INVARIANT FORMULATION FOR THE ZEROS OF COVARIANT VECTOR FIELDS

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ABSTRACT

Invariant formulation for the zeros of covariant vector fields is presented. It is shown that they can be determined at each stratum from certain cannonical equations in terms of invariants and the field components relative to a covariant basis.

1. Introduction

Let G be a finite group acting orthogonally on a carrier space $\mathbb{R}^n$ and let $f: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be a G-covariant vector field,

$$f(gx) = gf(x), \ \forall g \in G, \ \forall x \in \mathbb{R}^n . \tag{1}$$

Such fields and their zeros play an important role in many areas of physics. Determination of the zeros directly in the carrier space is pursued in another paper in the same proceedings[1]. An invariant, orbit space, approach will be sketched here. More detailed study will be presented elsewhere[2].

2. Orbit space approach

It is well known[3] that there is a finite G-covariant polynomial basis $e_s(x)$ such that every G-covariant polynomial field f(x) can be uniquely decomposed

$$f(x) = \sum_s q_s(\theta) e_s(x), \tag{2}$$

where q_s (θ) are polynomials in denominator invariants $\theta(x)$. (θ (x) are n algebraically independent G-invariant polynomials).

It is also well known[4] that at a particular stratum $\Sigma[L]$ (associated with a class $[L]$ of isotropy subgroups of G) a G-covariant vector field is tangential to the stratum. Therefore, we determine at each $\Sigma[L]$ the minimal set of basic G-covariant fields (say e'_t, $t=1,\ldots,\dim\Sigma[L]$) which are linearly independent at $\Sigma[L]$. The zeros of Eq. (2) can then be obtained from the equations of the stratum[5] and from the equations

$$f\cdot e'_t = \sum_s q_s(\theta)(e_s \cdot e'_t) = 0. \tag{3}$$

In all of these equations x-dependence is only implicite through denominator and numerator invariants (integrity basis). Thus, zeros of f(x) may be determined directly in the orbit space.

The basic fields e'_t and the equations (3) have been determined for all strata of all (finite and infinite) two- and three-dimensional point groups[2].

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