

by inspection. Some applications to wave equations in higher dimensional spaces are indicated.

<sup>1</sup> N. Kemmer, Proc. Cambridge Phil. Soc. 39, 189 (1943).  
<sup>2</sup> A. Klein, Phys. Rev. 82, 639 (1951).

**TA6. The Nonrelativistic Limit of Spin  $k + \frac{1}{2}$  Particle Wave Equations.** P. A. MOLDAUER AND K. M. CASE, *University of Michigan*.—The Dirac-Pauli-Fierz theory for half-integral spin particles is investigated in the nonrelativistic limit. Starting from the Rarita-Schwinger<sup>1</sup> formulation of the theory, the wave function  $\psi_{\mu_1\mu_2 \dots \mu_k}$  (symmetric) which describes a particle of spin  $k + \frac{1}{2}$  is reduced with respect to the three-dimensional rotation group and the superfluous components are eliminated. The Hamiltonian is calculated to order  $1/m^2$  and the intrinsic magnetic dipole and electric quadrupole moments are found. In addition the spin-orbit coupling and Darwin type terms are obtained.

<sup>1</sup> Rarita and Schwinger, Phys. Rev. 60, 61 (1941).

**TA7. Joint Probabilities and Transition Probabilities in Differential-Space Quantum Theory.** NORBERT WIENER, *Massachusetts Institute of Technology*, AND ARMAND SIEGEL, *Boston University*.—In our theory of quantum systems<sup>1</sup> the statistical predictions ordinarily obtained from the wave function are obtained from an ensemble of deterministic systems each of which has a sharp value for every observable. The values over this ensemble for two observables corresponding to two noncommuting operators in ordinary quantum mechanics are, necessarily, imperfectly correlated, and it is of interest to find their joint probability distribution. Suppose they correspond to quantum-mechanical operators  $R$  and  $S = \exp(-iK\lambda)R \exp(iK\lambda)$ ;  $K$  is Hermitian. We have obtained the joint probability to first order in  $\lambda$ , i.e. valid for  $S$  close to  $R$ : given a wave function  $\psi(x) = \sum a_i \varphi_i(x) = \sum a'_i \varphi'_i(x)$ ,  $\varphi_i(x)$  and  $\varphi'_i(x)$  eigenfunctions of  $R$  and  $S$ , respectively. Then for the ensemble corresponding to  $\psi(x)$ , prob.  $\{R=R_i, S=S_j\} = [ |a_i|^2 |a'_j|^2 - |a_j|^2 |a'_i|^2 ]_+$  for  $i \neq j$ ;  $[ \dots ]_+$  means the positive part of the quantity inside brackets. If  $K$  is the Hamiltonian and  $\lambda$  the time, this expression is  $|a_i|^2$  times the probability that a system (of the ensemble corresponding to  $\psi$ ) having  $R=R_i$  at time zero will make a transition after a (small) interval  $\lambda$  to  $R=R_j$ .

\* Work of this author supported by Office of Naval Research.  
<sup>1</sup> N. Wiener and A. Siegel, Phys. Rev. 91, 1551 (1953); also a forthcoming article "The differential-space theory of quantum systems."

**TA8. The Determination of the Scattering Potential from the Spectral Weight Function.\*** I. KAY AND H. E. MOSES, *New York University*.—By generalizing the method of Gelfand and Levitan<sup>1</sup> it is shown that in many cases the potential can be obtained uniquely from the spectral measure function, if we specify the asymptotic behavior in some representation of the eigenstates of the total Hamiltonian  $H$  associated with the measure function. In contrast to earlier treatments where one restricts oneself to an unperturbed Hamiltonian  $H_0$  and a representation such that the  $H_0$  is a second derivative operator and the potential is diagonal, the procedure described is quite general:  $H_0$  may have almost any character, and the potential need not be diagonal in the representation in which the asymptotic description of the eigenfunctions is specified. The transformation  $U$  which maps the eigenfunctions of  $H_0$  into those of  $H$  and  $U^{-1}$  are given triangular properties in the specified representation.  $U$  and  $U^{-1}$  then satisfy a Wiener-Hopf equation which involves the spectral measure function. In many cases the solution for  $U$ ,  $U^{-1}$  can be obtained explicitly in terms of the spectral measure function. The potential can then be obtained from these operators.

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<sup>1</sup> I. M. Gelfand and B. M. Levitan, Izvest. Akad. Nauk S.S.S.R. Ser. Math. 15, 309 (1951).

**TA9. Vacuum Polarization and Proton-Proton Scattering.\*** L. L. FOLDY,† *Case Institute of Technology*, AND E. ERIKSEN, *University of Oslo*.—An attempt has been made to detect the presence of effects of vacuum polarization in currently available experimental data on proton-proton scattering. In spite of the smallness of these effects and relatively large errors in the data, it appears that the data substantiate the predicted vacuum polarization effects in the electrostatic interaction of heavy charged particles, if one assumes the nuclear potential has a Yukawa shape. By correcting the available data for these effects, new values are obtained for the zero energy scattering length and effective range of the specifically nuclear interaction between two protons.

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 † Fulbright and Guggenheim Fellow during portion of this research.

**TA10. A Covariant Formalism Describing the Polarization of Spin One-Half Particles.** L. MICHEL, *Institute for Advanced Study*, AND A. S. WIGHTMAN, *Princeton University*.—We denote by  $u(p,s)$ , the state of a spin  $\frac{1}{2}$  particle of four momentum  $p$  and mass  $m \neq 0$ , whose spin is polarized along a space like four pseudo-vector  $s$ , such that  $s \cdot p = 0$ ,  $s^2 = -1$ . Then the projection operator onto  $u(p,s)$  may be written

$$P_{\alpha\beta}(p,s) = [\bar{u}u]^{-1} u_{\alpha} \bar{u}_{\beta} = [4m]^{-1} [1 - \gamma_5 s] [m + \not{p}]$$

(We use Feynman notation.)  $s_{\mu}$  is the generalization of the Stokes pseudo-vector and is the expectation value of  $\gamma_5 \gamma_{\mu}$ . For a mixture, the degree of polarization is  $(-s^2)^{\frac{1}{2}}$ . For  $m=0$ , we have

$$P_{\alpha\beta}' = [u^*u]^{-1} u_{\alpha} u_{\beta}^* = (4p_0)^{-1} [1 - \gamma_5 s(\tau + \xi)] p \gamma_0$$

where  $s_{\tau}$  is a transverse polarization vector and  $\xi$  is the amplitude for circular polarization. These formulas permit one to make covariant polarization calculations in terms of traces.

**TA11. A Problem in Shower Theory—"Approximation A."** R. C. O'ROURKE, *Naval Research Laboratory*.—The Bhabha-Heitler method of solving the shower equations in "approximation A" has been applied to the situation in which a beam of photons with a prescribed spectrum is incident upon a thick target. The mathematical aspects of the work are an immediate and direct extension of the classic work of Bhabha Heitler<sup>1</sup> and N. Arley,<sup>2</sup> who treated primary monoenergetic electrons and photons respectively. The contribution of the present work lies perhaps more in the numerical results which have been obtained on the Naval Research Laboratory electronic digital computer. The numerical work completed so far is for the ideal primary spectrum  $kN(o,k)$  equals a constant, i.e., the high energy Bremsstrahlung shape. The results can be usefully employed in the calculation of photonuclear yields in thick targets.

<sup>1</sup> H. J. Bhabha and W. Heitler, Proc. Roy. Soc. (London) A159, 432 (1937).  
<sup>2</sup> N. Arley and B. Eriksen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 17, No. 11 (1940).

**TA12. Wave Functions and Transition Probabilities for Light Atoms.\*** HUSEYIN YILMAZ,† *M.I.T.*—(Introduced by Philip M. Morse).—A new method of taking electronic correlations into account is presented. This method differs from conventional configuration methods in its generality, simplicity, and interpretation. However, due to the particular choice of perturbation series, there are substantial similarities. The computation is based on a function, the  $S$ -function, which was tabulated beforehand. This function is a bilinear Laplace transform of the Green's function and has various useful properties. The method is applied to the  $(1s^2 2s^2 2p^2)$  configuration of  $C_{II}$ ,  $N_{III}$ ,  $O_{III}$ , and  $F_{IV}$ . A few significant terms are identified which take care of 60-80 percent of the discrepancy between experimental and theoretical values of multiplet separations. The spin-orbit separations and nebular transition are calculated. With a slight modification of the  $2p$  wave

function can be obtained.  
 \* Work in progress.  
 † Now at ...

**TA13. HEH.\*** method hydrogen ing the expansion t Several t found to Wave fur results wi energy, u to be con function. rium inter by Toh, t accord wi electron p  
<sup>1</sup> Coulson  
<sup>2</sup> Toh, Proc

**VI. Rel tance.\* R. The modu an extern; transient l to the su conductan value in a more quai was studie sinusoidal response, 100 and 10**