# CHAPTER III

# COUPLING PROPERTIES OF NUCLEONS, MESONS AND LEPTONS

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# INTRODUCTION

With the discovery of the neutron (CHADWICK [1932]) and the positon (ANDERSON [1932]), all nucleons and leptons that we now know had been observed (the existence of the neutrino was assumed from $\beta^-$-radioactivity). The first successful quantum mechanical description of a non-electromagnetic interaction between elementary particles was given by FERMI [1934] for $\beta^-$-radioactivity; WICK [1934] in the same year showed the direct extension to the newly discovered $\beta^+$-radioactivity. However, this interaction was much too small to explain nuclear forces. The meson hypothesis of YUKAWA [1935] removed this difficulty, and suggested the coupling scheme between elementary particles according to Figure 1; it was then natural to classify particles as nucleons, mesons and leptons.

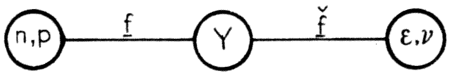


Fig. 1. Coupling scheme before 1947

The theory predicted (TOMONAGA and ARAKI [1940]) the absorption of negative mesons by nuclei, and this was verified in experiments with lead and iron, but CONVERSI *et al.* [1945, 47] showed, unexpectedly, that most negative mesons stopped in carbon decay. The validity of the scheme was then doubtful, and the discovery of the $\pi$-meson (LATTES *et al.* [1947]) destroyed it completely.

More recently, numerous schemes have been proposed (see for instance, surveys of the question in different years: ROSENFELD [1948b] and YUKAWA [1949]). Even the problem of nuclear forces has not yet been solved in terms of meson fields, and several new reactions, some involving new particles ($\pi^\circ$-mesons, heavy mesons), have been discovered in 1950.

## 1. THE PHYSICS OF PARTICLES

Any physical result is based on experimental data and theoretical hypothesis, and it is necessary to know fully its bases in order to appreciate its value and the limits of its validity. In the present state of physics, with the obvious specialization of physicists, it is often very difficult for experimental and theoretical people to know the exact significance and implications of their mutual discoveries. It is necessary for experimental cosmic ray workers to have right and precise ideas of the theoretical concepts they have to use to make qualitative deductions. (quantitative calculations are left to the theoreticians, and, moreover, form the weakest part of the theory !). In this paper it seems useful, after a short summary of experimental data on particles, to give a review of the framework of the theory as it is currently in use, and to show what is the basis of the main laws and theorems (conservation laws, Furry's theorem etc.) useful for the qualitative study of reactions and couplings between elementary particles. This, together with the trite, but essential, specification of units, is the aim of the present section.

## 1. 1 Experimental data

Experimentally, particles are at once classified into charged and neutral particles. Neutral quanta (photons, $\pi^{\circ}$-mesons) have strictly no charge, while within experimental accuracy (a few per cent.) charged particles have all the same amount of electric charge, with both signs of charge for electrons and mesons. This amount is supposed to be strictly the same for all charged particles, and this implies that neutrons and neutrinos have strictly no charge. Only for electrons has each charge conjugate state been named — negaton and positon. Particles are further experimentally characterized by their rest-mass. Table 1 gives the list of mass values and the notation used in this paper, and numerical results given by authors will be corrected by taking into account these values if necessary. Latest references of experimental data are indicated: see also POWELL's [1950] review on mesons.

There are mesons heavier than $\pi$-mesons: the existence of these particles is certain, but their masses are not yet known with any accuracy. The properties of these particles will be studied in § 2. 5 and § 2. 6 under the titles: "Neutral V-mesons" and "Charged V-mesons and $\tau$-mesons". We shall not, however, consider here varitrons (see DAUDIN [1950] for a critical survey) nor the photographic plate evidence of decays attributed to them (ALIKHANIYAN *et al.* [1949]).

Other characteristic properties of free particles: mean life, magnetic moment and spin, will be reviewed separately for each of them.

TABLE 1

| Particle | Mass notation | Mass value | Experimental data | |
|---|---|---|---|---|
| $\varepsilon$ electron | $m$ | unit | | |
| $\nu$ neutrino | $\nu$ | 0 | $< 0.002$ | CURRAN *et al.* [1949] |
| $\mu$ $\mu$-meson | $\mu$ | 213 | $210 \pm 4$ | SMITH *et al.* [1950] |
| | | | $215 \pm 4$ | RETALLACK, BRODE [1949] |
| | | | $212 \pm 4$ | PEYROU, LAGARRIGUE [1950] |
| $\pi$ $\pi^{\pm}$-meson | $\varkappa$ | 271 | $271.4 \pm 2.5$ | MOYER Harwell Conf. [1950] |
| $\pi$ $\pi^{\circ}$-meson | $\varkappa'$ | 261 | $261 \pm 4$ | MOYER Harwell Conf. [1950] |
| n, p nucleon | $M$ | 1837 | | |
| Difference $M_n - M_p$ | | 2.53 | $2.53 \pm 0.025$ | ROBSON [1951] |
| | | | $2.535 \pm 0.012$ | HORNYAK *et al.* [1950] |

## 1. 2 Theoretical methods

Many authors have attempted to develop fundamental theories giving the number of elementary particles, their rest-masses, and so on .... Others have tried to describe some particles as constituted of several other "elementary" particles (generally a pair of fermions). However, up to now the only far reaching attempt for the description of the properties of particles is the field theory: the wave function of a field satisfies equations defined for a given value of the spin, and which contain the mass of the particle as a parameter. Essays have been made for describing particles with several states of mass, but until now only particles with one mass and one value of spin have been widely used for explaining or forecasting experimental data, the values of spin being restricted to 0, $\frac{1}{2}$ and 1. An excellent report on non-interacting field theory has been given by PAULI [1941], since when no essential progress has been made on the subject. For fields in interaction, formal and substantial progress has been made since 1948 (see papers of TOMONAGA, SCHWINGER, FEYNMANN, DYSON and so on), but it is more or less restricted to electrodynamics.

The only relativistic equation for spin $\frac{1}{2}$ particles that has been widely used is the Dirac equation. There are two possibilities for describing spin 0 particles: the wave function can be either a scalar, $S$, or a pseudoscalar, $P$: this is also true for spin 1 particles, the wave function being either a vector, $V$, or a pseudovector, $A$. In the following we shall indicate these four possibilities for bosons by the letters $S$, $V$, $A$, $P$.

Charged particles are described by imaginary wave functions satisfying gauge invariant equations; imaginary wave functions are also necessary to describe neutral particles with a magnetic moment: their wave functions are directly coupled with the electromagnetic field, and not with the potential, and therefore gauge invariance is not imposed. In both cases there are two states of the particle, with opposite charge (for charged particles) and opposite magnetic moments (for spin $\neq 0$ particles). Most physicists believe in the existence of negative protons and antineutrons, but this question can ultimately only be solved by experiment [1].

Photons and $\pi^{\circ}$-mesons are described by real wave functions; it is not yet known whether neutrinos must be described with two states (called in the following "neutrino" and "antineutrino") or only one state (see § 3. 1 for discussion) as is possible with the Dirac equation (MAJORANA [1937]); in this case the wave function is real if a convenient representation of Dirac's matrices is used: $\tilde{a} = a^* = a$, $\tilde{\beta} = \beta^* = -\beta$. In the following, we shall refer to fermions with only one state as "Majorana particles".

It must be emphasized that there is actually no theoretical distinction between "kinds" of particles; the isotopic spin formalism allows one to speak of protons and neutrons as the same particle, the nucleon; electrons and neutrinos as the same particle, the lepton; but this is only a more condensed way of writing, and it does not correspond to a deeper theoretical concept [2].

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[1] Searches for negative protons have up to now led to negative results. However, they have mainly been based on the study of $\beta^-$-radioactive nuclei emitting delayed neutrons (since $M_n + m > M_p$), BRODA *et al.*[1947] SUN [1949], SCOTT and TITTERTON [1950], and in this case the theory forbids the spontaneous emission of negative protons, as was pointed out by the last authors (see also MICHEL [1950b]). We would expect the main source of antinucleons to be pair production of nucleon-antinucleon in the very energetic events of cosmic rays; they will disappear by annihilation.

[2] It seems possible, for fermions, to distinguish between "kinds" of particles by the anticommutation of the wave functions of the "same" kind

For the study of coupling properties between particles we shall study "reactions" between these particles; a reaction is defined by the nature and the "charge conjugate state" of the involved particles and by the set **M** of the values of the dichotomic variable M for each particle: $M = -1$ when the particle is absorbed, and $M = 1$ if the particle is emitted. In the following we shall also use the dichotomic variables L and K: if $Ce$ is the charge of the particle, for a charged particle $L = C$, for a neutral particle with a magnetic moment L is a function of its sign (here it will be chosen as the opposite of this sign because of the neutron), and for a neutral particle without magnetic moment (photons, probably $\pi^\circ$-mesons and perhaps neutrinos) L is arbitrary. K is defined by $K = LM$. A given reaction can be characterized by **L** and **M**, these notations representing the two sets of values of L and M for each involved particle.

## 1. 3 The conservation laws

The conservation laws summarised below, with the exception of the last, apply also in classical theory. They are expressed by group invariance, this condition restricting the number of possible terms in the Lagrangian. The information given by the application of these laws is absolutely independent of the particular hypothesis for the interactions between particles and even of the imperfect state of the theory. All physicists apply them in the same order for excluding impossible cases, but the selection rules due to the two last laws have not been well known.

*Conservation of charge*:* invariance under gauge group. This can be expressed for a reaction between $n$ particles by

(1) $$\Sigma M_i C_i = 0 = \Sigma' K_i$$

$\Sigma'$ indicating summation over the charged particles only.

*Conservation of momentum and energy*: invariance under Lorentz displacement group. If $\mathbf{k}_i$ is the four vector momentum-energy of the $i^{\text{th}}$ particle, the law is: $\Sigma M_i \mathbf{k}_i = 0$.

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of particles and the commutation of the wave functions of particles of "different" kind: see, for instance, NISHIJIMA [1950] who deduces certain selection rules for decay from such a distinction. But it can easily be shown that these two ways of writing are equivalent.

*Conservation of angular momentum.* This law requires that the number of fermions involved in a reaction must be even. For a given system of particles we shall call $J$ its total angular momentum: it is the sum of the angular momenta and the spins.

A system of two bosons with opposite momenta cannot have all possible values of $J$ in the following three cases:

(a) For two photons, $J \neq 1$: LANDAU [1948], YANG [1950a], PEASLEE [1950], and STEINBERGER [1949b] quotes also WIGNER for this result.

(b) For two identical spin 0 bosons, $J$ cannot be odd (see for instance PEASLEE [1950]).

(c) For one photon and one spin 0 boson, $J \neq 0$.

*Conservation of parity.* This law has no classical correspondence, its importance was shown by WIGNER [1927].

We can decompose the Lorentz group $\mathfrak{L}$ of transformations into $\mathfrak{L} = (1 + T)(1 + R)\mathfrak{L}_+^\uparrow$ (see, for instance, BHABHA [1949]) where $T$ changes $x, y, z, t$ into $x, y, z, -t$ and $R$ changes $x, y, z, t$ into $-x, -y, -z, t$. Invariance under the group $\mathfrak{L}_+^\uparrow$ gives the conservation of angular momentum. Time has an absolute orientation for us, therefore all physical laws cannot be invariant under $T$, the reversal of the time direction (however, this non-invariance under $T$ may be limited to statistical mechanics). But, outside biology, we cannot in nature make any distinction between right-handed and left-handed frames of reference; therefore no physical law allows us to distinguish between the two kinds of frame, and they must all be invariant under transformations $R\mathfrak{L}_+^\uparrow$ exchanging two frames of different kind, i.e. they must be invariant under the group $1 + R$. However, wave functions do not need to be invariant for $R$, thence the possible distinction between $S$ and $P$ mesons, $V$ and $A$ mesons.

Here we shall give the results of parity conservation only in the case of the spontaneous decay of a particle into integral spin particles: there is no need to specify which is the initial particle and which particles are secondaries. Decay is forbidden for the following sets:

$$SSP,\ PPP,\ SSA,\ PPA,\ SPV,\ SSSP,\ SPPP.$$

(see PEASLEE [1950] for the decay into two spin 0 mesons; MICHEL [1951] for all cases) [3].

These results are very fundamental, and the only hypothesis made for their validity is that there are only scalar coupling constants (no pseudoscalar coupling constants: see § 1. 6 and § 3. 6).

## 1. 4 Coupling between fields

The field equations mentioned in § 1. 2 are deduced by a variational principle from a Lagrangian which contains quadratically the field variables or, bilinearly, these variables and their first derivatives [4]. This Lagrange function must be a scalar. The field variables $\psi$ of non-interacting fields can, by quantization, be decomposed into plane waves which describe creation and annihilation of particles when they are applied to a state vector. There are interactions between the fields when there are, in the Lagrangian, terms formed with the $\psi$'s (or the first derivatives of one of them) of different fields. Field equations are deduced in the same way, but they cannot be solved exactly; an approximate method must be used, generally the perturbation theory where the coupling terms are considered as small and their presence perturbing only the $\psi$'s of free fields. This is why we can still speak in terms of particles, of their creation and their annihilation [5].

We use the notation $\psi_i^{K_i} = \psi_i$ if $K_i = -1$, and $= \psi_i^*$ if $K_i = 1$; we can define a coupling between fields by **K**, the set of values of the $K_i$ in the interaction terms of the Lagrangian; this coupling can

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[3] The list of these sets has been given by FUKUDA *et al.* [1950a] who found for them no Lorentz invariant terms of the S-matrix in particular lowest order perturbation calculations; see also papers of ONEDA, OZAKI and SASAKI quoted in § 2. 5.

[4] It is not impossible, indeed, to have a Lagrangian containing higher derivatives of the field variables, and the equations would then be partial differential equations of higher order, but the physical solutions taken are still restricted to plane waves for non-interacting fields.

[5] All the theoretical numerical results we shall use are obtained with this weak coupling method; there are other methods: strong coupling theory, more or less phenomenological methods for multiple meson production (Heisenberg, Fermi) etc., but they are not applied to the whole problem of coupling between particles. The first question to be answered is: can the usual theory give an acceptable solution to this problem? We restrict this paper to the ordinary weak coupling theory.

contain several terms if it is possible to form several independent scalars with the involved $\psi_i^{K_i}$ or the first derivatives of one of these $\psi_i^{K_i}$. The coefficient of each term is a coupling constant. The Lagrangian must also contain for any term which is not self conjugate, its hermitian conjugate: these conjugate terms correspond to the sets $-\mathbf{K}$ of the values of $K_i$. With the Majorana representation of Dirac matrices (see § 1. 2) and appropriate notations, the K defined here are identical with the K defined in § 1. 2: therefore the possible "first order" reactions **L**, **M** for a given coupling satisfy

$$(2) \qquad L_i M_i \quad K_i \quad \text{or} \quad L_i M_i \quad -K_i$$

We shall call a "class" the set of all possible first order reactions of a coupling $\pm\mathbf{K}$.

Now the operator which transforms the function representing the initial state into that representing the final state, i.e. the S-matrix or the perturbation Hamiltonian, is developed in increasing powers of the coupling constants, and a very intuitive and useful graphical representation of the perturbation theory has been introduced by Feynmann and Dyson [1949]. The physical reaction can be considered as the result of a sequence of "first order" reactions (each represented by a vertex of the graph); these intermediary reactions are virtual, and the particles absorbed in these reactions have been emitted by a previous one, if they are not the initial particles themselves. In the graph, virtual particles are represented by lines joining two vertices, real particles by lines touching only one vertex.

*Example* 1. We consider the two couplings $f$ and $e$ responsible for the first order reactions:

$$f,\ p^+ \rightarrow p^+ + \pi^\circ; \quad e,\ p^+ \rightarrow p^+ + \gamma.$$

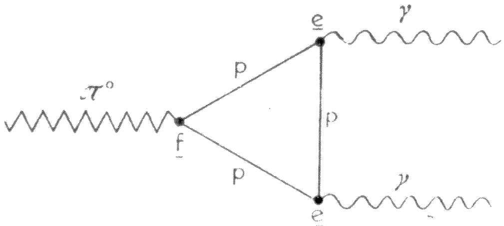



Fig. 2 Graph of $\pi^0 \rightarrow 2\gamma$

The three successive virtual reactions $\pi^\circ \to p^+ + p^-$, $p^- \to p^- + \gamma$, $p^+ + p^- \to \gamma$, give the real reaction $\pi^\circ \to 2\gamma$, and the graph of the processes is given in Fig. 2; an alternative for the second virtual reaction is $p^+ \to p^+ + \gamma$ and gives the same graph.

*Example* 2. A graph can be composed of disconnected parts; here is an example. The atomic masses of $A^{40}_{18}$, $K^{40}_{19}$, $Ca^{40}_{20}$ are respectively 39.9755, 39.9756, 39.9751 (ROSENFELD [1948a]), therefore the $\beta^-$-decay of $A^{40}_{18}$ is not possible, but the virtual reaction:

$$n \text{ (of } A^{40}_{18}) \to p^+ \text{ (of } K^{40}_{19}) + \varepsilon^- + \nu$$

followed by the reaction:

$$n \text{ (of } K^{40}_{19}) \to p^+ \text{ (of } Ca^{40}_{20}) + \varepsilon^- + \nu$$

gives a real reaction:

$$2n \text{ (of } A^{40}_{18}) \to 2p^+ \text{ (of } Ca^{40}_{20}) + 2\varepsilon^- + 2\nu.$$

The graph of this reaction is given in Fig. 3. If the neutrino is a "Majorana" particle (see § 1. 2), one can have either the graph of Fig. 3 or the graph of Fig. 4, corresponding to the reaction

$$2n \text{ (of } A^{40}_{18}) \to 2p^+ \text{ (of } Ca^{40}_{20}) + 2\varepsilon^-;$$

but if the neutrino has two states ("neutrino" and "antineutrino") only the graph of Fig. 3 is possible, as is easily seen.

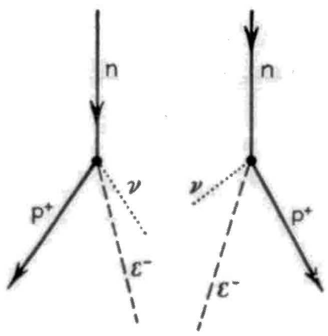



Fig. 3. Graph of $2n \text{ (of } A^{40}) \to 2p^+ \text{ (of } Ca^{40}) + 2\varepsilon^- + 2\nu$

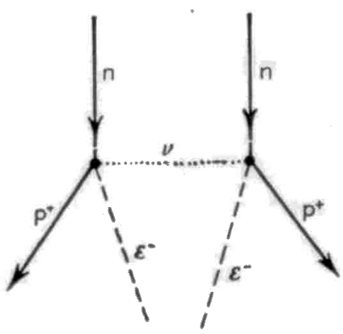



Fig. 4. Graph of $2n \text{ (of } A^{40}) \to 2p^+ \text{ (of } Ca^{40}) + 2\varepsilon^-$

The number of vertices in a graph gives the "order" $n$ of the phenomenon; this means that its probability is proportional to a homogeneous polynomial of degree $2n$ in the coupling constants involved [6].

It follows from the principles of perturbation theory that the graphs of minimum $n$ for a process give the main contribution, and the graphs with larger $n$ give only higher order (sometimes called "radiative") corrections.

From this formal study of coupling we deduce easily that the existence of a reaction **M**, **L** implies the existence of reactions −**M**, **L** (reversibility in microphysics), **M**, −**L** (charge conjugation) [7] and all the reactions (virtual or real) which have the same or opposite **K** ($K_i = L_i M_i$) from the observed one.

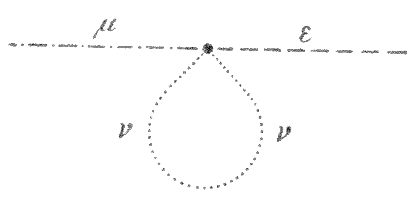
## 1. 5 Couplings commonly used

All possible couplings of the theory are not used for calculations; nearly all authors avoid the use of derivatives of Dirac wave functions and of imaginary and pseudoscalar coupling constants. A rapid review of the other (used and unused) possibilities is given here.

It is possible to form five invariants with two Dirac wave functions $\psi_1^{K_1}$ and $\psi_2^{K_2}$. With the Majorana representation (see § 1. 2) of the matrices, these invariants can be written $\psi_1^{K_1} F_i \psi_2^{K_2}$: $i = 1$ yields a scalar $s''$, $i = 2$ the four components of a vector $v''$, $i = 3$ the six independent components of a skew tensor $t''$, $i = 4$ the four components of a pseudovector $a''$, $i = 5$ a pseudoscalar $p''$. Among the 16 matrices $F$, 10 are symmetrical and 6 antisymmetrical; in the Majorana representation they are distributed so that

$$\widetilde{F}_i = -\theta_i F_i \text{ with } \theta_i = 1, -1, -1, 1, 1. \tag{3}$$

[6] Particle lines can only join at a vertex, therefore it is not coherent with the theory to use such a graph, for instance, for the coupling responsible for the reaction $\mu^- \to \varepsilon^- + \nu_+ + \nu_-$ ($\nu_+$ and $\nu_-$ are the two states of the neutrino), as was suggested by OGAWA and KAMEFUCHI [1950a] for making possible the reaction $\mu^- \to \varepsilon^-$ (in the presence of other particles intervening in the rest of the graph and assisting energy-momentum conservation).




[7] There is a complete symmetry between charge conjugate states of a particle; a correct use of Dirac's hole theory cannot give different results when one permutes the role of the holes and the particles.

If the two $\psi^{\mathrm{K}}$ are identical, from the anticommutation relations we immediately deduce:

(4) $$\psi^{\mathrm{K}} F_i \psi^{\mathrm{K}} = \tfrac{1}{2}\psi^{\mathrm{K}}(F - \widetilde{F})\psi^{\mathrm{K}} = \tfrac{1}{2}(1 + \theta_i)\psi^{\mathrm{K}} F_i \psi^{\mathrm{K}} \text{ i.e. } v'' = t'' = 0.$$

There is an arbitrary factor in the $F_i$'s; it can be removed in the coupling constants and it is then possible to have all the $F$ hermitian:

(5) $$F_i^{\dagger} = F_i$$

With the wave functions $s, v, a, p$ of $S, V, A, P$ bosons (see § 1. 2) and with the vector $\partial_\mu = \partial/\partial x,\ \partial/\partial y,\ \partial/\partial z,\ \partial/\partial t$, it is possible to form four other invariants $v_s', t_v', t_a', a_p'$.

*Case* 1. Coupling between one boson field and one fermion field. (a) charged fermion field: the boson field must be neutral, and we suppose that it is described by a real function. The only possible form of the coupling terms (see § 1. 4) is

(6) $$C = f_i \varphi_i \cdot \psi^{\mathrm{K}} F_i \psi^{-\mathrm{K}} + \text{h.c.}$$

where "·" indicates a scalar product, $\varphi$ either the boson wave function or its first derivatives and "h.c." the hermitian conjugate. Using equation (5), equation (6) can be written:

(7) $$C = (f + f^*)\varphi \cdot \psi^{\mathrm{K}} F \psi^{-\mathrm{K}},$$

and we are left with only the real part of the coupling constant; in other words, in this case the coupling constant must be real. $C$ is also charge conjugate, since (see equation (3)),

(8) $$C = \tfrac{1}{2}\varphi_i \cdot (f_i \psi^{\mathrm{K}} F_i \psi^{-\mathrm{K}} - f_i \psi^{-\mathrm{K}} \widetilde{F}_i \psi^{\mathrm{K}}) =$$
$$= \tfrac{1}{2}\varphi_i \cdot (f_i \psi^{\mathrm{K}} F_i \psi^{-\mathrm{K}} + \theta_i f_i \psi^{-\mathrm{K}} F_i \psi^{\mathrm{K}});$$

on account of the Majorana representation of matrices [8], the

[8] The Majorana representation of Dirac's matrices does not restrict the validity of the formalism; if it is not used, one has to introduce for charge conjugation a matrix C (see PAULI [1941]), the wave function of the Majorana particle is no longer real but there is imposed a more complicated condition, .... everything is, of course, equivalent, but written in a more complicated way. The Majorana representation is more convenient for all covariant calculations. Nevertheless the Dirac representation is much more used: historically it was the first, and it is more convenient for non-relativistic calculations (separation into large and small components).

second term is deduced from the first by changing **K** into $-$**K** (this changes **L** into $-$**L**, see § 1. 2 and § 1. 4) which is the charge conjugation, but we note that the coupling constants are multiplied by $\theta_i$. This important result will be used in § 1. 9.

We can now list all possible terms. There are eight. For scalar mesons: $f_S \varphi \psi^K F \psi^{-K} = f_S ss''$ and $\lambda f'_S \partial_\mu \varphi \cdot \psi^{\varkappa} F''_2 \psi^{-\varkappa} = \lambda f'_S v'_S \cdot v''$, these will be noted $Ss$ and $Sv$; for vector mesons: $f_V v \cdot v''$ and $\lambda f'_V t'_V \cdot t''$, noted $Vv$ and $Vt$; for pseudovector mesons: $f_A a \cdot a''$ and $\lambda f'_A t'_A \cdot t''$, noted $Aa$ and $At$; for pseudoscalar mesons: $f_P pp''$ and $\lambda f'_P a'_P \cdot a''$, noted $Pp$ and $Pa$. We have multiplied the coupling constants $f'$ (of the coupling terms containing derivatives of the meson wave function) by $\lambda = h/\varkappa c$, the wave length of the meson; then $f$ and $f'$ have the same dimensions since $\partial_\mu$ has the dimension of the reciprocal of a length.

If the boson field were described by an imaginary wave function, only the real part would be significant (as for $f$ in equation (7)); for this reason neutral mesons are described by neutral wave functions. From these theoretical considerations they must have no magnetic moment.

(b) neutral fermion field. If this is represented by an imaginary wave function, we can form exactly the same couplings as in (a); this is done for the neutron. There is another possibility with $f$ imaginary as well as the wave function; because of equation (4), this is only possible for $s$, $a$, and $p$ couplings. If the fermion is a Majorana particle, there is only one possibility which can be deduced from any of the previous cases by omitting the useless **K**; equation (4) shows that there are no $Sv$, $Vv$, $Vt$, $At$ couplings, and thus a Majorana particle cannot be coupled with vector mesons or with photons.

*Case* 2. Coupling between one boson field and two fermion fields. It would be significant here to use imaginary coupling constants, but it is possible by means of the isotopic spin formalism (see § 1. 8) to write the coupling terms in a form identical to Case 1(a), and if we restrict ourselves to this form we obtain the same limitations as for Case 1(a).

*Case* 3. Coupling between four fermion fields. Five invariants can be formed with the first two fields: $s''_1$, $v''_1$, $t''_1$, $a''_1$, $p''_1$, and five other invariants with the remaining two fields: $s''_2$, $v''_2$, $t''_2$, $a''_2$, $p''_2$;

by scalar products it is then possible to form five scalars $J_i$ where $J_1 = s_1'' s_2'' + \text{h.c.}$; $J_2 = v_1'' \cdot v_2'' + \text{h.c.}$ etc.... For this kind of coupling we shall use the notation $g$ for the coupling constants, and the most general coupling is a linear combination of five terms:

$$C = \Sigma g_i J_i. \tag{9}$$

It is possible here to take the $g$ imaginary, and this possibility is discussed in § 1. 6, but in this paper the $g$ will be taken as real.

One can show the following properties for the $J$ (MICHEL 1950a): if the order of the $\psi^{\mathrm{K}}$ in the $J_i$ is changed, the new $J_i$ are linear combinations of the old ones (see FIERZ 1937); if two (or more) $\psi^{\mathrm{K}}$ are identical, there are only three (or one) independent $J_i$: this result will be used in § 3. 2.

## 1. 6 Couplings not commonly used

(a) Couplings with derivatives of the fermion fields. Such a coupling was used for $\beta$-decay by KONOPINSKI and UHLENBECK [1935]; it was a direct coupling between n, p, $\varepsilon$, $\nu$ and it contained the first derivative of the neutrino wave function. It was put forward to fit experimental data, but it has since been rejected in the light of better experiments. No study has been made of meson-nucleon coupling involving derivatives of the nucleon wave functions. Some cases would be identical with the usual couplings, but most of them would give quite different results. Such couplings, with derivatives of nucleon wave functions, have probably not been used for the historical reason of correspondence with electrodynamics.

(b) Use of pseudoscalar coupling constants is allowed for the same reason as the use of pseudoscalar or pseudovector wave functions: the observable results are even functions only of coupling constants and of wave functions. Then, the coupling constants are formally written in the same way but their meaning is different. The first use of a pseudoscalar coupling constant was made (although not of intent) by FERMI [1934] in his paper on $\beta$-decay (direct coupling between four fermions); KONOPINSKI and UHLENBECK [1935] using scalar coupling constants found the same result as Fermi except for the terms containing the neutrino mass (which correspond to a change of sign of the neutrino mass), a discrepancy which has been explained by RACAH [1937]: this result can be shown to be quite general. There are no essentially new results if scalar $g$ and pseudoscalar $\hat{g}$ coupling constants are used in the same coupling, but their interference will give terms of the form $g\,\hat{g}\,\Delta$, where $\Delta$ is the only pseudoscalar which can be formed with four particles: it is the determinant of the four components of the four **k** fourvector momentum-energy of the particles; for four free particles $\Delta = 0$ on account of energy-momentum conservation ($\Sigma \mathrm{M}_i \mathbf{k}_i = 0$); for bound particles, inside a nucleus, for instance, these terms may give important results. Modifications to the transformation law for Dirac wave functions under space reflexions have been proposed by JARKOV [1950] and YANG and TIOMNO [1950]; these can be shown to be strictly equivalent to the possibility of using pseudoscalar ocupling constants. Such a use has been proposed practically by SCHOENBERG [1941]; there is no longer any distinction between scalar and pseudoscalar meson, vector and pseudovector meson. A spin 0 meson can have four coupling terms with nucleons: those already labelled $Ss$, $Sv$, $Pa$, $Pp$, while a spin 1 meson can have the four coupling terms $Vv$, $Vt$, $At$, $Aa$. In other words, spin 0 mesons can have properties of both $S$ and $P$ mesons, spin 1 mesons those of both $V$ and $A$ mesons (see calculations from ENATSU [1950]), but no other new physical effects occur when $\Delta = 0$. As we pointed out in § 1. 3, the use of pseudoscalar coupling constants would modify the form of results due to parity conservation.

(c) Imaginary coupling constants. We have seen in which cases coupling constants can be imaginary. It is easy to show that different observable results are obtained only for the terms containing the product of different coupling constants of the same coupling (as defined in § 1. 2, for instance $Ss$ and $Sv$, or the five $g_i J_i$). One can for instance have instead of the expression $g_i g_j E_{ij}$ for real $g$, the expression $|g_i| \cdot |g_j| \cdot E_{ij} \cos\theta$, $\theta$ being the phase difference between $g_i$ and $g_j$ in the complex plane. Thus, the use of imaginary coupling constants. when it is allowed, would introduce more arbitrary constants (phase differences) which would eventually decrease the importance of interference between the different terms of the same coupling; moreover, the domain of validity of Furry's theorem (see § 1. 9) would be modified.

In the following we shall use for the purposes of our discussion only the commonly used couplings of § 1. 5 and neglect the possible couplings described in the present subsection. Theory might be saved by the introduction of more arbitrary constants, but one has first to try the simplest hypotheses for fitting experimental data. We therefore explicitly make the following simplifying hypotheses for coupling: coupling constants scalar and real, derivatives of fermion wave functions excluded.

## 1. 7 Units

The best system of units for theoretical studies of coupling becomes apparent during computations. The method of introducing coupling constants $f$ and $f'$ in § 1. 5 is formally similar to the use of Heavyside units in electrodynamics, where $e^2/4\pi\hbar c = 1/137$. It is now usual to use the natural system of units where $\hbar = c = 1$, in which the product of a mass by a length is a pure number. Since $\hbar/mc = 3.85\ 10^{-11}$ cm in c.g.s. units, (electron mass, $m$) (1 cm) $= 1/(3.85\ 10^{-11})$. If $\varkappa$ is the mass of a nuclear (integral spin) meson, $(1/\varkappa)\partial_\mu$ is dimensionless (see § 1. 5, here $\lambda = 1/\varkappa$) and $f'$ as well as $f$ and $e$ is dimensionless. In this system, the values of $f^2$, $f'^2$ and $e^2$ are the values of the pure numbers $f^2/\hbar c$, $f'^2/\hbar c$ and $4\pi/137$.

Now we can express the dimensions of every quantity by a power of length or of mass by multiplying it by the requisite powers of $\hbar$ and $c$; here we choose mass, and, for instance, time has the dimensions $(\text{mass})^{-1}$. We have already chosen (§ 1. 1 table 1) the mass of the electron as our mass unit, and it is also the unit of energy and momentum. In the system where $\hbar = c = m = 1$, the unit of length is $3.85\ 10^{-11}$ cm (the Compton wavelength of the electron) and the unit of time $1.28\ 10^{-21}$ sec. (i.e. $\hbar/mc^2$); thus we multiply the values of time expressed in this system of units by $1.28\ 10^{-21}$ to bring them to seconds.

In the c.g.s system the $g$ constants are expressed in ergs.cm³. The pure number, $(g/mc^2)(\hbar/mc)^{-3}$, used by many physicists (see for instance BETHE [1947, p. 99]), is evidently their value in the system $\hbar = c = m = 1$.

## 1. 8 Isotopic variable

This name, proposed by ROSENFELD [1948a, p. 51], is better than the more usual "isotopic spin". As we pointed out in § 1. 3 this formalism is only a more condensed way of writing, which allows one formally to write the coupling of a charged and/or a neutral boson with two fermion fields (generally two nucleons or two leptons) in the same form as the interaction of one boson field with one fermion field (§ 1. 5).

(a) charged meson field $\phi$, proton field $\psi_p$, neutron field $\psi_n$; coupling,

$$C = f[\phi^* \cdot (\psi_n^* F \psi_p) + \phi \cdot (\psi_p^* F \psi_n)]. \tag{10}$$

Let us write $\phi = \phi_1 + i\phi_2$ with $\phi_1$ and $\phi_2$ real, and $\psi = \begin{pmatrix}\psi_n \\ \psi_p\end{pmatrix}$, and let us define the four hermitian matrices:

$$\tau_0 = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}, \; \tau_1 = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}, \; \tau_2 = \begin{pmatrix}0 & -i \\ i & 0\end{pmatrix}, \; \tau_3 = \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}. \tag{11}$$

Equation (10) can then be written (with $f_1 = f_2 = f$, and $\sigma = 1{,}2$):

$$C = \sum_\sigma f_\sigma \, \phi_\sigma \cdot (\psi^* \tau_\sigma F \psi). \tag{12}$$

(b) neutral meson coupled with the proton and neutron fields by the coupling constants $f_p$ and $f_n$. Let us define $f_3 = \frac{1}{2}(f_n - f_p)$ and $f_0 = \frac{1}{2}(f_n + f_p)$. The coupling term:

$$C' = f_n \phi \cdot (\psi_n^* F \psi_n) + f_p \phi \cdot (\psi_p^* F \psi_p), \tag{13}$$

can be written as in equation (12) with $\sigma$ taking the values 0 and 3. With $\sigma$ taking the values 1, 2, 3, 0 all these coupling terms can be condensed into the form of equation (12) with the condition $f_1 = f_2$ only. The extension to the $f'$ is obvious.

## 1. 9 Furry's theorem

This theorem applies to the graphs containing closed loops of fermions. For instance, let us take the graph of Fig. 2. It represents elements of the S-matrix proportional to $f\,e^2$. We have seen (§ 1. 4, example 1), that this graph represents two possible alternatives obtained from each other by exchange of $p^-$ and $p^+$. Because of the charge conjugation of the formalism, this shows that the values of the corresponding terms of the S-matrix are obtained from each other by the exchange of $e$, $f$ and $\theta_2 e$, $\theta_i f$, ($\theta_2 = -1$, see equations (3), (8)), therefore if $f$ is the constant of one of the couplings *Sv*, *Vv*, *Vt*, *At*, the sum of the terms represented by this graph is equal to 0 (since $1 + \theta_i = 0$).

Now it is easy to show for a reaction between real bosons through intermediary fermion fields, that when these real bosons introduce an odd number of couplings of given kinds, any graph contains at least one closed loop of fermion lines with an odd number of these couplings (because virtual particles introduce only an even number of couplings of any kind, and any partition of an odd integer contains at least one odd integer). Thence Furry's theorem:

*T*1. A reaction between neutral bosons through one intermediary fermion field is forbidden if the total number of $v$ and $t$ couplings is odd.

However, we must emphasize the following restriction: only one coupling term is allowed for $S$ and $A$ bosons; $V$ and $P$ bosons can have both their coupling terms ($Vv$ and $Vt$, $Pp$ and $Pa$) since they are equivalent for this theorem, evidently $Vv$ and $Vt$ or $Pp$ and $Pa$ of the *same* meson are only counted as one. This theorem was proved by FURRY [1937] for electrodynamics, and its evident extension to neutral fields was indicated.

The isotopic variable formalism suggests an extension. The coupling $f_3$ (see § 1. 8) will be equivalent for this theorem to the $v$ and $t$ couplings, when one exchanges protons and neutrons in the graph considered, since $f_n = -f_p$ for neutral mesons with a pure $f_3$ coupling ($f_0 = 0$). Therefore, with the supplementary restriction that, for neutral mesons, either $f_3$ or $f_0$ are equal to 0, we transform *T*1 into *T*2:

*T*2. A reaction between charged and neutral mesons through the nucleon field is forbidden if the number of $f_3$, $v$, $t$ couplings is odd.

This extension of *T*1 was first enunciated by FUKUDA and MIYAMOTO [1949c] (see also VAN WYCK [1950b]); they pointed out that *T*2 cannot be applied to photons (as *T*1) since for photons $f_n = 0$, i.e. $-f_3 = f_0 = 0$. They also extend *T*1 to the case of neutral mesons interacting with nucleons, but we disagree with their result: it is true that charge conjugation gives *T*1; moreover, to each closed loop of protons we have to add the same closed loop of neutrons, this contribution being deduced from the first by multiplication by $(-1)^x$, $x$ being the number of $f_3$ couplings.

*T*3. A reaction between neutral mesons through the nucleon field is forbidden if either, the number of $f_3$ couplings is odd, or, the total number of $v$ and $t$ couplings is odd.

We must emphasize that the selection rules due to Furry's theorem are valid to *any* order of the perturbation calculation.

It is of course possible to extend the theorem to $g$ couplings: for instance, if we consider the direct coupling leptons-nucleons or $\mu$, $\nu$, n, p, we can still apply $T2$, replacing "charged mesons" by "charged mesons, electron-neutrino pairs and $\mu$-meson-neutrino pairs". The coupling constants $g$ do not change by charge conjugation, but the proof of $T2$ is based on the exchange, for the isotopic variable, of n and p, and the exchange of n and p change $g_i$ into $\theta_i g_i$. Therefore the extension of $T2$ is:

$T4$. A reaction between charged and/or neutral mesons and pairs of one charged and one neutral fermion through the nucleon field is forbidden if the total number of $v$, $t$, $f_3$, $g_2$, $g_3$ couplings is odd.

For example, the decay of the $\pi$-meson can be represented by the graph Fig. 5 from two classes of reactions:

$$\pi^+ + \text{n} \rightarrow \text{p}^+, \quad \text{n} \rightarrow \text{p}^+ + \varepsilon^- + \nu.$$

There are selection rules for this decay (valid to all orders) arising from $T4$, see § 4. 1.

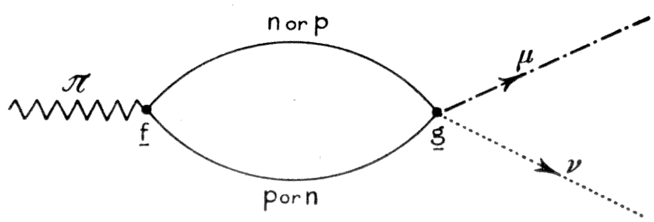



Fig. 5. Graph of $\pi^\pm \rightarrow \mu^\pm + \nu$

## 1. 10 Equivalence theorem, radiative corrections, renormalization and regularization

It can be shown (Nelson [1941], Dyson [1948], le Couteur and Rosenfeld [1949], Case [1949b]) that in most calculations to the second order in nucleon-meson coupling, the coupling $Pa$, with a coupling constant $f_p'$, can formally be replaced by a coupling $Pp$ with a coupling constant $f_p = f_p' (M_n + M_p)/\varkappa$; and the coupling $Sv$ (with constant $f_s'$) replaced by the coupling $Ss$ with coupling constant $f_s = f_s' (M_n - M_p)/\varkappa$. This property is generally called the "equivalence theorem". It is reasonable to put $M_n = M_p = M$, and then the $Sv$ coupling gives no contribution when the equivalence theorem applies (for instance for second order nuclear forces).

In recent years, developments of electrodynamics have been completely successful in explaining the Lamb shift and the anomalous magnetic moment of the electron (SCHWINGER [1948]); these results were obtained with the ideas of charge and mass renormalization which enable one to separate small physical effects from the divergences inherent in the present state of the quantum field theory. This handling of infinite quantities is quite delicate, and it is only successful in electrodynamics and neutral *Vv* meson theory. PAULI and VILLARS [1949] have proposed a "safer" and more powerful, but more arbitrary, way of handling these mathematically meaningless quantities: regularization. For many decay processes, the results we shall quote have been obtained with the regularization procedure, but this will always be specified.

One of the simplest radiative corrections in meson theory leads to the magnetic moments of nucleons; for pseudoscalar meson theory, calculations have been carried out (in the lowest order) both with old and recent methods, all give the same result (LUTTINGER [1949a], SLOTNICK and HEITLER [1949], CASE [1948, 1949a], BOROWITZ and KOHN [1949]), but this result does not fit with experimental data. LUTTINGER [1949b] however showed that the values of the magnetic moments of n and p can be found for a mixture of *Vv* and *P* mesons, but with very large values of the coupling constants. In any case, these results cannot be a test for meson theories.

On the other hand, results obtained by regularization must not be too firmly "believed" (see §§ 2. 2, 2. 5, 2. 6 and 4. 1): they can be quite different from those obtained by older methods (when these work) and they are sometimes ambiguous. Broadly, regularization has failed in its aim: there is at present no *a priori* method able to handle all divergence difficulties which can be encountered in calculations. As we shall see, many conclusions can be drawn from much safer theoretical grounds.

## 2. π-MESONS AND HEAVIER MESONS

### 2. 1 Nuclear forces

π-mesons can safely be identified with the particles of Yukawa's hypothesis for explaining nuclear forces, but the best experimental confirmation of the theory does not come from nuclear forces. Up to now, meson theories have failed to explain high energy nucleon-nucleon scattering. They are not even completely successful for low energy scattering or for the consideration of the constitution of nuclei, but in these cases the particle aspect of the meson is not so important, and these phenomena do not allow one to choose between the various meson theories. Successful attempts to explain "magic numbers" of protons and neutrons in nuclei have used a shell model involving strong spin-orbit coupling. Now only vector mesons can give large spin-orbit interaction between two nucleons, and they would therefore lead to a strong spin-orbit coupling between an "external" nucleon and the core of the nucleus. But the converse argument is not true: the existence of strong spin-orbit coupling (not yet accounted for) in the shell model of nuclei does not necessarily imply the existence of such an interaction between two nucleons; it is actually very doubtful if such a direct relation exists (see notes, added in proof, of CASE and PAIS [1950]).

The most important bearing of nuclear forces on meson theories comes from the equality of n—n, p—p, n—p forces observed for low energy (S wave) scattering and from the study of mirror nuclei. This leads to only two possibilities for the meson theory:

*pure neutral theory*: couplings given by equation (12) with $f_1 = f_2 = f_3 = 0$; there are only neutral mesons, coupled to protons and neutrons with the same strength ($f_n = f_p = f_0$);

*symmetrical meson theory*: in equation (12) put $f_1 = f_2 = \pm f_3$ and $f_0 = 0$; there are charged and neutral mesons, the coupling constants ("mesic" charge of the nucleons) of the neutral meson and of the charged meson have the same absolute value, but for the neutral meson $f_n = -f_p$.

It is not possible to have a mixture of symmetrical and neutral theory with the *same* neutral meson, since then $f_n$ and $f_p$ are different, i.e. n—n and p—p forces are different (see ROSENFELD [1948b] p. 137). Of course one cannot exclude *a priori* a mixture of charged and neutral mesons of different kinds, with coupling constants such that the charge independence of nuclear forces is satisfied in its experimental domain of validity, on the ground that such an hypothesis is unaesthetic! But as we shall see (§ 2. 4), experimental data on $\pi^{\pm}$ and $\pi^0$ mesons (without excluding the possible existence of heavier nuclear mesons), outside the field of nuclear forces, are strongly in favour of charged and neutral pseudoscalar mesons.

Pseudoscalar meson theory has particular features, not shared to the same extent by the other theories: relativistic effects and, notably, higher order effects are very important. For the second order interaction between nucleons, the equivalence theorem is valid. However, the numerical values of the coupling constants for the second order still depend on the method of calculation; even the exact solution of the relativistic equations of the deuteron for the second order potential (see VAN HOVE [1949] for this potential) gives a logarithmic divergence (LEVY [1951]). The value of $f_p'$ (*Pa* coupling) necessary to explain low energy scattering and the binding energy of the deuteron is about $f_p'^2 = 0.15$, and this coupling is formally equivalent to *Pp* coupling with $f_p = (2M/\varkappa)f_p'$, which gives for $f_p^2$ about 40. Now the fourth order interaction between nucleons gives rise to divergences for *Pa* coupling, but it has been calculated for *Pp* coupling by BETHE [1949] and by WATSON and LEPORE [1949], and these authors have shown the importance of the fourth order terms (and terms of still higher order), which decrease the value of $f_p^2$ to about 4 when they are taken into account.

## 2. 2 The nature of the $\pi^\circ$-meson

The neutral $\pi^0$-meson has been observed through its decay products (two photons) since the end of 1949, both in cosmic rays (POWELL, Edinburgh Conf. Nov. 1949) and produced at Berkeley (CRANDALL, MOYER and YORK, American Phys. Soc. meeting, Stanford Dec. 1949); under both conditions it was produced in nucleon-nucleon collisions. The mass has been measured (Table 1) and the mean life evaluated to be between 1 and $5 \times 10^{-14}$ second (CARLSON *et al.* [1950]). Production by photons on nuclei has also been observed (STEINBERGER *et al.* [1950]). The existence of this particle had been assumed from observations (FRETTER [1948, 1949], CHAO [1949] and others) and from theoretical studies (for example HEITLER and POWER [1947], GREISEN [1948]) of the cosmic ray soft component and of extensive and mixed showers;

see also KAPLON *et al.* [1949] and MARSHAK [1949c] for the observation and study of a very large star (in a photographic emulsion) produced by an energetic $\alpha$-particle and containing a narrow shower of about 23 charged mesons and 35 $\gamma$-rays.

The discovery of the neutral $\pi^0$-meson is an important success for the meson theory which had long predicted its existence and its decay into photons. In § 1. 3 we have seen that the conservation of angular momentum forbids the decay of a spin 1 particle into two photons, therefore we conclude that the $\pi^0$-meson has spin 0. But the quantitative theoretical results, based on the graph Fig. 2, are not so attractive. Calculations were first made by FINKELSTEIN [1947] for *Pp* mesons, who found a lifetime of $10^{-16}$ sec. TANIKAWA [1947] using damping theory found $10^{-24}$ sec. and FUKUDA *et al.* [1949b] showed that some terms were not gauge invariant, and proposed to suppress them. The use of the regularization procedure is very necessary in this case, but the results are ambiguous. FUKUDA and MIYAMOTO [1949a] and STEINBERGER [1949b] have made independent calculations; both give alternative values and they are not completely comparable. Steinberger's results, with the values of $M$ and $\varkappa'$ given in Table 1, give for *Ss* and *Pp* mesons: $\tau_s = f_s^{-2}\, 10^{-14}$ sec., $\tau_p = f_p^{-2} \cdot 4\; 10^{-15}$ sec.; in both $\tau$ is proportional to $\varkappa'^{-3}$. For the pseudoscalar meson $\tau$ is too small; however it is not known if the equivalence theorem applies in this case, and the author gives another alternative for *Pa* coupling, where $\tau$ is proportional to $\varkappa'^{-7}$ and $\tau \cdot (2Mf'/\varkappa')^2 = 2\; 10^{-11}$ sec. One dare not argue from these results concerning the nature of the $\pi^0$-meson, (except that *Sv* cannot decay into two photons, being forbidden by Furry's theorem).

There is a possibility of deciding the nature of $\pi^0$-mesons which is based only on fundamental laws (parity conservation): for a scalar meson, the two secondary photons will always be polarized in parallel planes, while for a pseudoscalar meson the planes of polarization of the two secondary photons will always be perpendicular. This can be tested by experiment, and YANG [1950b] has proposed to study the angular distribution of pairs produced by these two photons in order to determine the parity (even for *S*, odd for *P*) of the $\pi^0$-meson.

We shall resume this discussion in § 2. 4, after examining evidence about the nature of the $\pi^{\pm}$-meson.

## 2. 3 The nature of the $\pi^{\pm}$-meson

The reaction $\pi^- + p^+ \rightarrow n + \gamma$ shows that the $\pi^-$-meson is a boson. It was pointed out by WENTZEL [1949] that $\pi^{\pm}$-mesons produced in the Berkeley cyclotron might show a polarization effect for star production or for decay into $\mu$-mesons if they have spin 1. Theoretical calculations by FEER [1949b] have been made on the polarization of mesons produced by photons at the energy threshold; here too polarization effects would give information on the nature of the coupling $\pi$-meson to nucleon. Anisotropic effects do not seem to have been observed either for artificially produced mesons or for cosmic ray mesons (the latter, however, are probably irrelevant, since polarization effects decrease as energy increases).

An interesting phenomenon for deciding the spin of $\pi$-mesons seems to be their production by photons on nucleons. Experiments were first reported by MCMILLAN *et al.* [1949a, b]: in the laboratory system, meson production by a $dE/E$ photon energy spectrum (Bremsstrahlung photons) is nearly isotropic between 45° and 135°, and the total cross-section decreases slightly with increasing meson energy. BRUECKNER [1950a] showed from semi-classical arguments as well as from perturbation theory (lowest order) that the experimental distribution is in agreement with calculation for $A$ or $P$ mesons; furthermore a very characteristic feature of spin 1 mesons is the rapid increase of their production cross-section with energy, and thus the properties of a spin 1 $\pi^{\pm}$-meson seem to be incompatible with the experimental data. The ratio of $\pi^-/\pi^+$ production (about 1. 7 in the reported experiments) is also more in favour of a $P$ meson than an $S$ meson (see also ARAKI [1950] and BENOIST–GUEUTAL *et al.* [1950] for this last point). FERRETTI and GALLONE [1950] have proposed the study of the low energy scattering of $\pi$-mesons by deuterium as compared with scattering by hydrogen in order to decide the parity of spin 0 mesons. A more decisive experiment, however, is possible. WIGHTMAN [1950] has shown that $\pi^-$-mesons moderated in hydrogen or deuterium fall into the K-orbit before decaying or being captured, and this has been verified by experiment. Then the capture of a meson by deuterium giving two neutrons:

$$\pi^- + D^+ \rightarrow n + n \tag{14}$$

will be very different for $S$ and $P$ mesons, because of the parity conservation law. If $p$ is the parity of the meson at rest, the parity of the system $D^+ + \pi^-$ (in the K-orbit) is $p$, since D is even and $L$ the orbital momentum $= 0$: the total angular momentum is therefore $J = 1$ (deuteron spin), and the final state which is formed of two neutrons must also have $J = 1$. According to the exclusion principle this state is odd, and the initial state must therefore also be odd [9], i.e. $p$ is odd, and the meson cannot be scalar if (14) takes place.

The absorption of $\pi^-$ by deuterium has been observed at Berkeley (AAMODT *et al.* [1950]): among the three possible reactions (14), (15), (16):

(15) $$\pi^- + D^+ \rightarrow n + n + \gamma$$

(16) $$\pi^- + D^+ \rightarrow n + n + \pi^0$$

(16) does not occur to the limits of experimental accuracy, (15) has been observed directly while (14) has not been observed directly. However, the existence of (14) can be deduced by subtraction, and the branching ratio (14)/(15) calculated in this way is about 2. The most extensive theoretical comparative study of these reactions has been made by TAMOR and MARSHAK [1950]; results depend chiefly on the parity and spin of the meson, and not on more subtle points of meson theories. For $S$ and $V$ mesons the ratio (14)/(15) is respectively 0 and 55, and these two possibilities can be confidently excluded. Only in the case when $\pi^-$ has spin 0 and the same parity as $\pi^0$ is (16) very improbable ((16)/(14) about $10^{-4}$), in the other cases: $\pi^-$ spin 1, or $\pi^-$ spin 0 while $\pi^-$ and $\pi^0$ of opposite parities, the occurrence of (16) is comparable to that of (15). Therefore, although the existence of an $A$ $\pi^-$-meson is not strictly impossible, these experiments are strongly in favour of $\pi^-$ and $\pi^0$ both pseudoscalar. Moreover, in this case the equivalence theorem does not apply, and pure $Pa$ and pure $Pp$ coupling give respectively for the ratio (14)/(15), the values 2.1 and 4.1; the second of these seems therefore to be ruled out.

[9] A similar argument shows that the existence of the reaction $p^+ + p^+ \rightarrow D^+ + \pi^+$ (see PETERSON [1950]), if it is confirmed at the threshold of energy, would be unfavourable to the hypothesis of a scalar $\pi^+$-meson. On the other hand, the observation of the angular distribution of the products would give an indication of the spin of the meson, PRENTKI [1951].

According to recent calculations by CHESTON [1951] the absorption of a $\pi^+$-meson by deuterium at sufficiently high energy (25 MeV.) will give $\pi^+ + D^+ \to 2p^+$ with a not too small cross-section (of the order $10^{-25}$ cm$^2$), and the observation of the ratio of the intensities of secondary protons at 90° and 180° (centre of mass system) would lead to a definite conclusion for the spin of the meson, and, in the case of spin 0, for its parity.

### 2. 4 Are $\pi^\pm$ and $\pi^\circ$ mesons symmetrical pseudoscalar mesons?

The existence of the reaction: $p^+ + \pi^-$ (in K-orbit) $\to n + \pi^0$ also shows that if $\pi^-$ has spin 0, $\pi^-$ and $\pi^0$ have the same parity (application of parity conservation only).

Summing up the discussion, we have seen that:

$\pi^0$ has spin 0 (decay into two photons),

$\pi^-$ cannot be $S$ or $V$ or, very probably, $A$ (absorption by deuterium),

$\pi^\pm$ cannot have spin 1 (slightly decreasing cross-section of meson production by photons with energy).

We therefore conclude that $\pi^\pm$ is probably pseudoscalar, and it is then implied that $\pi^0$ is also pseudoscalar. (Several other arguments support the same conclusion.) Then, as we have seen in § 2. 1, the charge independence of nuclear forces requires a symmetrical set of $\pi^\pm$, $\pi^0$ mesons.

The comparable production of charged and neutral mesons by nucleon-nucleon collisions in accelerators (CRANDALL *et al.* [1950]) and in cosmic rays (CARLSON *et al.* [1950] give for the production ratio in large stars $N(\pi^0)/N(\pi^\pm) = 0.45 \pm 0.10$) shows that the coupling strengths of charged and neutral mesons are comparable. However, there is one case in which this ratio of production would be expected to be small (about $\varkappa'^2/M^2$) according to the theory: this is meson production by photons on nuclei. But the ratio has been found experimentally to be about 1 for Bremsstrahlung photons from the 320 MeV. accelerator (STEINBERGER *et al.* [1950], STELLER and PANOFSKY [1951]). This experimental result has been discussed by BRUECKNER and WATSON [1950b]. We have already seen how important the contribution of higher terms in symmetrical pseudoscalar meson theory can be for nuclear forces (§ 2. 1); by a consideration of higher order effects for meson production by photons, Brueckner and Watson have shown that pseudoscalar theory (and this theory only, among symmetrical theories) is not necessarily incompatible with experiments.

There is an experiment which, besides a precise determination of the mass $\varkappa'$ of the neutral $\pi$-meson, can give more information on its coupling constants; this is the capture of $\pi^-$-mesons in hydrogen, which has been examined by PANOFSKY *et al.* [1950a, b] at Berkeley. The following reactions have been observed:

(17) $$\pi^- + p^+ \rightarrow n + \gamma$$

(18) $$\pi^- + p^+ \rightarrow n + \pi^0 \quad (\text{and } \pi^0 \rightarrow 2\gamma).$$

From equation (18), it has been found that $\varkappa - \varkappa' = 10.6 \pm 2.0$. The study of these reactions has been carried out by MARSHAK and WIGHTMAN [1949b] (there are several errors, however), OGAWA and YAMADA [1950b] (for spin 0 mesons only), and more extensively by MARSHAK *et al.* [1950]. The branching ratio (18)/(17), observed to be $0.96 \pm 0.20$, is a function of either $f_3$ or $f_0$ (see § 1. 8) of the neutral meson. However the authors have not made calculations for the case of pseudoscalar symmetrical theory. The more refined calculations of AIDZU *et al.* [1950] give for pure *Pp* coupling (symmetrical theory) $f_p^2 = 2.8$ (we have seen in § 2. 1 that nuclear forces give $f_p^2$ = about 4). The equivalence theorem does not hold in this case. A promising way for deciding between *Pa* and *Pp* coupling is to study the dependence of the cross-section of the $\pi$-meson reactions in hydrogen with energy; this was proposed by ASHKIN *et al.* [1950].

What is the value of pseudoscalar symmetrical theory for explaining nuclear forces? For n — p and p — p scattering above 50 MeV., meson theories have not yet explained the experimental data, and this is not surprising if $\pi$-mesons are pseudoscalar. In pseudoscalar theory, as we have already stated (§§ 2. 1 and 2. 4), higher order corrections are very large, and it is probably illusory to expect more detailed quantitative results from the weak coupling perturbation method; it is necessary to look for more appropriate methods in meson field theories for relativistic calculations. One may hope that the great importance of the higher order terms of the perturbation theory is directly related to "multiple" meson production in nucleon-nucleon collisions, which received strong experimental support in 1950 from particular stars observed in photographic emulsions, and which can be studied by more phenomenological methods (LEWIS *et al.* [1948], HEISENBERG, FERMI).

## 2. 5 Neutral V-mesons

The existence of a neutral particle heavier than the $\pi$-meson was first reported by ROCHESTER and BUTLER [1947]. Now its existence is well established: the particles are observed, chiefly in cosmic ray penetrating showers, by recognition of the two charged products of decay, which appear as V-shaped tracks in the cloud chamber. For a survey of the status of experimental investigations on these particles and their decay products, we refer to the article by BUTLER in this volume. We observe, however, that it is probably not at present possible to identify the meson secondaries certainly as $\pi$-mesons or as $\mu$-mesons. On account of the origin of the V-particles in nuclear events, and their relatively great abundance, it is unlikely (and probably incompatible with the observed smallness of the interaction between $\mu$-mesons and nucleons), however, that $\mu$-mesons are a product of decay of these particles.
$V'^0$. We shall label V′ the particle which would decay according to:

(19) $$V'^0 \to p^+ + \pi^-.$$

$V'^0$ would have half integral spin, and can be considered as an unstable state of the neutron comparable to those predicted in strong coupling theories (see, for instance, ROSENFELD [1948a] p. 19), or in a theory of particles of several possible rest-masses.
$V^0$. We similarly denote by V the particle which would decay according to:

(20) $$V^0 \to \pi^+ + \pi^-.$$

The most likely hypothesis is to consider $V^0$ as a nuclear meson decaying through its nuclear coupling. To explain such a reaction in a coherent way, we must show that other likely reactions due to the same hypothetical coupling are not more probable: these reactions could, very probably, only be decays into two particles:

(21) $$V^0 \to 2\gamma$$

(22) $$V^0 \to 2\pi^0$$

(23) $$V^0 \to \gamma + \pi^0$$

(24) and even $$V^0 \to \varepsilon^- + \varepsilon^+.$$

The conservation laws (§ 1. 3) and Furry's theorem (§ 1. 9) can then give very interesting results: these results are independent of the nature of $\pi^{\pm}$ and of a spin 0 $\pi^0$-meson, since (with the restriction only that they do not have both *Ss* and *Sv*, *Aa* and *At* or $f_3$ and $f_0$ couplings, see § 1. 9) $\pi^+ + \pi^-$ or $2\pi^0$ introduce an even number of $v$, $t$, and $f_3$ couplings. Furry's theorem (T2) does not give any selection rules for equation (20) but gives the nature of the coupling of V° ($f_3$ or $f_0$); then T3 gives selection rules for equation (22). A summary of the results is given in Table 2.

TABLE 2

| Nature of $V^0$ | *Ss* | *Sv* | *Vv* | *Vt* | *At* | *Aa* | *Pa* | *Pp* | |
|---|---|---|---|---|---|---|---|---|---|
| (21) $V^0 \rightarrow 2\gamma$ | . | T1 | A | A | A | A | . | . | |
| (22) $V^0 \rightarrow 2\pi^0$ | . | T3 | A | A | A | A | P | P | |
| (23) $V^0 \rightarrow \gamma + \pi^0$ | A | A | . | . | . | T1 | A | A | |
| (24) $V^0 \rightarrow \varepsilon^+ + \varepsilon^-$ | T1 | ? | . | . | . | T1 | T1 | T1 | |
| (20) $V^0 \rightarrow \pi^+ + \pi^-$ | . | ? | . | . | P | P | P | P | (spin 0 $\pi^{\pm}$) |
| | . | ? | . | . | . | . | . | . | (spin 1 $\pi^{\pm}$) |
| Coupling of $V^0$ | $f_0$ | $f_3$ | $f_3$ | $f_3$ | $f_3$ | $f_0$ | $f_0$ | $f_0$ | (by T2) |

A forbidden by angular momentum conservation
P ,, ,, parity conservation
T ,, ,, Furry's theorem T1, T2, or T3
? ,, in lowest order if equivalence theorem is valid
Order of application: A, P, T and ?.

We see in Table 2 that if $\pi^{\pm}$ had spin 1 and V° were an Aa meson, only reaction (20) would be allowed and all the others would be forbidden. However, we have concluded in § 2. 4 that $\pi^{\pm}$ is probably pseudoscalar. We are then left with two possibilities: V° is either *S* or *V*.

V° *scalar.* The coupling *Sv* forbids (21), (22), (23) and is acceptable. (This coupling is never very attractive to theoretical physicists, because it does not contribute to nuclear forces in the lowest order). The coupling Ss allows decay into $2\gamma$ or $2\pi°$. The second decay gives no difficulty, the lifetime for the processes (20) and (22) being roughly equal and twice the apparent lifetime of V°, but we know from the observations on the $\pi°$-meson that the decay of a neutral nuclear meson into two photons is very rapid. Even if all charged particles in penetrating showers were $\pi^{\pm}$-mesons, the relative abundance $V°/\pi°$ is $\geqslant 0.06$; hence, if $F$ is the nuclear coupling constant of V°, neglecting the difference of mass of V° and $\pi^\circ$ (the heavier mesons would be relatively more difficult to produce), we have $F^2/f^2 \geqslant 0.06$. On the other hand, a heavy meson decays more easily into two photons than a lighter meson with the same coupling (in § 2. 2 we have seen that for an *Ss* meson $\tau$ is proportional to $\varkappa'^{-3}$); even neglecting this, we find for the two photon decays $\tau_V < \tau_\pi(1/0.06) = 16\ \tau_\pi$, thus there is a difficulty with this hypothesis, since experimentally $\tau_V$ is already about $10^4$ times larger than $\tau_\pi$.

V° *vector*. The reaction (23) is also very rapid. We are obliged, for studying this question to rely on quantitative predictions of the theory, but it seems better to draw no conclusions! With the new methods, extensive calculations for all reactions have been made by FUKUDA *et al.* [1950a, b, d] and by ONEDA, OZAKI, SASAKI (see OZAKI *et al.* [1949], ONEDA *et al.* [1950], SASAKI *et al.* [1950] and OZAKI [1950]). The reaction (23) has also been studied by FINKELSTEIN [1947], who gives a lifetime $10^{-18}$ sec., and by VAN WYCK [1949] for charged mesons (his result corresponds, for neutral mesons, to a lifetime $4.\ 10^{-16}/F^2f^2$ sec.). We can say from these calculations that (20) is more probable than (23), at least for *V* V°-mesons and *Pp* $\pi^\circ$-mesons, but its calculated mean life is too small. The best conclusions from the very interesting and extensive calculations of the Japanese physicists are to be drawn on the value of regularization; many results were divergent and they applied regularization, in some cases the divergence is reduced to zero instead of a finite result and the reaction is forbidden (in this order), in other cases the procedure gives ambiguous results. Moreover, if the regularization procedure is applied to all cases (divergent or finite) it also modifies finite results and leads to more ambiguity. Reaction (24) has been studied by FUKUDA and MIYAMOTO [1949d] and by STEINBERGER [1949b]; it is relatively slower.

Summing up, if reaction (20) is observed with the lifetime $10^{-10}$ sec., it favours V° being *Sv*, $f_3$ or *V*(*t* or even *v*), $f_3$ or even, but much less likely, *Ss*, $f_0$.

## 2. 6 Heavier charged mesons, V and $\tau$

The existence of heavier charged mesons has been reported for several years, the first by LEPRINCE–RINGUET and L'HÉRITIER [1944] who measured a meson mass about 1000. Now there is considerable experimental evidence; however these particles appear in three different kinds of phenomena, and for each the experimental data is very scarce. Here is a summary of the data, (for details, see the paper by BUTLER in this volume).

*1st kind of events*: one charged secondary of decay, observed in the same conditions as V°. We have still less knowledge of $V^{\pm}$ than of V°. The lifetime of the charged particle is probably rather shorter than that of the neutral particle (SERIFF *et al.* [1950]); the charged secondary is probably a $\pi$-meson, and mass of about 1000 or 1200 would be consistent with two particle decay.

*2nd kind of events*: a charged meson at low energy produces a star in a photographic emulsion, among the prongs is a $\pi$-meson. The mass of the initial meson is accordingly intermediate between $M$ and $\varkappa$, and is estimated by the authors reporting the three recorded examples (LEPRINCE–RINGUET [1949], WAGNER and COOPER [1949], FORSTER [1950]) to be greater than 700.

*3rd kind of events*: decay into three charged particles. One case reported by BROWN *et al.* [1949], two by HARDING [1950]. Among the secondaries there is certainly one $\pi^-$-meson, and the other two particles are probably $\pi$-mesons also. If this is so, the kinetic energies of secondaries are measurable in two cases, and an accurate value of mass of the primary follows: about 940 (using $\varkappa = 271$). From the energy and range of the initial particle (3 mm in the event observed at Bristol), the mean life cannot be shorter than about $10^{-11}$ sec.

The heavy mesons responsible for the second and third kind of events are usually described as $\tau$ mesons. The similarity of the first kind of events with the observations on neutral V′-mesons of course suggest the reaction

$$V'^{+} \rightarrow n + \pi^{+}, \tag{25}$$

$V'^{+}$ being an excited state of the proton; since there are some negative V′ particles antinucleons must also be introduced. However, this hypothesis is unable to explain events in the other two categories, while all three kinds of events suggest that $V^{\pm}$ may be a nuclear meson. Is it possible to explain these different events in a single scheme? The hypothesis of the reaction

$$V^{\pm} \rightarrow \pi^{\pm} + \pi^{\circ} \tag{26}$$

would lead to difficulties, since then the reaction

$$V^{\pm} \rightarrow \pi^{\pm} + \pi^{\pm} + \pi^{\mp} \tag{27}$$

involving one more meson would be less probable and not observed. The existence of (27) implies the existence also of

$$V^{\pm} \rightarrow \pi^{\pm} + 2\pi^{\circ} \tag{28}$$

which can be used to explain events of the first kind. Since it is understandable that decays with only one charged particle may be missed or not identified in photographic emulsions (and some observations have been reported which could be interpreted as (28)), it would be noteworthy if events (28) had been identified in the cloud chamber without the detection of reaction (27), which would be equally probable.

However, the Manchester group (ARMENTEROS *et al.*, reported by BLACKETT at the Harwell conference 1950) has obtained one cloud chamber picture suggesting

$$V^{\pm} \rightarrow V^{\circ} + \pi^{\pm} \tag{29}$$

while BRODE stated at the same conference that he also had obtained evidence supporting this reaction. It is then conceivable to identify the three kinds of $V^{\pm}$ decay as due ot the same particle, if (26) is forbidden while (27), (28) and (29) are allowed. Furry's theorem gives this result with the restriction only that $\pi^{\pm}$ and $\pi^{\circ}$ form a symmetrical theory (see § 1. 8).

Table 3 is drawn up with the supplementary hypothesis that $\pi$-mesons are pseudoscalar (§ 2. 4). It shows that if we accept (27) for $V^{\pm}$, this meson cannot decay into two particles according to (26) but only according to (29). It shows that the $V^{\pm}$ meson must be either a $P$ (pseudoscalar) or an $Aa$ meson. If it is $P$, the reaction

$$V^{\pm} \rightarrow \pi^{\pm} + \gamma \tag{30}$$