The present status of CP, T and CPT invariance

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Last time I came to Sweden, cars were driven on the left side of the roads. This week most cars I saw were driven on the right side. I am very grateful to the Nobel Foundation for this opportunity to observe this *P* violation. I may have found the cause of this *P* violation by looking at buses or trucks with the same inscription on both sides; they are not symmetrically written. The first word is near the front on the left side and near the rear on the right side of the car.

I have also made a counter-experiment to verify this theory. In Tokyo I found an equal probability for cars to be driven on the left side or the right side of the streets and I did observe that identical texts were placed symmetrically on buses and trucks: The first character is near the front part on both sides so the same text must be read from left to right on one side and from right to left on the other side.

Since I have never been yet in a car undergoing a spontaneous *CP* transformation, I will be able to give you today my report on *CP*, *T* and *CPT* invariance.

1. *History*

The invariance under the Poincaré group is the corner-stone of the present analysis of high energy experiments (energy-momentum balance, mass, spin and parity determination of resonances, polarization measurements, etc.). Dirac's equation (1) has been extremely useful and together with Maxwell's equations (the first relativistic equations!) it developed into the very successful theory of quantum electrodynamics.

The main paper on fundamental aspects of particle kinematics is that of Wigner (2) on the up-to-a-factor unitary irreducible representations of the Poincaré group (i.e. the inhomogeneous Lorentz group) (3). We denote by $\mathcal{P}$ the connected component of this group.

If a theory is invariant under a group $\mathcal{G}$, then Aut $\mathcal{G}$, that is the automorphism group of $\mathcal{G}$, by its action on $\mathcal{G}$, its invariants, covariants and linear representations, etc. transforms the theory into other theories. Are those theories physically equivalent or inequivalent? In the former case Aut $\mathcal{G}$ is an invariance group of the physical theory.

As you well know, Aut $\mathcal{P}$ is generated by $\mathcal{P}$, the space reflexion P, the time reflexion T and the space-time dilatations D. Only theories for mass zero particles are D-invariant; so dilatations are simply "passive" symmetries in physics (4).

Invariance under P and T for quantum theories were both emphasized and first studied by Wigner (5). T invariance corresponds to reversal of motion and is represented by anti-unitary operators.

I will also speak of C, the particle-antiparticle conjugation, an internal symmetry discovered by Dirac, who predicted the positive electron (6). C is not a geometrical symmetry but it is intimately related to T invariance by the CPT theorem (7).

Proposed by Lee & Yang (8), P violation was verified (9) in January 1957. It was soon realized that P violations were related to the existence of two-component neutrinos (10). These particles seem to exist only in a totally circularly polarized state so that P or C mirror states do not occur but existing states are related by CP transformation from each other. "Maximal" CP violation is well understood in processes involving leptons while the situation is not so clear for purely hadronic processes.

It was in such a hadronic decay $K_L^0 \rightarrow \pi^+\pi^-$ that CP violation was unexpectedly discovered nearly four years ago (11). Since then a frantic search for new effects of C, CP, T violation is going on. None have been found outside K^0-decays.

2. *Can we define P, C, T?*

Of course we can speak of C, P, T because we can define these operations in particle physics. I do not want it to happen to you what happened to me once: to hear a one hour lecture on C and P and T violations with the remark at the end that these operations cannot be defined. Since P and T act on energy, momentum and polarization, P and T violations can be observed. Of course one unique experiment is not enough to decide if the observed P violation is intrinsic to the observed phenomenon or occurs in the interaction with the "observing" apparatus (which has to use physical interactions!). But from the many different experiments performed over the last ten years, we can say that, in the approximation where we can disentangle strong, electromagnetic, weak and gravitational effects in a physical process, no violation of P has been observed outside weak couplings. Four recent remarkable Soviet experiments using kilocuries of radioactive nuclei have shown that γ's emitted spontaneously from nuclei are slightly circularly polarized. This should be expected from what we know from weak interaction and these experiments will be precious for knowing the relative amount of $\Delta I=0$ and $\Delta I=1$ weak interaction between nucleons (12).

C is not a geometrical operation; it does not act on energy, momentum or angular momentum. But, by definition of C as introduced historically by physicists and as such used in this report, C anti-commutes with all conserved charges. So the electron and positron are C conjugated. Hypercharge Y is preserved by strong and electromagnetic interactions. In the excellent approximation with weak interaction radiative corrections being neglected, K^0 and $\bar{K}^0$ are C conjugated when produced in the reactions $K^+ + n \rightarrow K^0 + p$ and $K^- + p \rightarrow \bar{K}^0 + n$. As you know this is no longer true after the production because neither Y nor C commute with the time evolution operator.

3. *Evolution of a K^0-beam*

We will now review the K^0 decay experimental situation. Elegant experiments (13) on interference between $K_L^0 \rightarrow \pi^+\pi^-$ and regenerated $K_S^0 \rightarrow \pi^+\pi^-$ exclude the possibility that the rare $\pi^+\pi^-$ decay mode comes from another particle (resulting from a cascade of decays from K^0, for instance) and seem to show that the Weisskopf–Wigner (14) approximation yields a good description of the temporal evolution of a K^0-beam (15). (*Note.* Historically a clear picture of this evolution has been obtained as the sum of efforts by many people (15*a*, *b*, *c*, *d*). Consequences of P, C, T violations for a K^0-beam were neatly studied by Lee, Oehme & Yang (15*e*) assuming CPT invariance and by Sachs (15*f*) in the most general case. (See also (15*f*) for a complete list of earlier references.) After the discovery of a CP violation in $K_L \rightarrow \pi^+\pi^-$ several phenomenological analyses have appeared (16*a*, *b*, *c*, *d*, *e*) of the K^0-beam time evolution. It is strange that (16*b*)—in no way inferior to the others—is never quoted, not even in the most complete review (16*f*) of K^0-beam physics. For a more complete numerical analysis, see (16*g*, *h*, *i*). However, to read these nine papers is disheartening for the following reasons: they all used different notations; some of them have an extremely cumbersome way of handling 2×2 matrices and they all introduce non-observable quantities and then make arbitrary, unnecessary—and sometimes inexplicit—phase assumptions. So I felt it necessary to assume no knowledge of these papers for the understanding of my report). K^0-states are vectors of $\mathcal{H} \otimes \mathcal{H}_2$ where $\mathcal{H}$ is the Hilbert space of spin zero one particle states and $\mathcal{H}_2$ is the two-dimensional Hilbert space corresponding to the hypercharge coordinate of the K-meson. To simplify, one can just consider the projection of the linear operators of $\mathcal{H} \otimes \mathcal{H}_2$ on those of $\mathcal{H}_2$. Then we have to study the equation:

$$i\frac{d}{dt}|\varkappa\rangle = Z|\varkappa\rangle \tag{3.1}$$

where Z is a 2×2 matrix which depends on the medium (vacuum, matter) and the K-meson wave number.

We can decompose Z into hermitean parts:

$$Z = M - i\frac{\Gamma}{2}, \quad \text{with } M = \tfrac{1}{2}(Z^* + Z), \quad \tfrac{1}{2}\Gamma = \frac{1}{2i}(Z^* - Z) \tag{3.2}$$

The positivity of the energy corresponds to $M \geqslant 0$ while the requirement that (3.1) describes absorption or decay phenomena is

$$\Gamma > 0 \tag{3.2'}$$

The case of interest for us is the general case where Z has two distinct eigenvalues (such that $M > 0$, $\Gamma > 0$)

$$z' = m' - i\gamma'/2, \quad z'' = m'' - i\gamma''/2; \quad m', m'', \gamma', \gamma'' > 0 \tag{3.3}$$

Let S and L be the hermitian projectors on the corresponding eigenvectors

$$S^2 = S = S^* \quad L^2 = L = L^* \tag{3.4}$$

$$ZS = z'S \quad ZL = z''L \tag{3.4'}$$

Let us assume first the general case where Z is not a normal operator

$$[Z^*, Z] = i[M, \Gamma] \neq 0 \tag{3.5}$$

This means that $SL \neq 0 \neq LS$ and from the positivity of S and L and the Schwartz' inequality one finds that

$$\mathrm{Tr}\, SL = \sigma^2 = \mathrm{Tr}\, LS, \quad 0 < \sigma < 1 \tag{3.5'}$$

(Note that in that case S and L do not project on eigenvectors of M and Γ and m', m'' and γ', γ'' are not eigenvalues of M and Γ respectively. However, we always have $\mathrm{Tr}\, MS = m'$, $\mathrm{Tr}\, ML = m''$, $\mathrm{Tr}\, \Gamma S = \gamma'$, $\mathrm{Tr}\, \Gamma L = \gamma''$.)

It is easy to compute

$$I = (1-\sigma^2)^{-1}(S + L - SL - LS) \tag{3.6}$$

$$Z = (1-\sigma^2)^{-1}[z'S(I-L) + z''L(I-S)] = z'SS' + z''LL' \tag{3.6'}$$

where SS' and LL' are non-hermitian commuting projectors of sum I

$$(SS')^2 = SS', \; (LL')^2 = LL', \; SS'LL' = 0 = LL'SS', \; SS' + LL' = I \tag{3.7}$$

So, for instance

$$e^{-iZt} = e^{-iz't}SS' + e^{-iz''t}LL' \tag{3.8}$$

Note also that

$$\Gamma=(1-\sigma^2)^{-1}[\gamma' S+\gamma'' L-\gamma(LSe^{i\varphi}+SLe^{-i\varphi})] \tag{3.9}$$

where

$$\gamma>0, \quad \gamma e^{i\varphi}=\tfrac{1}{2}(\gamma'+\gamma'')+i\mu, \quad \mu=m''-m' \tag{3.10}$$

$$\Gamma>0 \Leftrightarrow \sigma\gamma<(\gamma'\gamma'')^{\frac{1}{2}} \tag{3.10'}$$

Let $R_i(0)$ be the density matrix of a state at time $t=0$

$$R_i(0)=\alpha_i' S+\alpha_i'' L+\frac{\alpha_i}{\sigma}(e^{i\varphi_i}SL+e^{-i\varphi_i}LS) \tag{3.11}$$

$$R_i^*(0)=R_i(0)>0 \Rightarrow \alpha_i'\geqslant 0, \quad \alpha_i''\geqslant 0, \quad 0\leqslant\alpha_i\leqslant(\alpha_i'\alpha_i'')^{\frac{1}{2}} \tag{3.11'}$$

with the last equality for a pure state:

$$\alpha_i=(\alpha_i'\alpha_i'')^{\frac{1}{2}} \tag{3.11''}$$

$$\mathrm{Tr}\, R_i(0)=1 \quad \alpha_i'+\alpha_i''+2\alpha_i\sigma\cos\varphi_i=1 \tag{3.11'''}$$

At time t the state has developed into (see eq. (3.1))

$$\begin{aligned} R_i(t)&=e^{-iZt}R_i(0)\,e^{iZ^*t}\\ &=e^{-\gamma' t}\alpha_i' S+e^{-\gamma'' t}\alpha_i'' L+e^{-\frac{1}{2}(\gamma'+\gamma'')t}\alpha_i\sigma^{-1}(e^{i(\mu t+\varphi_i)}SL+e^{-i(\mu t+\varphi_i)}LS) \end{aligned} \tag{3.12}$$

An observation at time t of the observable $A=A^*$ yields the expectation value $\mathrm{Tr}R(t)A$. For instance, the probability density for the initial state $R(0)$ to disappear (by absorption or decay) at time t is

$$\mathrm{Tr}R_i(t)\Gamma=e^{-\gamma' t}\alpha_i'\gamma'+e^{-\gamma'' t}\alpha_i''\gamma''+2e^{-\frac{1}{2}(\gamma'+\gamma'')t}\alpha_i\gamma\sigma\cos(\mu t+\varphi_i-\varphi) \tag{3.13}$$

(*Note.* The conservation of probability requires $\int_0^\infty \mathrm{Tr}\, R(t)\Gamma dt=1$ for all $R(0)$ which satisfy (3.11′) and (3.11‴). This is equivalent to $\int_0^\infty e^{iZ^*t}\Gamma e^{-iZt}dt=I$ which is equivalent to $\Gamma>0$.)

4. K^0-*decay*

Since we are interested in the spontaneous decay of K^0 in vacuum, from now on we shall assume that Z governs the evolution of a K^0-beam in *vacuum*. All considerations refer to the K *rest-frame*. So m', m'' and γ', γ'' are respectively the mass and inverse lifetime of the short-lived and long-lived neutral K-mesons. Γ is the sum of the contribution for all decay modes k. We can write

$$\Gamma=\sum\Gamma_k, \quad \Gamma_k=\int\Gamma_k(\omega)\,d\omega_k>0 \tag{4.1}$$

Table 1. *List of all observables in spontaneous decays of a neutral K-beam*

In order to simplify the writing, we have used the integrated Γ_k instead of the density $\Gamma_k(\omega)$

For all decays	γ', γ'', μ	
For a given initial state	$(\alpha_i', \alpha_i'', \alpha_i) \geqslant 0, \varphi_i$	
For a given decay mode	$(\gamma_k', \gamma_k'', \gamma_k) \geqslant 0, \varphi_k$	
After all possible experiments:	$0 \leqslant \sigma < 1$	
These quantities satisfy	$\alpha_i^2 \leqslant \alpha_i' \alpha_i''$	(3.11′)
	$\alpha_i' + \alpha_i'' + 2\alpha_i \sigma \cos \varphi_i = 1$	(3.11‴)
$\gamma' = \sum \gamma_k', \quad \gamma'' = \sum \gamma_k'', \quad \sigma\left(\frac{\gamma'+\gamma''}{2} + i\mu\right) = \sum \gamma_k e^{i\varphi_k}$; $\gamma_k^2 \leqslant \gamma_k' \gamma_k'' \Leftrightarrow \Gamma_k \geqslant 0$		(4.2)

where the integral with positive measure $d\omega_k$ corresponds to phase space and, maybe, spin summation (e.g. 2γ decay).

We define

$$\operatorname{Tr} \Gamma_k(\omega) S = \gamma_k'(\omega), \operatorname{Tr} \Gamma_k(\omega) L = \gamma_k''(\omega), \operatorname{Tr} \Gamma_k(\omega) LS = \sigma_k(\omega) e^{i\varphi_k(\omega)} \tag{4.2}$$

and similarly

$$\operatorname{Tr} \Gamma_k S = \gamma_k' = \int \gamma_k'(\omega)\, d\omega_k, \operatorname{Tr} \Gamma_k L = \gamma_k'', \operatorname{Tr} \Gamma_k LS = \sigma \gamma_k e^{i\varphi_k} \tag{4.3}$$

Then the probability density for the state $R_i(0)$ to decay at time t into the decay mode k is

$$\operatorname{Tr} R_i(t) \Gamma_k = e^{-\gamma' t} \alpha_i' \gamma_k' + e^{-\gamma'' t} \alpha_i'' \gamma_k'' + 2e^{-\frac{1}{2}(\gamma'+\gamma'')t} \alpha_i \gamma_k \cos(\mu t + \varphi_i - \varphi_k) \tag{4.4}$$

The list of all observables which can be measured in spontaneous decays (after phase space integration) is given in Table 1.

Note that the condition $\Gamma_k > 0$ for each k (and not only their sum $\Gamma > 0$) gives stricter condition on σ than (3.10′). Indeed,

$$\sigma\gamma = \left|\sum_k \gamma_k e^{i\varphi_k}\right| \leqslant \sum_k \gamma_k \leqslant \sum_k (\gamma_k' \gamma_k'')^{\frac{1}{2}} \tag{4.5}$$

The observables $\alpha_i', \alpha_i'', \alpha_i, \varphi_i, \varphi_k, \sigma$ are pure numbers. The value of the others will be given in the unit system where $\hbar = c = 1 = \gamma'$ (in sec., $\gamma'^{-1} = (.874 \pm .011) \cdot 10^{-10}$). The measured quantities are found in Table 2 and Table 3, this last table for quantities which yield direct evidence of CP violation.

Table 2

Experimental values in unit $\gamma' = 1$ of $\gamma'' = 1.65 \times 10^{-3}$, $\mu = .48 \pm .02$

Decay mode	$\pi^+\pi^-$	$\pi^0\pi^0$	$\pi\mu\nu$	$\pi e\nu$
γ'	$\gamma'_{+-} = .68$	$\gamma'_{00} \simeq .32$	$\gamma'_\mu \sim \gamma''_\mu$?	$\gamma'_e \sim \gamma''_e$
γ''	See Table 3	See Table 3	$\gamma''_\mu = .45 \times 10^{-3}$	$\gamma''_e = .58 \times 10^{-3}$
Decay mode	$\pi^+\pi^-\pi^0$	$3\pi^0$	2γ	
γ'	$\gamma'_{3\pi} \lesssim \gamma''_{3\pi}$	$\gamma'_{3\pi^0} \ll \gamma''_{3\pi^0}$	$\gamma'_{2\gamma} \gtrsim \gamma''_{2\gamma}$	
γ''	$\gamma''_{3\pi} = .18 \times 10^{-3}$	$\gamma''_{3\pi^0} = .41 \times 10^{-3}$	$\gamma''_{2\gamma} \simeq 10^{-6}$	

Note that

$$\gamma_k^2 = \gamma'_k \gamma''_k \quad \text{for} \quad k = \pi^+\pi^-,\ 2\pi^0, \quad \text{or} \quad \pi^+e^-\nu + \pi^-e^+\nu$$

We surely have the inequality $\gamma_k^2 < \gamma'_k \gamma''_k$ for $k = \pi^+\pi^-\pi^0$, 2γ (photons). The inequality is possible for $\pi^\pm\mu^\mp\nu$ or $\pi^\pm e^\mp\nu$ because there is more than one form factor when $\Delta S/\Delta Q = 1$ is violated, while the preservation of this rule implies the equality $\gamma_{l\pm}^2 = \gamma'_{l\pm}\gamma''_{l\pm}$ with $l = \mu$ or e. Since the $3\pi^0$ from a K-decay do not differ much from a state with $CP = -I$, the equality $\gamma_{3\pi_0}^2 = \gamma'_{3\pi^0}\gamma''_{3\pi^0}$ cannot be much violated.

Note that only $\varphi_{+-} - \varphi_0$ where φ_0 is the phase of a K^0-beam, has been measured (18). All papers on the subject agree, at least up to a sign, on the

Table 3. *Summary of experimental data for direct evidence of CP violation*

Put γ_{+-}, γ_{00} for $\gamma_{\pi^+\pi^-}, \gamma_{2\pi^0}$

$$\frac{\gamma''_{+-}}{\gamma'_{+-}} = 3.6 \pm 2 \cdot 10^{-6} \quad (17\,\text{a}) \qquad \Leftrightarrow \sqrt{\frac{\gamma''_{+-}}{\gamma'_{+-}}} = |\eta_{+-}| = (1.89 \pm .09) \cdot 10^{-3} \quad (17\,\text{b})$$

$$\varphi_{+-} - \varphi_0 = 60 \pm 12^0 \quad (17\,\text{a}) \qquad = 65 \pm 11^0 \quad (17\,\text{b}) \qquad = 60 \pm 18^0 \quad (17\,\text{c})$$

$$|\eta_{00}|^2 = \frac{\gamma''_{00}}{\gamma'_{00}} = \begin{cases} 23 \pm 13 \times 10^{-6} & (17\,\text{d}) \\ 24 \pm \ 5 \times 10^{-6} & (17\,\text{e}) \\ 17 \pm \ 3 \times 10^{-6} & (17\,\text{f}) \\ -1 \pm \ 6 \times 10^{-6} & (17\,\text{g}) \end{cases}$$

$$\frac{\gamma''_{\mu+}}{\gamma''_{\mu-}} = 1.0081 \pm .0027 \quad (17\,\text{h}) \qquad \frac{\gamma''_{e+}}{\gamma''_{e-}} = 1.0047 \pm .0007 \quad (17\,\text{k})$$

definition of the observable quantities which are

$$\eta_{+-} = \left(\frac{\gamma''_{+-}}{\gamma'_{+-}}\right)^{\frac{1}{2}} e^{i(\varphi_{+-}-\varphi_0)}, \quad \eta_{00} = \left(\frac{\gamma''_{00}}{\gamma'_{00}}\right)^{\frac{1}{2}} e^{i(\varphi_{00}-\varphi_0)}$$

(*Note.* It is a pity that most experimental papers do not even reproduce the rate equation corresponding to (4.4) that they use, in order to specify the sign of the definition of the angles φ_i and φ_k. They prefer to give it in terms of phases of amplitudes which can be defined through convention, often made implicitly and different from paper to paper. For example two groups from the same laboratory (CERN) in the same journal (18*a*, 18*b*) give their value of $\varphi_{+-}-\varphi_0$ with opposite sign.)

We should study the special case when Z is normal, hence $\sigma=0$. Our formulae for the transition rates (such as (4.1), (4.4)) are still valid in the limit $\sigma\to 0$. (Note that (4.4) does not contain σ!) Of course α_i, φ_i, γ_k, φ_k have then to be defined in a chosen basis and φ_i and φ_k are no longer observables, *only their difference* $\varphi_i-\varphi_k$ is.

CP preservation is equivalent to $[Z, CP]=0$, so Z and CP have a common pair of eigenvectors. Since $CP \neq I$ and since it is hermitean ($(CP)^2=I$) these two eigenvectors are orthogonal and Z is normal. Furthermore a spin zero $\pi^+\pi^-$ or $2\pi^0$ state has a well defined $CP=I$. Therefore only one of the two eigenstates can decay into $\pi^+\pi^-$ or $2\pi^0$. The decays $K_S\to\pi^+\pi^-$ and $2\pi^0$ were well known. The Princeton group (11) was first to observe $K_L\to\pi^+\pi^-$. The decay $K_L\to 2\pi^0$ has also been observed but its rate is very poorly known (see Table 3).

So, if CP were preserved, K_L would be a $CP=-I$ eigenstate and it would have the same probabilities $\gamma''_{e^+}=\gamma''_{e^-}$ and $\gamma''_{\mu^+}=\gamma''_{\mu^-}$ to decay into the CP conjugated modes $\pi^+l^-\bar{\nu}$ and $\pi^-l^+\nu$ where the lepton l is either e or μ. However, charge asymmetries have been observed in both cases (see Table 3). This is also a direct proof of CP violation.

5. *Discussion of experimental data on 2π-decays*

To what extent has the situation changed since the last conference, that of Heidelberg, half a year ago, where Okun and Rubbia gave an excellent and detailed report on CP violation (19, 20)?

The main new experimental fact is that we know less today. A new value of $|\eta_{00}|=(\gamma''_{00}/\gamma'_{00})^{\frac{1}{2}}$ by V. L. Fitch *et al.* (see Table 3, ref. 17*g*) is not compatible with that of the previous experiments.

There is of course a possible, remarkable value for $|\eta_{00}|$, i.e. $(\gamma''_{00}/\gamma'_{00})=$

$|\eta_{00}|^2 = |\eta_{+-}|^2 = (\gamma''_{+-}/\gamma'_{+-})$ (equality up to phase space corrections and Coulomb effects in $\pi^+\pi^-$, see for instance ref. 16*f* for details). This means that K_S and K_L states have the same branching ratio $(\gamma'_{00}/\gamma'_{+-}) = (\gamma''_{00}/\gamma''_{+-})$ and the simplest explanation of this equality would be $[CP, \Gamma_{+-}] = 0 = [CP, \Gamma_{00}]$ i.e. CP is not violated in the transition $K \to 2\pi$ but K_S and K_L are not CP eigenstate so it is their $CP = I$ component that we observe to decay into 2π. For K_S this component is nearly one, for K_L it is nearly zero. Examples of theories which predict $|\eta_{00}| = |\eta_{+-}|$ are: Sachs (21) (CP violation due to interference between $(\Delta S/\Delta Q) = 1$ and $(\Delta S/\Delta Q) = -1$, so it does not occur in a purely hadronic process), Wolfenstein's superweak interaction (22), Gürsey & Pais (23) cosmogonic interaction with a pseudoscalar field. Although the opposite opinion is often expressed and printed (e.g. ref. 16*f*, p. 539) I believe these two theories predict the same effects in K meson physics.

What can the six real and measurable numbers $\gamma'_{+-}, \gamma'_{00}, \gamma''_{+-}, \gamma''_{00}, \varphi_{+-}, \varphi_{00}$, tell us? The $\Delta I = \frac{1}{2}$ rule predicts $\gamma'_{+-} = 2\gamma'_{00}$. This is in good agreement with experimental results. As we saw the inequalities $\gamma''_{+-} \neq 0 \neq \gamma'_{+-}$ are evidence of CP violation. What about the angles?

Equation (4.1) can be rewritten:

$$\Gamma = \Gamma_{\pi^+\pi^-} + \Gamma_{2\pi^0} + \Gamma_{\pi^+\pi^-\pi^0} + \Gamma_{3\pi^0} + (\Gamma_{\pi^+\mu^-\nu} + \Gamma_{\pi^-\mu^+\nu}) + (\Gamma_{\pi^+e^-\nu} + \Gamma_{\pi^-e^+\nu}) + \Gamma^+_{2\gamma} \dots \tag{4.1'}$$

If we take the trace with LS, we note that all but the two first terms on the right hand side have rather small moduli. Indeed, because the CP violation is small, $\gamma'_{3\pi^0}$ is rather small. From the $\Delta I = \frac{1}{2}$ rule the yet unmeasured $\gamma'_{\pi^+\pi^-\pi^0}$ seems smaller than $\gamma''_{\pi^+\pi^-\pi^0}$; the sum of two matrices in each bracket is nearly a multiple of I, so its contribution is of the order $2 \cdot 10^{-3}\sigma\gamma'$. $\mathrm{Tr}\,\Gamma_{2\gamma} LS$ and $\mathrm{Tr}\,LS$ with the omitted terms are even smaller. Thus

$$\mathrm{Tr}\,\Gamma LS = \sigma\left(\frac{\gamma' + \gamma''}{2} + i\mu\right) = \sqrt{\gamma'_{+-}\gamma''_{+-}}\, e^{i\varphi_{+-}} + \sqrt{\gamma'_{00}\gamma''_{00}}\, e^{i\varphi_{00}}$$

plus terms with rather small moduli. Therefore

$$\frac{\sigma}{2}\left(1 + \frac{2\mu}{\gamma'} i\right) = (0.68\,\eta_{+-} + 0.32\,\eta_{00})\, e^{i\varphi_0} + \dots \tag{5.1}$$

which gives an approximate relation between η_{+-} and η_{00}. Frequently, one defines (24)

$$\varepsilon_0 = \tfrac{2}{3}\eta_{+-} + \tfrac{1}{3}\eta_{00}, \quad \varepsilon_2 = \tfrac{1}{3}(\eta_{+-} - \eta_{00}) \tag{5.2}$$

So one sees that ε_0 cannot be too different from

$$\frac{\sigma}{2}\left(1 + \frac{2\mu}{\gamma'} i\right) e^{-i\varphi_0} \sim e^{-i\varphi_0}\sigma(1+i)/2 \sim \varepsilon_0 \tag{5.2'}$$

(*Note.* We use here the notations of ref. 19. See p. 304 for the relations between the different notations in refs. 19, 16*a*, 16*d*, 16*e*, 16*f*, 20. With a convention on amplitude phases, ε_0 is essentially the amplitude ratio

$$A(K_L \to 2\pi, I=0)/A(K_S \to 2\pi, I=0)$$

and $\varepsilon_2 = 2^{-\frac{1}{2}} e^{i(\delta_2 - \delta_0)} i \,\mathrm{Im}\,[A(K_L \to 2\pi, I=2)/A(K_S \to 2\pi, I=0)]$

where δ_2 and δ_0 are the π–π phase shifts in the isospin states $I=2$ and $I=0$ so $\mathrm{Arg}\, \varepsilon_2 = (\pi/2) + \delta_2 - \delta_0$.)

6. *Experimental difficulties*

Let us recall some facts about the difficulties of the experiments. The density matrices of K^0 and $\bar{K}^0$ are

$$K^0 = \tfrac{1}{2}(I + Y) = \alpha' S + \alpha'' L + \frac{\sqrt{\alpha'\alpha''}}{\sigma}(SL\, e^{i\varphi_0} + LS\, e^{-i\varphi_0}) \tag{6.1}$$

$$\bar{K}^0 = \tfrac{1}{2}(I - Y) = \bar{\alpha}' S + \bar{\alpha}'' L + \frac{\sqrt{\bar{\alpha}'\bar{\alpha}''}}{\sigma}(SL\, e^{i\bar{\varphi}_0} + LS\, e^{-i\bar{\varphi}_0}). \tag{6.1'}$$

Y is the hypercharge operator,

$$\mathrm{Tr}\, Y = 0,\ Y^2 = I, \tag{6.2}$$

$$\alpha' + \alpha'' + 2\sigma\sqrt{\alpha'\alpha''}\cos\varphi_0 = 1 = \bar{\alpha}' + \bar{\alpha}'' + 2\sigma\sqrt{\bar{\alpha}'\bar{\alpha}''}\cos\bar{\varphi}_0 \tag{6.2'}$$

$$K^0 + \bar{K}^0 = I \Leftrightarrow (1-\sigma^2)^{-1} = \alpha' + \bar{\alpha}' = \alpha'' + \bar{\alpha}'',\ \sqrt{\alpha'\alpha''}\, e^{i\varphi_0} + \sqrt{\bar{\alpha}'\bar{\alpha}''}\, e^{i\bar{\varphi}_0} = -\frac{\sigma}{1-\sigma^2} \tag{6.2''}$$

Since the strong and electromagnetic interactions conserve hypercharge the evolution matrix in matter is (in the K^0 rest frame)

$$Z_{\mathrm{matter}} = Z + \lambda_1 I + \lambda_2 Y \tag{6.3}$$

where the complex numbers λ_1, λ_2 depend on the velocity of the K and on the nature and density of the nuclei (25). (*Note.* The part $\lambda_1 I + \lambda Y_2$ represents both absorption and coherent scattering effects. This was first studied by Good (25):

$$\lambda_1 = -\pi N k^{-1} v(1-v^2)^{-\frac{1}{2}}(f(0) + \bar{f}(0)),\ \lambda_2/\lambda_1 = (f(0) - \bar{f}(0))/(f(0) + \bar{f}(0))$$

where k is the K-beam wave number, v its velocity, N the density of nuclei, $f(0)$ and $\bar{f}(0)$ the forward scattering amplitude by nucleus of, respectively, K^0 and $\bar{K}^0$ mesons.)

All performed experiments have been made from K^0-beams, but it is difficult to be near the source of an intense K-beam, so to observe the oscillation in

$e^{-\frac{1}{2}(\gamma'+\gamma'')t}$ one had to use a regenerator (exception, ref. 17*c*, Table 3) and λ_1 and λ_2 have to be partly measured (from K^+, K^- cross sections when the matter has I_3 = third component of isospin = 0) partly computed (optical theorem, phase shift analysis, $I_3 \neq 0$). Note also that one had to use a regenerator for measuring the sign of μ, the mass difference, since in equation (4.4), for vacuum, μt is in a cosine argument.

K_L beams are easy to prepare: One observes neutral K's far enough from the source (for instance $>20 K_S$ mean paths). Finally, the main difficulty is the study of rare modes because of the large background: $\gamma''_{\pi^+\pi^-}/\gamma''_{3\pi^0} \sim \frac{1}{200}$, so $\gamma''_{2\pi^0}$, of the same order of magnitude as $\gamma''_{\pi^+\pi^-}$ is extremely difficult to measure.

7. *Consequence of CPT for neutral K-decays*

With the notations of this report

$$CPT \text{ invariance} \Rightarrow \mathrm{Tr}\ YZ = 0 = \mathrm{Tr}\ YZ^* \tag{7.1}$$

When $\sigma \neq 0$ this is equivalent to

$$Y = \eta\sigma^{-1}(I - S - L), \quad \eta = \pm 1 \tag{7.2}$$

We deduce from (6.2′) and (6.2″)

$$\varphi_0 - \bar{\varphi}_0 = \pi, \quad \varphi_0 = 0 \text{ or } \pi \quad \mathrm{mod}\ 2\pi \tag{7.3}$$

$$\alpha' = \alpha'' = \tfrac{1}{2}(1-\sigma^2)^{-1}(1+\eta\sigma), \quad \cos\varphi_0 = -\eta \tag{7.3′}$$

$$\bar{\alpha}' = \bar{\alpha}'' = \tfrac{1}{2}(1-\sigma^2)^{-1}(1-\eta\sigma), \quad \cos\bar{\varphi}_0 = +\eta \tag{7.3″}$$

If $(\Delta S/\Delta Q) = 1$ is an exact rule for semileptonic decays $\bar{K}^0 \to \pi^- + l^+ + \nu$ and $K_0 \to \pi^+ + l^- + \bar{\nu}$ are forbidden. Therefore

$$\frac{\Delta S}{\Delta Q} = 1 \text{ exact } \Rightarrow \Gamma_{l^+\pi^-\nu} = \lambda_l K^0, \Gamma_{l^-\pi^+\bar{\nu}} = \bar{\lambda}_l \bar{K}^0 \tag{7.4}$$

With CPT invariance

$$\lambda_l = \bar{\lambda}_l = \frac{2\gamma''_{l^+\pi^-\nu}}{1-\eta\sigma} = \frac{2\gamma''_{l^-\pi^+\bar{\nu}}}{1+\eta\sigma} \tag{7.5}$$

hence

$$\frac{\gamma''_{l^+\nu\pi^-}}{\gamma''_{l^-\bar{\nu}\pi^+}} = \frac{1-\eta\sigma}{1+\eta\sigma} \sim 1 - 2\eta\sigma$$

This is to be compared (Table 3) with $1.0047 \pm .0007$. (We take only the electron experiment because with $\Delta S/\Delta Q \neq 1$ the discussion of the μ-experiment might be more elaborate. Note that the two experimental results are compatible.) Therefore

$$\sigma = (2.3 \pm .4)\, 10^{-3} \tag{7.6}$$

and $\eta = -1$ so

$$\varphi_0 = 0, \quad \bar{\varphi}_0 = \pi, \quad Y = \sigma^{-1}(L + S - I) \tag{7.7}$$

and equation (5.2′) reads

$$\varepsilon_0 \sim 1.7 \times 10^{-3} e^{i45^\circ} \tag{7.8}$$

and the experimental value $\eta_{+-} = 1.9 \times 10^{-3} e^{i60^\circ}$ yields (see Fig. 1) approximately

$$\eta_{00} \sim 1.6 \cdot 10^{-3} e^{i10^\circ} \tag{7.8′}$$

This implies for the $\pi - \pi$ phase shifts (26) $\delta_2 - \delta_0 \sim 30°$ or 210° while the experimental value (28) is $-15° - 35° = -50°$.

There might also be some $(\Delta S/\Delta Q) = -1$ transitions. If x is the complex ratio of the $(\Delta S/\Delta Q) = -1$ and the $(\Delta S/\Delta Q) = 1$ amplitude, then assuming that x is constant on the phase space, equation (7.4) is changed into

$$\frac{\gamma''_{e^+\nu\pi^-}}{\gamma''_{e^-\bar{\nu}\eta^+}} \sim 1 - 2\eta\sigma \frac{1 - |x|^2}{|1 - x|^2} \tag{7.9}$$

This relation is compatible with any value of σ and therefore of η_{00}. So we are lead to the question: what do we know about the validity of the rule $\Delta S = \Delta Q$ for semi-leptonic interactions? The parameter x can be measured from the time evolution of $\pi^+ l^- \bar{\nu}$ and $\pi^- l^+ \nu$ decay rates in a neutral K-beam. The latest such experiment by Hill *et al.* (26) (where refs. to the six previous experiments are given) yields for x:

$$|x| = .26^{+.08}_{-.11}, \quad \mathrm{Arg}\, x = 50^{\circ}{}^{+25^\circ}_{-27^\circ}$$

This is in agreement with the previous experiments (see their Fig. 3): all experimental values of x put it in the complex plane!) Such a value of x does not change η_{00} very much (it can decrease its modulus by a factor 1 to 1.5; see our Fig. 1). The value $x = 0$ is not excluded by those experiments but there exist other rare events (e.g. $\Sigma^+ \to \eta + \mu^+ + \nu$) which violate $\Delta S = \Delta Q$ (27).

Experiments like $K \to \pi^\pm + \mu^\mp + \nu$ are more difficult to analyse but they can give the value of the μ-polarization perpendicular to the decay plane 0.003 ± 0.014 (28).

From the present experimental situation one can conclude that there are conflicting experiments, or that CPT is violated (see in Fig. 1 the preferred region for η_{00} from $\pi - \pi$ phase-shift experimental value). Maybe $\eta_{+-} = \eta_{00}$ can reconcile everything (the $\pi - \pi$ phase-shift is then irrelevant!).

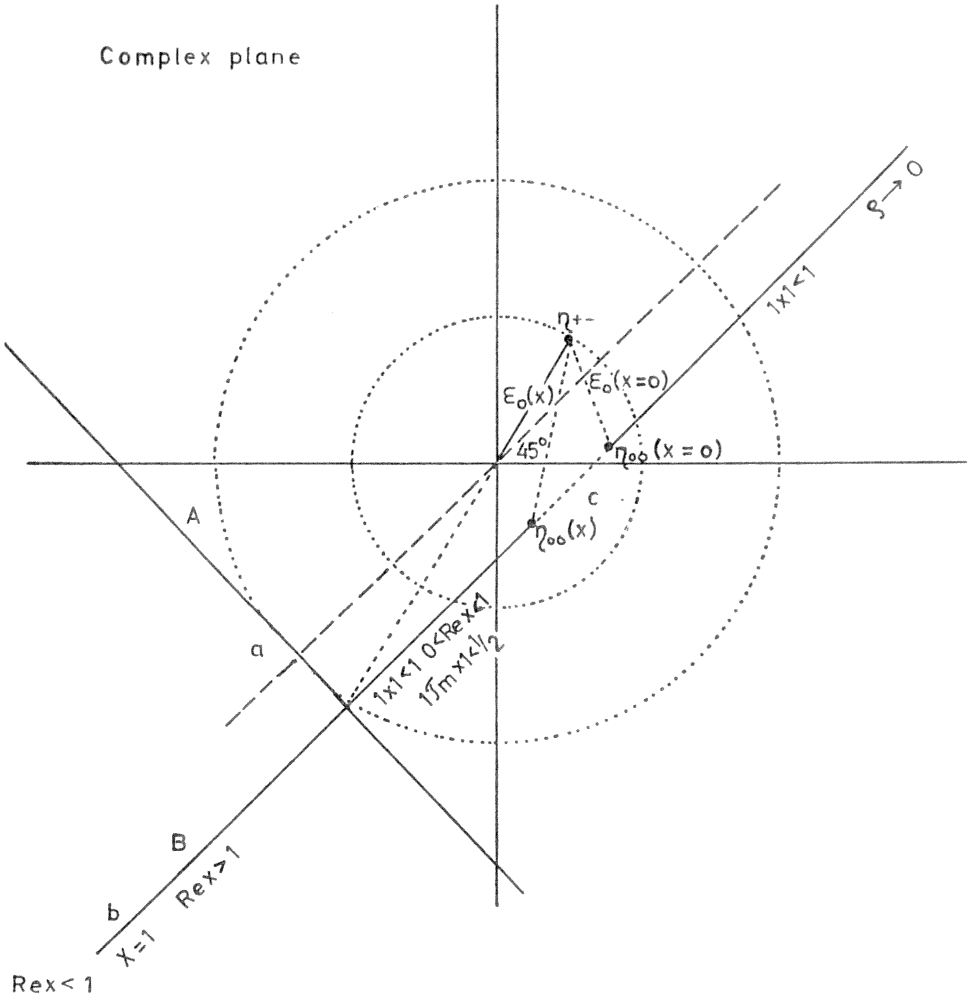

Fig. 1. This diagram should be considered as qualitative only. It is based on the roughly approximated eq. (5.2′) and experimental errors are not taken into account. From the measured value of η_{+-} and μ/γ' and the assumption of either T or CPT invariance the complex number η_{00} must lie either on the line A or B which cross at $-2\,\eta_{+-}$. The present experimental data on $\pi-\pi$ phase shifts favour the two regions a and b for η_{00}.

If CPT is assumed, to each point of the locus of η_{00}, corresponds a value of $\varrho=(1-|x|^2)/|1-x|^2$ (where x is the ratio of the $\Delta S=-\Delta Q$ over. $\Delta S=\Delta Q$ amplitudes in $K\to\pi e\nu$) because the charge asymmetry a in $K_L^0\to e^{\pm}\nu\pi^{\mp}$ is a measure (when (5.2′ holds) of $2\sqrt{2}|\varepsilon_0|\varrho$. Then $\eta_{00}=-2\eta_{+-}+3(1+i)a/4\varrho$. The experimental values of x (combined with those of η_{+-} and a) favor a region c for η_{00} *incompatible* with that favored by the $\pi-\pi$ phase-shift values. Note however that for $\eta_{+-}=\eta_{00}$ the phase-shift condition disappears.

If T is assumed, the relation between a and η_{00} depends on the model.

The most honest conclusion is that more experiments are needed. $|\eta_{00}|$ will be measured again, maybe also its phase. There are also experiments on 3π, $2\pi\gamma$, 2γ decays. I will not discuss here the information one will obtain from them, as well as that from K^+ decays compared to K_L^0 decays.

8. *Is CPT invariance a sacred cow?*

It is true that we have no decent relativistic quantum field theory which is not *CPT* invariant. However, evidence of a small *CPT* violation could mean that we just have to relax weak locality. I would like also to present an alternative for which CP, T, CPT are strict invariances of physics while there would be an apparent violation of CP, CPT (but not of T) in neutral K-decays.

We have already pointed out in section 5 that the Gürsey & Pais (23) cosmological field gives identical effects to that of the superweak theory (22). In that case CPT is preserved and CP and T appear violated. However, since the universe is not CP symmetric, the influence of the cosmogonic spin zero field could simulate CP and CPT violation, while T would be preserved in K-decays. Note that this is the case for the strong and electromagnetic interactions of neutral K in matter (matrix Z_{matter}, eq. 6.3). T invariance is preserved (because Y is conserved) while CP appears violated ($CPY-YCP\neq 0$; indeed $CPY+YCP=0$) and so appears CPT. This manifests the non CP-symmetry of matter.

Therefore, we are led to study the implications of T. It is very easy to obtain for the K^0 and $\bar{K}^0$ angles the conditions

$$\varphi_0+\bar{\varphi}_0=0, \quad \pi<|\varphi_0-\bar{\varphi}_0|<2\pi \quad \bmod 2\pi \tag{8.1}$$

which are quite different from equation 7.3 imposed by CPT. For small CPT violation we can introduce a small angle $\delta>0$ ($\delta\sim 10^{-3}$) and

$$|\varphi_0|=\frac{\pi}{2}+\delta; \quad |\varphi_0-\bar{\varphi}_0|=\pi+2\delta \tag{8.1'}$$

The second equality cannot be easily distinguished by experiments from $\varphi_0-\bar{\varphi}_0=\pi$ imposed by CPT and generally proposed as a direct test of it (see also ref. 29). The first equality will be a clear cut test if the angles φ_0, $\bar{\varphi}_0$ can be directly measured instead of the differences $\varphi_0-\varphi_k$. Equation (3.13) does not depend on φ_k. However, it would be difficult to measure a σ-proportional oscillating term for the complete rate of disappearance of K-mesons.

Another possible experiment could be performed with a mixture of K^0 and $\bar{K}^0$ unpolarized in the "charge" space (30) (Density matrix $\frac{1}{2}I$). Such a mixture could be obtained by a p^- beam on hydrogen. Note that high energy neutral K beams produced by π or p^+ on matter are really a mixture containing $\zeta=5$, 10 or even 20% of K^0. If CPT is preserved, this just decreases the coefficient of the oscillation term which is multiplied by a factor $(1-2\zeta)/(1+2\zeta)$ (where $\zeta>0$). If CPT is not preserved, the apparent value of φ_k is modified. The probability density for the decay mode k to appear at time t is

$$\tfrac{1}{2}\mathrm{Tr}\, e^{-iZt}e^{iZ^*t}\Gamma_k=\frac{1}{2(1-\sigma^2)}\left(e^{-\gamma' t}\gamma_k'+e^{-\gamma'' t}\gamma_k''-2e^{-\frac{1}{2}(\gamma'+\gamma'')t}\sigma\gamma_k\cos(\mu t-\varphi_k)\right) \tag{8.2}$$

From this measurement of φ_k and the ordinary measurement of the difference $\varphi_0-\varphi_k$ we can deduce φ_0.

However, the T preserving, CPT violating cosmological interpretation we propose here might be already ruled out by the present experimental situation. As we have seen in section 5, such a theory would imply $\eta_{+-} \simeq \eta_{00}$ and by (5.2′) this gives:

$$\text{Arg}\,\eta_{+-} = \frac{\pi}{4} \pm \frac{\pi}{2} = 135^\circ \quad \text{or} \quad -45^\circ$$

which seems excluded by experiments (see Table 3). On the other hand a *T* preserving, slightly *CPT* violating theory is not excluded from a phenomenological study of the present experimental data (see Fig. 1).

If *CPT* invariance can be tested, it should be tested (29). The most often proposed test is the reversal of sign implied by *CPT* in the oscillation term when one observes a decay, starting from a K^0- and a $\bar{K}^0$-beam. As we saw, *T* may simulate the same effect when the *CPT* violation is small. It may also happen that a detailed analysis, assuming *CPT*, of all experimental data in *K*-meson physics may reveal a contradiction. Most unlikely *CPT* would then be one of the last abandoned hypotheses.

Of course an inequality between the mass, lifetime and absolute value of magnetic moment for particles and antiparticles would be a proof of *CPT* (and *C* and *CP* and *CT*) violation. However, the relative equality of those quantities up to 10^{-3} proves nothing for the validity of *CPT* although the opposite is often claimed (31). Since no *CP* violation of more than 10^{-3} has been observed, one should expect such equality up to 10^{-3}. And I must confess that I do not know of a feasible test where a *CPT* violation could be observed while *CP* is preserved.

9. *Miscellaneous on C, P, T*

No *CP* or *T* violation has been observed in the stronger electromagnetic interactions. We refer to the refs. 19 and 20 for a detailed survey. We just give here the most recent evidences:

(a) η-decay

M. J. Bazin, A. C. Goshaw, A. R. Zacher & C. R. Sun in "An evaluation for the searches of *C* non-conservation in η-decay" (32) review all performed experiments made on T. D. Lee's suggestion and conclude that there is no compelling evidence (branching ratio of the mode $\eta^0 \to \pi^0 e^+ e^- < 2.10^{-4}$ no asymmetry in $\eta^0 \to \pi^+\pi^-\pi^0$ or $\pi^+\pi^-\gamma$).

(b) Neutron electric dipole moment

These experiments seem to be the most precise ones. A neutron electric dipole ν would imply *P* and *T* violation in physical laws (*P* is already known!) (33, 34).

Note that much less precise limits are known for p, Λ, and μ. Stein *et al.* (35) and previous authors quoted there, have tried to observe atomic electric dipoles. This yields for the electron dipole moment an upper limit: $(1.7 \pm .5)10^{-23} e \times \text{cm}$

(c) *T invariance in weak decays*

The most precise measurements of the relative phase of V and A couplings have been obtained from $Ne^{19} \to F^{19} e^{+} \nu$ by F. P. Calaprice, E. D. Commins, H. M. Gibbs, G. L. Wick & D. A. Dolson (36) namely $179.6° \pm 2°$.

However, it should be noted that even a maximal T violation would not give a larger effect in such a transition and in other nuclear transitions because isoparity (conserved up to 1%) forbids a phase $\neq 0°$ or $180°$. (*Note.* Isoparity, due to the combination of charge conjugation and isotopic spin, was introduced by the author (37*a*) where reference is also given to the first paper (37*b*) giving weak coupling selection rules due to C and isospin. When Lee & Yang (37*c*) made a lot of advertisement for some parts of my paper (their only contribution was to change the significant name "isotopic parity" into the dull G-parity) they seem to have missed the applications to weak couplings—and also the useful formula $G = C(-1)^{I}$. However, they did later apply isotopic parity to weak couplings (37*d*).)

For $|\Delta S| = 1$ transitions, time reversal is not well checked (38) when the measured relative phase between P and S wave in $\Lambda^0 \to p^+ \pi^-$ is $9^0 \pm 1.5$ instead of the expected value of $6.5° \pm 1.5$ from $p^+ \pi^-$ phase shift.

10. *Conclusion*

During the last four years about thirty "theories" have been proposed for explaining the observed CP violation. Due to the lack of accuracy of the relevant data in neutral K decay, few died. The lack of evidence of CP or T violation effects outside K^0 decays (or C for non-weak decays) gave a lethal blow to a few of them (some survived through drastic surgery such as change of heart which is fashionable these days). A quite deadly experiment might be the measurement of the neutron electric dipole moment $e \cdot$(the neutron length $= 2.10^{-14}$ cm)$\cdot$(weak coupling $= 10^{-5}$)$\cdot$(CP violation $= 10^{-3}$) $= 2.10^{-22}\, e \cdot \text{cm}$. Ramsey and his collaborators hope to reach a precision of $10^{-24}\, e \cdot \text{cm}$ within two years.

Of course, since CP violation has been seen only in K^0 decay, which is a $\Delta S \neq 0$ transition, a theory which predicts CP violation only with $|\Delta S| = 1$ is presently safe with respect to the outcome of this experiment (prediction $\sim 10^{-27} e \times \text{cm}$). If $\eta_{+-} = \eta_{00}$ the theories quoted in refs. 22 (superweak) and 23 (cosmogonic) might stay safe for a long time.

It is often fashionable to classify a theory according to the nature of the interaction made responsible for the *CP* violation. The theory proposed by Good *et al.* (39) is then alone in its class. No interaction is separately responsible for *CP* violation, the cooperation of semi-strong (giving a factor 10^{-1}) electromagnetic (factor 10^{-2}) and weak interactions is required. In other words, although the full Lagrangian is not *CP* invariant, this Lagrangian minus either the semi-strong part, or the electromagnetic part, or the weak part, has a *CP* automorphism. This theory has $\Delta I = \frac{1}{2}$ and $(\Delta S/\Delta Q) = 1$ built in selection rules and also has the ambition to understand the $\mu - e$ mass difference, the separate (μ, e) lepton charge conservation, the apparent absence of a neutral leptonic current and of *CP* violation in pure leptonic or $\Delta S = 0$ semi-leptonic processes. It does predict neutral leptonic currents for pure leptonic processes, and either a 10^{-3} *CP* violation for $|\Delta S| = 1$ processes, or *maximal CP violation* for rare phenomena with $|\Delta S| = 1$ such as $K^{\pm} \to \pi^{\pm} + \mu^{+} + \mu^{-}$ (but not for $K^{\pm} \to \pi^{\pm} + e^{+} + e^{-}$!) or $K_L^0 \to \mu^{+} + \mu^{-}$ with a branching ratio comparable to the weak-electromagnetic Dalitz pair creation.

The theories of *CP* violation which are still alive are joined about every month by a new-born theory. They all await to be hurt by new experiments. I hope it might not take long, although it requires much more effort from our colleagues to make new experiments than for us to make new theories.

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Discussion

Gell-Mann

I am very interested in your remarks about a scalar interaction due to the rest of the universe, because what we would like to do really is to connect the *CP* violation in the Fitch–Cronin experiment to the known temporal asymmetry in the universe. And so you would like to have something like the processes in the stars, which are associated with the arrow of time in the universe, generating an influence which would cause $K_2^0 \to 2\pi$. Suppose you try to do that by considering the rate of change of I_z, the *z*-component of the isotopic spin. In the stars the protons are being converted into neutrons and by this process I_z is changed. If you try to induce a transition between $K_1^0 - K_2^0$ caused by the same forces, then you don't find the right rate by a factor of 10^{20}. Did you say there was a successful theory of this kind, that took some known process in the world, coupled a spinless field to it and predicted the rate of $K_2^0 \to 2\pi$?

Michel

I said there was an unpublished paper that I did not read. But as the authors are here, I suppose they will answer your question.

Gürsey

No, it is unfortunately not so successful. It is just an ad hoc theory. You just cook up an extra spin zero cosmological field with a coupling strength adjusted to give the effect. That is why, actually, the paper was not published. The predictions are the same as in the superweak theory.

Michel
It is not more cooked up than the superweak. Both are cooked up. Both theories avoid questions.

Gürsey
What we had in mind was to relate the effective coupling strength of the cosmological field to known or possible feature of the universe at large. So far we have not succeeded in this program.

Pais
We agree indeed in detail with superweak also in regards to the phase. The superweak asks for a phase of 43°. This is also what we have, for both 2π modes. Incidentally, if the spin is one the phase would be 90° different from that. But there are much stronger reasons, of course, to be against spin one anyway.

Salam
I assume the same objections applied to the old vector version of this theory.

Michel
Well, the vector is out.

Ne'eman
In answer to Salam's last question Steven Weinberg has calculated the possible effects of a long-range vector field. He has shown that you get a ratio of some 10^{-19} between the rate you would expect from such a field and the actual *CP* violations. It was published in Phys. Rev. Lett. (*13*, 495, 1964).

Sudarshan
Is there a possibility that angular momentum might not be conserved to the same degree in which the *CP* violation is observed? It appears to me that the test of angular momentum conservation is nowhere near as good as the degree to which *CP* is conserved.