"Structure of the Images of the Irreducible Linear Representations of the Crystallographic Little Space Groups"

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Many problems in solid state physics require the use of the irreps (= irreducible linear representations) of the space groups G_j (j = 1 to 230). A representation of a group is determined by its kernel and its image. Most of its properties depend only on the image (e.g. the invariants). An irrep of G_j is characterized by a wave vector k and is induced from an irrep of $G_{j,k}$, the little group of k in G_j. There is an infinite set of irreps of the $G_{j,k}$'s. We have established that the image of each one is generated by one of only 25 images of irreps of a Schur covering $S(P_k)$ of the little point group P_k and a set of phases forming a U(1) subgroup. In the most physically important cases of finite image, these phases form a cyclic group of order n, this integer depending on k. We have characterized these 25 images.

We can also say that the images of the irreps of $G_{j,k}$ are also the images of the irreps of P(j,k), central extensions of P_k by Imk (i.e., $P_k = P(j,k)/Imk$, and Imk ⊂ center of P(j,k)) where k maps the translation subgroup τ of G_j into U(1) $t \mapsto e^{-ik\cdot t}$. Note that P_k and Imk are independent of "j" in any of the 73 arithmetic classes and we proved that they are so related:

$P_k = C_i, C_{2h}, C_{4h}, D_{2h}, S_6, C_{6h}, D_{3d}, D_{4h}, D_6, D_{6h}, T_h, O, O_h$	Imk = {1}, Z_2
$P_k = D_2, D_{2d}, S_4, D_4, T, T_d$	Imk = {1}, Z_2 or Z_4
$P_k = D_3, C_{3h}, D_{3h}$	Imk = {1}, Z_2, Z_3 or Z_6

P_k =	C_6, C_{6v}	C_3, C_{3v}	C_2, C_{2v}, C_4, C_{4v}	C_s	C_1
Imk =	Z, Z_m	ZxZ_3, Z, Z_m	ZxZ_2, Z, Z_m	Z^2xZ_2, Z^2, ZxZ_m, Z_m	Z^3, Z^2xZ_m, ZxZ_m, Z_m m≥1

The G_j's are extensions corresponding to the cohomology group $H^2_\Delta(P,\tau)$, $P \xrightarrow{\Delta} \text{Aut } \tau$ similarly the $G_{j,k}$'s correspond to $H^2_{\Delta'}(P_k,\tau)$ where $\Delta' = \Delta|_{P_k}$ and the P(j,k) correspond to $H^2_o(P_k,Imk)$ where o means trivial action. The P(j,k) are defined by the functorial homomorphisms: $H^2_\Delta(P,\tau) \xrightarrow{a_k} H^2_{\Delta'}(P_k,\tau) \xrightarrow{k} H^2_o(P_k,Imk)$. For the 10 cyclic P_k's the P(j,k) are Abelian. For $P_k = C_1, C_i, S_4, S_6, C_{3h}, D_3$, P(j,k) = Imk x P_k. Finally all P(j,k) are of the form A(j,k) x Q(j,k) where A(j,k) is Abelian. We will give in a forthcoming publication the list of the 27 (?) nonisomorphic Q(j,k) and of all Abelian A(j,k)'s.