ_{10} , ρ_{21} , and ρ_{2-1} are zero, and the density matrix in Table 5.2(a) has a "checker-board pattern" , while for helicity quantization the imaginary parts of ρ_{10} , ρ_{21} , ρ_{1-1} , ρ_{20} , ρ_{2-1} and ρ_{2-2} are zero, and the density matrix is real.

TABLE 5.2. - Relation between different even polarization parameters of spin 2 particle

(a) Even part of the density matrix for spin 2 particle

$$\rho = \begin{vmatrix} \rho_{22} & \rho_{21} & \rho_{20} & \rho_{2-1} & \rho_{2-2} \\ \bar{\rho}_{21} & \rho_{11} & \rho_{10} & \rho_{1-1} & -\rho_{2-1} \\ \bar{\rho}_{20} & \bar{\rho}_{10} & \rho_{00} & -\rho_{10} & \rho_{20} \\ \bar{\rho}_{2-1} & \bar{\rho}_{1-1} & -\bar{\rho}_{10} & \rho_{11} & -\rho_{21} \\ \bar{\rho}_{2-2} & -\bar{\rho}_{2-1} & \bar{\rho}_{20} & -\bar{\rho}_{21} & \rho_{22} \end{vmatrix} \qquad \rho_{00} = 1 - 2(\rho_{22} + \rho_{11})$$

(b) Relation between the matrix elements in (a) and the multipole parameters in Table 4.1

$$\begin{array}{ll:l}
 & r^{(2)}_{0} = \sqrt{\frac{5}{14}}\,(2\rho_{22} - \rho_{11} - \rho_{00}) & \\
 & r^{(4)}_{0} = \sqrt{\frac{1}{14}}\,(\rho_{22} - 4\rho_{11} + 3\rho_{00}) & \text{(BH)} \\
 & r^{(2)}_{2} = \sqrt{\frac{20}{7}}\,\mathrm{Re}\,\rho_{20} + \sqrt{\frac{15}{14}}\,\mathrm{Re}\,\rho_{1-1} & r^{(2)}_{-2} = -\sqrt{\frac{20}{7}}\,\mathrm{Im}\,\rho_{20} - \sqrt{\frac{15}{14}}\,\mathrm{Im}\,\rho_{1-1} \\
 & r^{(4)}_{2} = \sqrt{\frac{15}{7}}\,\mathrm{Re}\,\rho_{20} - \sqrt{\frac{10}{7}}\,\mathrm{Re}\,\rho_{1-1} & r^{(4)}_{-2} = -\sqrt{\frac{15}{7}}\,\mathrm{Im}\,\rho_{20} + \sqrt{\frac{10}{7}}\,\mathrm{Im}\,\rho_{1-1} \\
 & r^{(4)}_{4} = \sqrt{\frac{5}{2}}\,\mathrm{Re}\,\rho_{2-2} & r^{(4)}_{-4} = -\sqrt{\frac{5}{2}}\,\mathrm{Im}\,\rho_{2-2} \\
\hdashline
 & r^{(2)}_{1} = \sqrt{\frac{30}{7}}\,\mathrm{Re}\,\rho_{21} + \sqrt{\frac{5}{7}}\,\mathrm{Re}\,\rho_{10} & r^{(2)}_{-1} = -\sqrt{\frac{30}{7}}\,\mathrm{Im}\,\rho_{21} - \sqrt{\frac{5}{7}}\,\mathrm{Im}\,\rho_{10} \\
\text{(BT)} & r^{(4)}_{1} = \sqrt{\frac{5}{7}}\,\mathrm{Re}\,\rho_{21} - \sqrt{\frac{30}{7}}\,\mathrm{Re}\,\rho_{10} & r^{(4)}_{-1} = -\sqrt{\frac{5}{7}}\,\mathrm{Im}\,\rho_{21} + \sqrt{\frac{30}{7}}\,\mathrm{Im}\,\rho_{10} \\
 & r^{(4)}_{3} = \sqrt{5}\,\mathrm{Re}\,\rho_{2-1} & r^{(4)}_{-3} = -\sqrt{5}\,\mathrm{Im}\,\rho_{2-1}
\end{array}$$

(BH) For B-symmetry and helicity quantization, the parameters in this column are zero.

(BT) For B-symmetry and transversity quantization, the parameters in these three lines are zero.

(Continuation of Table 5.2)

(c) Relation between the multipole parameters for transversity and helicity quantizations

$$H_r(2)_0 = -\sqrt{\frac{1}{4}}\; T_r(2)_0 - \sqrt{\frac{3}{4}}\; T_r(2)_2$$

$$H_r(2)_2 = -\sqrt{\frac{3}{4}}\; T_r(2)_0 + \sqrt{\frac{1}{4}}\; T_r(2)_2$$

$$H_r(2)_1 = -\; T_r(2)_{-2}$$

$$H_r(4)_0 = \sqrt{\frac{9}{64}}\; T_r(4)_0 + \sqrt{\frac{5}{16}}\; T_r(4)_2 + \sqrt{\frac{35}{64}}\; T_r(4)_4$$

$$H_r(4)_2 = \sqrt{\frac{5}{16}}\; T_r(4)_0 + \sqrt{\frac{1}{4}}\; T_r(4)_2 - \sqrt{\frac{7}{16}}\; T_r(4)_4$$

$$H_r(4)_4 = \sqrt{\frac{35}{64}}\; T_r(4)_0 - \sqrt{\frac{7}{16}}\; T_r(4)_2 + \sqrt{\frac{1}{64}}\; T_r(4)_4$$

$$H_r(4)_1 = \sqrt{\frac{1}{8}}\; T_r(4)_{-2} + \sqrt{\frac{7}{8}}\; T_r(4)_{-4}$$

$$H_r(4)_3 = \sqrt{\frac{7}{8}}\; T_r(4)_{-2} - \sqrt{\frac{1}{8}}\; T_r(4)_{-4}$$

(B)

$$H_r(2)_{-2} = T_r(2)_1$$

$$H_r(2)_{-1} = -\; T_r(2)_{-1}$$

$$H_r(4)_{-2} = -\sqrt{\frac{1}{8}}\; T_r(4)_1 - \sqrt{\frac{7}{8}}\; T_r(4)_3$$

$$H_r(4)_{-4} = -\sqrt{\frac{7}{8}}\; T_r(4)_1 + \sqrt{\frac{1}{8}}\; T_r(4)_3$$

$$H_r(4)_{-1} = \sqrt{\frac{9}{16}}\; T_r(4)_{-1} + \sqrt{\frac{7}{16}}\; T_r(4)_{-3}$$

$$H_r(4)_{-3} = \sqrt{\frac{7}{16}}\; T_r(4)_{-1} - \sqrt{\frac{9}{16}}\; T_r(4)_{-3}$$

(d) Relation between the multipole parameters for two different transversity quantizations (rotation on the normal of angle ϕ_{ba})

$$ {}_bT_r(L)_0 = {}_aT_r(L)_0 \qquad L = 2,\ 4$$

$$ {}_bT_r(L)_M = \cos(M\phi_{ba})\; {}_aT_r(L)_M - \sin(M\phi_{ba})\; {}_aT_r(L)_{-M} \qquad M = 1,2,\ldots L$$

$$ {}_bT_r(L)_{-M} = \sin(M\phi_{ba})\; {}_aT_r(L)_M + \cos(M\phi_{ba})\; {}_aT_r(L)_{-M} \qquad \text{(BE)}$$

(B) For B-symmetry, the following parameters are zero.

(BE) For B-symmetry, M = 2, 4.