is forbidden. One can expect that (since $V^{\pm}$ is probably less strongly coupled with nucleons than π), (27) is only a little slower than (29) and so the three kinds of event could be attributed to the same particle. Calculations have been carried out for the

TABLE 3

Nature of V	*Ss*	*Sv*	*Vv*	*Vt*	*At*	*Aa*	*Pa*	*Pp*
(26) $V^{\pm} \to \pi^{\pm} + \pi^0$	T2	.	.	.	P	P	P	P
(29) $V^{\pm} \to V^0 + \pi^{\pm}$	P	P	T2*	T2*	T2	.	.	.
(27) $V^{\pm} \to \pi^{\pm} + \pi^{\pm} + \pi^{\mp}$	P	P	T2	T2	T2	.	.	.
(28) $V^{\pm} \to \pi^{\pm} + 2\pi^0$								
(30) $V^{\pm} \to \pi^{\pm} + \gamma$	A	A	.	.	.	.	A	A

A forbidden by angular momentum conservation,
P forbidden by parity conservation,
T2 forbidden by Furry's theorem; T2* also forbidden by parity conservation if V is scalar.

reactions of Table 3 by POWER [1949] (*P* mesons only), FUKUDA *et al.* [1950b, c, d], ONEDA *et al.* [1950] and OZAKI [1950]; since the calculations are divergent they require regularization and all authors find coherent values which can agree with experiment (10^{-10} to 10^{-11} sec.) for the lifetime of (27); the decay (30) into a meson and a photon is very rapid when it is allowed (see § 2. 5 and also POWER [1949] and FUKUDA and MIYAMOTO [1950d]), and this rules out the possibility $V^{\pm}$ pseudovector. But when $V^{\pm}$ is pseudoscalar, even the reaction

$$V^{\pm} \to \pi^{\pm} + 2\gamma \tag{31}$$

would be a little more rapid than (28): by a factor 2 according to POWER [1949] (see also VAN WYCK [1950a]). As we have seen in § 2. 5, there are important difficulties for V° decay (and also for (29)) if these calculations using regularization are taken too seriously. There is another difficulty: the τ-meson of the third kind of event appears to have a mass about 950, which gives for V° and $V^{\pm}$ smaller mass values than those commonly reported.

Summing up, we can say that the hypothesis of a charged *P* (or even *Aa*) meson of mass about 1000 is not ruled out for explaining *all* experimental data, up to now, on heavy charged mesons. There are also arguments for the existence of nuclear mesons heavier

than π-mesons from the study of nuclear forces [10]; however, relativistic calculations on neutron-proton scattering (MICHEL, 1950, unpublished) at the second order of perturbation show that a mixture of scalar or vector (Vv) V°-meson and of pseudovector (Aa), or pseudoscalar $V^{\pm}$-meson (according to the results of §§ 2. 5 and 2. 6, but with more arbitrary masses) and of symmetrical pseudoscalar π-meson cannot fit the experimental data on n — p scattering at 90 MeV (see also MARTY and PRENTKI [1950a], MARTY [1950b]), but it must be remembered that even relativistic calculations are not reliable at the second order for pseudoscalar meson theory.

3. INTERACTION BETWEEN FOUR FERMIONS

3. 1 The neutrino

The existence of the neutrino was postulated by PAULI to preserve the existence of the conservation laws (momentum, energy and angular momentum) in β-decay. Up to now the neutrino has escaped detection, i.e. no neutrino induced effects have been observed [11]. It is possible to make an energy-momentum balance for a few β-activities (see CRANE [1948] for a review of the search for the neutrino): the (not yet very precise) experimental relations between the missing momentum and energy can be satisfactorily explained by the emission of a zero rest mass particle, which is called neutrino. Angular momentum conservation requires for it spin $\frac{1}{2}$.

When a π-meson decays into a μ-meson, some energy and momentum are also missing; this can be satisfactorily explained by the simultaneous emission of a zero rest mass particle.

[10] For instance, the range of nuclear forces at low energy corresponds to a heavier mass (about 330) than those of $\pi^{\pm}$ and π° mesons, and this may point to a contribution of heavier mesons with attractive sign (ROSENFELD at the Harwell Conference). From high energy data, JASTROW [1951] has proposed the existence of a short range repulsive core in the nuclear potential, which also could be an effect of heavier mesons. Unhappily, these arguments are contradictory!

[11] Cross-sections for neutrino induced β-decay are of the order 10^{-44} cm^2. However, it is possible that such an effect will in time be detected with the always increasing sources of neutrinos constituted by the large piles (PONTECORVO, lecture at Manchester, 1949).

O'CEALLAIGH [1950] has searched for effects induced by this particle along its path and has shown that it cannot be a photon.

When a μ-meson decays to one electron, the lacking energy and momentum cannot be explained by the simultaneous emission of one particle, since the secondary electrons of μ-decay at rest have a continuous spectrum of energy between 0 and 55 Mev. (see LEIGHTON *et al.* [1949], LAGARRIGUE and PEYROU [1951] and also for qualitative work STEINBERGER [1948, 1949a] and DAVIES *et al.* [1949]). On the other hand, when the electron is emitted with maximum energy, the particle (or particles) capable of carrying away the missing energy and momentum must have negligible rest mass (or rest masses). Therefore μ-meson decay can be coherently explained by the emission of an electron and two zero rest mass particles. The nature of these particles has been investigated by several physicists, and the work of HINCKS and PONTECORVO [1950] has shown that they are not photons (observed photons are shown to be created by Bremsstrahlung of the electrons).

Since, in these three decays the emission of neutral particles with zero rest mass "characterized" by the absence of detectable effects is assumed, the simple hypothesis is that they are in each case the same particle, the already named neutrino. This assumption implies that the μ-meson is a fermion like the electron, and that the π-meson is a boson, in agreement with the other experiments on π-mesons (§ 2. 3) and on μ-mesons (CHRISTY and KUSAKA [1941]).

We have seen in § 1. 2 that there are two theoretical possibilities for spin $\frac{1}{2}$ neutral particles: they can be described either with two "charge conjugate" states or with only one (Majorana particle). The existence of a magnetic moment for the neutrino would eliminate the second possibility. The experiment of NAHMIAS [1935], according to BETHE's [1935] calculations, gives for the neutrino magnetic moment an upper limit of 1/5600 Bohr magneton (see also BARRETT [1950] who gives a smaller cross-section for neutrino interaction with atomic electrons, but for less energetic neutrinos). We have seen (§ 1. 4) that if the neutrino is a Majorana particle, double β-decay is possible without the emission of neutrinos (Fig. 4). This was first pointed out by FURRY [1939], who also showed that the double β-decay without the emission of neutrinos is about 10^{10} times more rapid than the other possibility (Fig. 3).

FIREMAN's [1949] experiment was in favour of the rapid process (Fig. 4), but all other more recent experiments do not support it (INGHRAM and REYNOLDS [1949, 1950], LEVINE *et al.* [1950], LAWSON [1951]). Except for μ-meson decay, in all the following there is theoretically no possible distinction between the two alternatives; reactions will be written without distinguishing the two possible states of the neutrino.

3. 2 μ-meson decay

It can easily be shown that:

$$\mu^{\pm} \to \varepsilon^{\pm} + \nu + \nu \tag{32}$$

cannot be implied by the existence of any known reaction. This will appear very clearly in § 4. The indirect coupling (μ, ν)—π—(ε, ν) would probably be the most rapid; calculations can be made from our knowledge of the lifetime of the π-meson for the two spontaneous decays:

$$\pi^{\pm} \to \mu^{\pm} + \nu \tag{33}$$

$$\pi^{\pm} \to \varepsilon^{\pm} + \nu. \tag{34}$$

Measurements of the lifetime $\tau_{\pi\mu}$ of equation (33) have been made by RICHARDSON [1948] $0.89 < \tau < 1.42$; MARTINELLI and PANOFSKY [1950] $1.80 < \tau < 2.13$; KRAUSHAAR *et al.* [1950] $1.32 < \tau < 1.98$; CHAMBERLAIN *et al.* [1950] $2.47 < \tau < 2.77$ (values given in units 10^{-8} sec.) [12]. The decay (34) has not yet been observed, and so a lower value of its mean life can be given, $\tau_{\pi\varepsilon} > 10^{-6}$ sec. (according to the results of FRIEDMAN and RAINWATER [1951], see also POWELL [1950]). Therefore the lifetime of the μ-meson for decay (32) through a virtual π-meson would be much too great (to be measured in months!). We are thus led to the hypothesis of a direct coupling between the four fermions $\mu, \varepsilon, \nu, \nu$. We have seen in § 1. 5 that the most general (used) coupling is a linear combination of five terms $g_i J_i$. Calculations have been made for some particular couplings by TIOMNO *et al.* [1949a] and for the general coupling by MICHEL [1949] [13].

[12] Authors seem to consider only statistical errors, and to neglect possible systematic errors.

[13] In both papers the effect of the acceleration of the electric charge,

Michel's results are as follows: if E is the energy of the electron, $1 \leqslant E \leqslant W = (\mu^2 + 1)/2\mu$, the energy spectrum is given, (omitting negligible terms), by

$$P(E) = (1/24\,\pi^3)\mu E(E^2 - 1)^{\frac{1}{2}}(WQ_1 - EQ_2) \tag{35}$$

where Q_1 and Q_2 are two quadratic forms of the g. The mean life of the meson is $\tau_\mu = 2.15 \pm 0.07 \times 10^{-6}$ sec. (CONVERSI and PICCIONI [1946]), and gives the condition:

$$\int_1^W P(E)\,dE = 1/\tau_\mu. \tag{36}$$

If we suppose that only one g is $\neq 0$, this condition gives its value, of the order 10^{-12} (or 10^{-49} erg. cm^3); this is also the magnitude of the Fermi constant, as was pointed out by KLEIN [1948]. We shall study this interesting fact in § 3. 3. For the comparison of equation (35) with experimental data, (36) gives the scale of each theoretically possible curve. It is easy to see that (36) imposes a linear condition between Q_1 and Q_2, and therefore the whole family of theoretical curves depends only on a linear parameter ϱ (a function of the g_i); all the curves pass through a single point (at $E = 3W/4$) and sweep a certain area when ϱ takes all possible values. The agreement with experimental data is very satisfactory (see MICHEL [1950a] and LAGARRIGUE and PEYROU [1951]), the experimental curves of Leighton *et al.* and Lagarrigue and Peyrou pass through the imposed point and are shaped like the theoretical curves. Of course, other hypotheses can fit the experimental data, (see TIOMNO [1949c] where the μ-meson is assigned spin 0 and a new spin 0 neutral particle is introduced), but the hypothesis studied here is the simplest, for it does not involve any new particle, and it assumes the simplest possible coupling.

Now we have to distinguish between two cases: (i) the two emitted neutrinos in μ-meson decay are distinguishable (one "neutrino" and one "antineutrino" are emitted, probably with opposite magnetic moment); (ii) the two emitted neutrinos are

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carried successively by the meson and by the electron, has been neglected. This will probably lead to the emission of a photon with energy a few per cent of the electron energy, according to the preliminary results of ABRAGHAM and HOROWITZ [1951], and is negligible for the following discussion (compare FEER [1949a] who makes calculations for a two particle decay of the μ-meson, or HANAWA and MIYAZIMA [1950] for $\pi - \mu$ decay).

identical and must satisfy the Pauli exclusion principle. From the remark in the last paragraph of § 1. 5, we know that in case (ii) there are only three independent J_i (when the order $\mu, \varepsilon, \nu, \nu$ is chosen for the wave functions in the J_i, equation (4) shows that $J_2 = J_3 = 0$), and in this case the area swept by all possible curves is smaller than, and included within, the area for case (i). If the experimental curves had been in the part of the area (i) outside the area (ii), it would have proved that there are two (charge conjugate) states of the neutrino, but this is not the case, and the experimental curve certainly lies in the common area.

Knowledge of the spectrum gives the value of the linear parameter, ϱ, i.e. a quadratic relation between the g. This is important only if other restricting hypotheses are made, as in § 3. 3.

3. 3 The radioactivity of the neutron

The fact that the same coupling constant could explain both β-radioactivity and μ-meson decay is attractive. When meson theory was developed, and the (wrong) meson and its instability discovered in cosmic rays, β-radioactivity was explained through a virtual meson (see Fig. 1), and the (earlier) direct coupling went out of fashion. However in the present state of affairs there are difficulties in describing β-radioactivity through intermediary virtual nuclear mesons (see § 4. 3), and on the other hand we have seen in § 3. 2 strong arguments for a direct coupling between $\mu, \varepsilon, \nu, \nu$, in order to explain μ-meson decay. The similarity of the coupling strength with that of β-radioactivity invites a closer comparison between these two phenomena.

The study of β-radioactivity is not quantitatively simple because n and p are not free particles but are bound in the initial and final nuclei, except in the case of neutron decay. The large production of neutrons in piles has now made a study of the radioactivity of the neutron possible (SNELL *et al.* [1950], ROBSON [1950, 1951]).

We shall therefore directly compare the decay (32) with

$$\mathrm{n} \to \mathrm{p}^+ + \varepsilon^- + \nu, \tag{37}$$

explaining them by the same coupling $g_i J_i$. From the remark in the last paragraph of § 1. 5, we know that the J_i depend on the order in which the four wave functions are written in these expressions; more precisely, if we do not pay attention to the per-

mutations of this order which change only the sign of the J_i, we have only to specify how the four wave functions are paired with each other by the two operators F_i. For β-radioactivity it is natural to have the two nucleons (n, p) with one operator, and the two leptons (ε, ν) together. Since reaction (32) has a common pair (ε, ν) with reaction (37), we are led to compare the pair (μ, ν) with the nucleons (n, p). Passing from the case $\mathrm{n} \leftrightarrow \nu$, $\mathrm{p} \leftrightarrow \mu$ to the case $\mathrm{n} \leftrightarrow \mu$, $\mathrm{p} \leftrightarrow \nu$ changes only the sign either of g_2 and g_3, or of g_1, g_4 and g_5 [14].

With this chosen order, a pure g_5 or a pure g_1 interaction for μ-meson decay seems ruled out by experimental data. The next subsection will also be devoted to β-radioactivity, we need at the moment only the two following preliminary results concerning the nature of the interaction.

A pure g_5 (the socalled "pseudoscalar") interaction is ruled out [15]. Because of the low energy of the nucleons, the dependence of the lifetime in β-radioactivity on the g is quite different for g_5 and for the other coupling terms, and for a pure g_5 interaction the lifetime depends chiefly on the change of the kinetic energy of the nucleon during the decay. This variation of kinetic energy is quite irregular among the different nuclei, and it would lead to quite irregular values of "$F\tau$" (for a definition of $F\tau$, see equation (39)). Moreover, this variation is small in the case of the neutron as compared with other nuclei, and a value of g_5 based on the lifetime of some families of nuclei would give a neutron mean life of several days or months (about 10^4 too long).

The neutron decay spectrum, as far as it is known, is of the allowed shape; knowledge of the shape of spectrum for an allowed transition gives a quadratic relation between the g when one uses a linear combination of the five terms $g_i J_i$. This was done first by FIERZ [1937], who found, besides the main term responsible for

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[14] Different authors use different conventions for the sign of the J_i, since there are no theoretical reasons for the choice of this sign (and even of the relative phases of the $g_i J_i$ if one does not reject the use of imaginary coupling constants). Therefore comparison of results in different papers sometimes involves a change of sign of some g. Here, the signs of the J_i have been implicitly defined by the notation of § 1. 5.

[15] This was kindly pointed out to me by Dr KOFOED–HANSEN; see also MARSHAK [1949a] footnotes 8 and 11.

the shape of the allowed spectrum, the term $\pm(g_1g_2 - 3g_3g_4)/E$, where E is the energy of the electron and $\pm$ refers to $\beta^{\pm}$-decay. This term would seriously modify a part of the allowed spectrum, and would then be in contradiction with experimental data, if it were more than a few per cent of the main term. In the following it will be neglected.

With these two remarks on the nature of the coupling, the condition corresponding to equation (36) is, for the case of the neutron:

$$(g_1^2 + g_2^2 + 3g_3^2 + 3g_4^2) = (2\pi)^3/4F\tau_n \tag{38}$$

where F is the integral of the energy distribution, $(1 \leqslant E \leqslant W)$:

$$F(W) = \int_1^W (E^2 - 1)^{\frac{1}{2}} (W - E)^2 \, dE. \tag{39}$$

We see that for the neutron the product $F\tau$ depends only on the coupling constants. For complex nuclei it also depends on the nuclear matrix element, the product $F\tau$ is therefore the criterion for the classification of nuclei according to their β-decay. (Of course the charge Ze of the nuclei cannot be neglected for not too small Z, then F is a more complicated function of E and Z and is different for β^+ and β^- decays.) In Table 1 we have given $W = 2.53$ and the corresponding value of F is 1.63; (it must be noted that for the neutron F is very sensitive to the value of W, the older value of which, (2.47), gives $F = 1.39$).

For μ-meson decay we saw (§ 3. 2) that two cases must be considered. We now make comparison with these.

(i) *distinguishable neutrinos.* The relation (36) is written

$$g_1^2 + 4g_2^2 + 6g_3^2 + 4g_4^2 + g_5^2 = 6\,(2\pi)^3/(\tfrac{1}{2}\mu)^5\tau_\mu = (2\pi)^3/2280\,\tau_\mu, \tag{40}$$

(it happens to be independent of the order of the particles in the J_i). Among all possible direct couplings there is one remarkable one: it is the determinant made with the four indices of the four wave functions (it is therefore invariant under any permutation of the order of the four particles); it was first proposed by CRITCHFIELD and WIGNER [1941], and in the notation of this paper it is defined by

$$g_1 = -g_4 = g_5 = g_0, \text{ and } g_2 = g_3 = 0. \tag{41}$$

With the value $\tau_\mu = 2.15 \times 10^{-6}$ sec., equation (40) gives $g_0 = 3.3 \times 10^{-12}$ (or 1.5×10^{-49} erg.cm^3), and for the mean life of the neutron (from equation (38))

$$\tau_0 = 18.8 \text{ min.} \tag{42}$$

This value is quite acceptable. For the study of the other possible couplings, we define λ by

$$\tau_n = \lambda\tau_0 = 18.8\,\lambda \text{ min.} \tag{43}$$

According to the experiments of SNELL *et al.* [1950], and of ROBSON [1950], the "half-life" τ_n' of the neutron is between 10 and 25 minutes; i.e. the mean life $\tau_n = \tau_n'/\ln 2$ is between 14 and 36 minutes, the corresponding values of λ being $0.75 < \lambda < 1.92$. The experimental determination of λ will give (from equations (38) and (40)) the following quadratic relation between the g:

$$(2 - 3\lambda)g_1^2 + (8 - 3\lambda)g_2^2 + 3(4 - 3\lambda)g_3^2 + (8 - 9\lambda)g_4^2 + 2g_5^2 = 0, \tag{44}$$

(the terms discarded in this formula are less than 3 % of those kept).

(ii) *identical emitted neutrinos.* The values of g_0 and τ_0 are not changed, but equation (44) is replaced by:

$$\begin{cases} \tfrac{3}{4}(1 - 4\lambda)g_1^2 + 3(2 - \lambda)g_2^2 + 9(1 - \lambda)g_3^2 + 3(2 - 3\lambda)g_4^2 + \tfrac{3}{4}g_5^2 \\ + g_1(2g_2 - 3g_3 - 2g_4) + 4g_2g_4 + g_5(\tfrac{1}{2}g_1 + 2g_2 + 3g_3 - 2g_4) = 0 \end{cases} \tag{45}$$

It is remarkable that mean lives differing by a factor 10^9 can be explained by the same coupling. We have then to make assumptions about the nature of the (admitted) direct coupling terms g_iJ_i responsible for the β-decay of nucleons (this is the object of the next subsection), and then test, using equations (35), (36) and (44) or (45), the hypothesis of common coupling between $(\mu, \nu)(\varepsilon, \nu)$ and (n, p)(ε, ν).

3. 4 Nature of the coupling of β-radioactivity

We suppose a direct coupling, and note (§ 3. 3) that:

g_5 cannot be much larger than the other g, (46)

and FIERZ's [1937] term must be small; here we shall assume:

$$g_1g_2 = 0 \qquad g_3g_4 = 0. \tag{47}$$

For complex nuclei Fierz's term is $\pm (A\, g_1 g_2 - B\, g_3 g_4)$, where A and B are functions of the nuclear matrix elements: since A/B varies with the nuclei, the smallness of Fierz's term implies $g_1 g_2$ and $g_3 g_4$ separately small.

Now we must proceed from the simplest hypothesis: can a single g give a satisfactory interaction? Historically, this question was first raised for the g_2 ("vector") interaction, that proposed by FERMI [1934]. Using the isotopic variable formalism (see § 1. 8), let us write ψ and ψ^* the initial and final states of the nucleon field, and φ and φ^* the corresponding functions of the lepton field. Because of the low velocity of the nucleons, the density of the interaction Hamiltonian reduces to

$$h = g_2 J_2 = g_2(\psi^*\psi)(\varphi^*\varphi). \tag{48}$$

For nuclei, ψ^* and ψ differ from zero only inside the nucleus, neglecting the Coulomb field of the nucleus, φ and φ^* are normalized plane waves describing the free leptons. Their wave lengths are much larger than the radius of the nucleus, and so inside the nucleus $\varphi^*\varphi$ is nearly constant and equal to 1. If the nucleus changes its spin and/or its parity during the transition, the initial and final states are orthogonal, and therefore

$$\int h\, dv = g \int \psi^*\psi\, dv = 0. \tag{49}$$

It is easily seen that the $g_1 J_1$ term gives the same result. Therefore β-decay of nuclei is allowed for g_1 or g_2 interaction only if

$$\Delta I = 0, \text{ no} \tag{50}$$

(I, spin of nucleus, "no" means no change of parity). Even with the same spin and parity, the initial and final states of the nucleus can have quite different structures (change in super multiplet, WIGNER [1939]) then $\int J_2\, dv$ is small compared with unity, these transitions are called "unfavoured allowed". However, for nuclei having the same structure ("mirror" nuclei) this term can have nearly its maximum value.

When equation (49) is satisfied, the main contribution for the transition comes from neglected terms, (a) the term in $g_2 J_2$ proportional to the velocity, which gives a contribution of about 1 % when the main term is $\neq 0$, and (b) the first term of the development of $(\varphi^*\varphi - 1)$, which gives about the same contri-

bution. Terms (a) and (b) give the main contribution to the "first forbidden" transitions, and (a) still gives an allowed shape for the spectrum. If these terms are also equal to zero, one must consider the second term of the exponential $(\varphi^*\varphi - 1)$ or the product of the first forbidden terms to get the main contribution to the "second forbidden" transitions, and so on ... (see KONOPINSKI [1943] for a general survey of β-radioactivity).

As we said in § 3. 3, the value of $F\tau$ is the criterion for recognizing the degree in which a transition is forbidden. We have the value of F for the neutron, and therefore for the neutron

$$F\tau_0' = 1275, \tag{51}$$

τ_0 has been defined in equation (42), and since $F\tau'$ the relative half-life is more widely used than $F\tau$ the relative mean life, we recall here that

$$\tau_n' = \lambda\tau_0' = 13\,\lambda \text{ min.} \tag{52}$$

Many nuclei are known with $F\tau'$ of some 10^3. Few spins of nuclei (chiefly of unstable nuclei) and still fewer parities have been determined experimentally, but other theoretical considerations can give this information (shell structure for nuclei has had great success). One of the best established theoretical rules is:

(53) "nuclei with even numbers of both n and p have spin 0"

and these nuclei have even parity.

Several examples are known of "allowed" transitions between nuclei for which we would have expected $\Delta I = 1$. A historic transition is

$$He^6 \rightarrow Li^6 + \varepsilon^- + \nu; \tag{54}$$

He^6 has spin 0 according to (53) while Li^6 has spin 1 (measured by MANLEY and MILLMAN [1937]), but $F\tau' = 590$, the smallest value known! (All values of $F\tau'$ given here, except that of the neutron, have been calculated by TRIGG and are given by HORNYAK *et al.* [1950]). This and other transitions led GAMOW and TELLER [1936] to propose, instead of a g_2 interaction, a g_3 or g_4 ("tensor" or "pseudovector") interaction, since the corresponding selection rules for allowed transitions would then be

$$\Delta I = 0 \text{ or } \pm 1 \text{ (except } 0 \leftrightarrow 0), \text{ no.} \tag{55}$$

There have, however, been some difficulties, according to theoretical expectations, for these G—T selection rules also. A well known example is

$$\text{Be}^{10} \rightarrow \text{B}^{10} + \varepsilon^- + \nu, \tag{56}$$

expected to be very similar to reaction (54), since there is one α-particle more in each nucleus, but here $F\tau' = 7.9 \times 10^{13}$. Spin measurements for B^{10} give 3, and so $\Delta I = 3$. On this basis, MARSHAK [1949a] made a theoretical study of the spectrum of this reaction, and predicted a unique possible shape for the spectrum; this has been well confirmed by experiments (it also necessitates G—T selection rules if the theoretical prediction of even parity for B^{10} is reliable). Other cases of uniquely predicted spectra have also been well verified, notably for the family found since 1949, chiefly among fission products: Y^{91}, Y^{90}, Sr^{89}, Sr^{90}, Sr^{91} ... (see WU [1950] for general survey); this family has β-decay, according to shell structure predictions, $\Delta I = 2$, yes; and this imposes G—T selection rules. Another difficult case is

$$\text{C}^{14} \rightarrow \text{N}^{14} + \varepsilon^- + \nu, \tag{57}$$

of the same form as (54) and (56). C^{14}, an even-even nucleus has spin 0 (see (53)) and even parity, the spin of N^{14} has been measured and found = 1 while its parity is expected to be even; the transition is therefore expected to be allowed, but $F\tau' = 9.3 \times 10^8$, too high even for an unfavoured allowed transition. (GERJUOY [1951] has proposed measurements of the parity of N^{14} since an odd parity would solve the difficulty; on the other hand, the ground state of N^{14} is a mixture of $^3S_1$ and $^3D_1$; the difficulty would be solved if the contribution of the ^3S part is negligible, for instance, destroyed by interference).

The β-radioactive O^{14} discovered by SHERR *et al.* [1949] is the mirror nucleus of C^{14}. The structures of C^{14} and O^{14} are: an α-particle + all $P_{3/2}$ states filled + a pair of $P_{1/2}$ neutrons (for C^{14}) or of $P_{1/2}$ protons (for O^{14}). The higher energy of O^{14} is due to a larger Coulomb interaction. There is also an excited state of N^{14}, denoted here $*N^{14}$, which also has the same configuration (α-particle + all $P_{3/2}$ states filled + one $P_{1/2}$ neutron and one $P_{1/2}$ proton), see Fig. 6, and the Coulomb energy calculations fit very well with the data. SHERR *et al.* [1949] found that O^{14} decays to the state $*N^{14}$, they

observed the 2.3 Mev. γ-ray and found for the $F\tau'$ of O^{14} the value 3000, which is normal for an allowed transition between mirror nuclei. The three nuclei C^{14}, $*N^{14}$, O^{14} have spin 0 and the same parity (even), but $0 \leftrightarrow 0$ transitions are forbidden for G—T selection rules (55) and allowed, on the contrary, by the Fermi selection rules (50) (see HORNYAK *et al.* [1950]). The transition $O^{14} \rightarrow N^{14}$ (ground state) has not been observed, and this gives for the corresponding $F\tau'$ the value 2.10^6, difficult to explain with G—T selection rules; however, this is not a new difficulty since the expected $F\tau'$ of this reaction must be about 10^9, i.e. the value of the $F\tau'$ for the identical reaction (57).

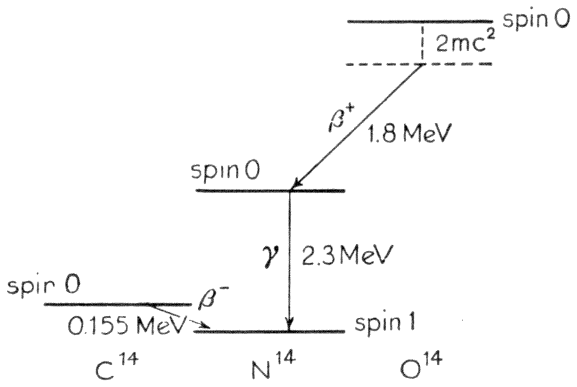

Fig. 6. C^{14}, N^{14}, $*N^{14}$, O^{14} nuclei

Must we admit a mixture of F (Fermi) and G—T selection rules? According to equation (47), the only possible mixtures are: g_1, g_3; g_2, g_3; g_2, g_4; g_1, g_4, and if a third term is assumed it can only be g_5. The necessity of such a mixture has been claimed by LONGMIRE *et al.* [1949] for the reaction

$$Cl^{36} \rightarrow A^{36} + \varepsilon^- + \nu. \tag{58}$$

The spin of Cl^{36} has been measured and found $= 2$; no γ-rays are observed, and therefore the transition is into the ground state of A^{36} which is expected to have spin 0, according to (53), and even parity. The authors say that no calculated spectra for $\Delta I = 2$, "no" (and even "yes") will fit the experimental curve. They interpret this by a mixture of g_1, g_3; g_2, g_4 or even g_2, g_3 but not g_1, g_4. From the published curves it does not seem to be altogether convincing that a pure g_2 or g_3 coupling is ruled out, but in any case the conclusions, that *either* g_2 *or* g_3 *are necessary* and that any

mixture of g_1, g_4, g_5 is ruled out, seem well established. More recent study, LANGER and MOFFAT [1951], for the decay of Cs^{137}, expected $\Delta I = 2$, also rules out any combination of g_1, g_4, g_5. A pure g_3 interaction could explain this spectrum, but the decay of O^{14} has ruled out pure G—T selection rules.

There is still another kind of experiment which can give information on the nature of the coupling terms; it is the experiments on the recoil of the nucleus which give information on the angular correlation between the emitted electron and neutrino (BLOCH and MØLLER [1935]). Theoretical calculations have been made by HAMILTON [1947] for allowed and first forbidden transitions on the hypothesis of only one coupling term. The following is the correlation function for neutron decay and for the most general coupling satisfying conditions (46) and (47):

$$(59)\quad \begin{cases} C(\theta) = 1 + \alpha v \cos\theta, \\ \text{with } \alpha = (-g_1^2 + g_2^2 + g_3^2 - g_4^2)/(g_1^2 + g_2^2 + 3g_3^2 + 3g_4^2), \end{cases}$$

(θ is the angle between the two leptons, v is the electron velocity in c units). For complex nuclei the corresponding formula is given by equation (50) in DE GROOT and TOLHOEK [1950]. (See also TOLHOEK's thesis for proposed experiments on the polarization of β-rays to give more information on the nature of the interaction). A review of experimental data has been given by CRANE [1948]; experiments are still difficult and the results sometimes contradictory (for example, difficulties arise when the source is on solid material). Recent experiments with a gaseous source, He^6, performed by ALLEN *et al.* [1949] give an a which can be from 0 to $-1/3$ or $-1/2$; a result which seems to be against a mixture of g_2 and g_3. Very recently, systematic calculations of forbidden decays with the general direct coupling (linear combination) have been announced by TRIGG [1951].

Summing up, we see how recent and still inconclusive the experimental data are which allow us to make hypotheses on the nature of the terms of the (direct) coupling responsible for β-radioactivity. To-day, the best hypothesis seems to be a mixture of g_1, g_3 or g_2, g_4 or even perhaps g_2, g_3; but there are still difficulties not overcome by the theory. However, the situation is very promising: better recoil experiments and more precise determinations of the neutron

half-life can be expected, more measurements of spin (and even of parity) together with improvements of the shell model will permit fruitful analysis of forbidden spectra; transitions with ΔI equal to the degree of forbiddeness, will give information on the mixture of coupling terms. We can already say that the good experimental agreement with uniquely predicted spectrum shapes for allowed and for some forbidden transitions, and also K-capture lifetimes (not reviewed here) are strong verification of the hypothesis of coupling terms of the form $g_i J_i$ (with the g_i constants and not a function of the energy, as happens for some coupling terms with an intermediate virtual meson as in Fig. 1 scheme). Further, the introduction in the J's of derivatives of the wave functions (as proposed by KONOPINSKI and UHLENBECK [1935] for explaining inaccurate experimental data) are completely ruled out.

3. 5 The capture of μ-mesons by nuclei

We noted in the introduction that μ-meson capture was observed in Pb and Fe, but not in light nuclei. Since this behaviour was unexpected (for nuclear mesons!), many experiments were carried out with various materials from Be to S, by SIGURGEIRSSON and YAMAKAWA [1947, 1949], VALLEY [1947], TICHO [1947a, 1948b, c], NERESON [1948], KISSINGER and COOPER [1948], TICHO and SCHEIN [1947b, 1948a], VALLEY and ROSSI [1948]. It was discovered by the last four authors that while the lifetime of the μ^+-meson was constant, the lifetime of the μ^--meson decreases with increasing Z (charge of nucleus); the quantitative results verified the theoretical predictions of WHEELER [1947] who showed that the μ^--meson first falls into a Bohr orbit of the nucleus and then must be captured without electron emission (see also WHEELER [1949]) with a lifetime of capture $\tau_c \sim Z^{-4}$, giving the μ-meson an apparent mean life:

$$\tau = \left(\frac{1}{\tau_c} + \frac{1}{\tau_\mu}\right)^{-1}. \tag{60}$$

Since τ_μ (for decay into electron) is independent of Z, τ_c can be given by:

$$\tau_c = \tau_\mu (Z_0/Z)^4 \tag{61}$$

where, from experiments, Z_0 has been found to be about 11.

That the μ-meson falls into a Bohr orbit of the nucleus was directly shown by the observation of the photons it emits during its jumps to lower quantum number orbits, CHANG [1949]; (HINCKS [1951] gives, for the total energy of the γ-rays emitted for each meson, about 15 MeV). How is the meson captured from the lowest orbit? No nuclear explosions are observed [16], and this rules out the hypothesis of transfer of all the meson rest energy to the nucleus. We must therefore assume that a light neutral particle is emitted, carrying away most of the available energy (about 90 MeV). No high energy photons have been observed (PICCIONI, [1948]) and it is natural to identify this neutral particle with the neutrino (PONTECORVO, [1947]) as we have already done in three processes (§ 3. 1) where energy (and momentum and also angular momentum) is missing; this hypothesis avoids the introduction of a new particle and it is coherent, since it gives spin 1/2 for the μ-meson.

We have therefore to assume the reaction

$$\mu^- + \mathrm{p}^+ \to \mathrm{n} + \nu. \tag{62}$$

Even with the emission of the 0 rest-mass neutrino, the neutron receives a non-negligible energy (about 10 to 20 MeV). Such an excitation of the nucleus has been studied by TIOMNO and WHEELER [1949b] and ROSENBLUTH [1949], who showed that the most probable event is the loss of one neutron or sometimes two. GROETZINGER and MCCLURE [1948], SARD *et al.* [1948, 1949], have detected these neutrons by coincidence methods. More recent experiments give quantitative results for the number of emitted neutrons by captured mesons: 1.90 ± 0.24 (CROUCH, [1951], 1.40 ± 0.30 (CONFORTO and SARD, [1951]).

What is the nature of the coupling responsible for reaction (62)? This reaction can be explained from the two classes of reactions

$$\mathrm{p}^+ + \pi^- \to \mathrm{n} \text{ and } \pi^- \to \mu^- + \nu. \tag{63}$$

by the graph of Fig. 7 (virtual π-meson). The value of the coupling constants f and $\check{f}$ are determined from nuclear forces and from the

[16] μ-mesons can produce nuclear disintegrations (EVANS and GEORGE [1949], see also COCCONI and COCCONI–TONGIORGI [1951]) but this is another phenomenon due to the (probably electromagnetic) interaction of the nuclei with fast, instead of bound, μ-mesons.

lifetime of the π-meson; LODGE [1948], PUPPI [1948, 1949], LEE *et al.* [1949], TAKETANI *et al.* [1949] have shown that these values of f and $\check{f}$ give quantitative agreement with the experimental data for the reaction (62). On the other hand, reaction (62) can be attributed to a direct coupling between n, p, μ, ν, and as has been pointed out by these authors, and by TIOMNO and WHEELER [1949b], the necessary coupling strength is about the same as for β-radioactivity (by a direct coupling) and for μ-meson decay: 10^{-12} (i.e. 10^{-49} ergs.cm.^{3}).

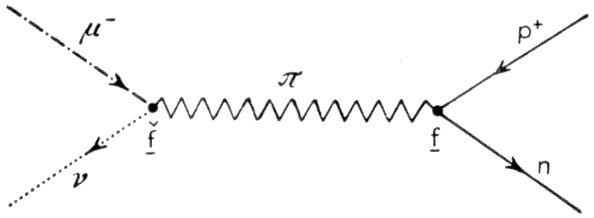

Fig. 7. Graph of $\mu^- + p^+ \rightarrow n + \nu$

3. 6 Universal direct coupling between four fermions?

Is it only fortuitous that a direct coupling with the same strength (within experimental results) can explain β-radioactivity, μ-meson decay and μ-meson capture by nuclei? It is natural to make the hypothesis that the *same* coupling is responsible for the three kinds of phenomena and this has been proposed independently by PUPPI [1948, 1949], LEE *et al.* [1949], TIOMNO and WHEELER [1949b]. Of course, the final decision lies with experiment, and we shall discuss some consequences of this hypothesis in § 4. 1; however, since such a decision is pending we can look for the "meaning" of this hypothesis of the *same* coupling between different kinds of particles.

As we emphasized in § 3. 2, when defining the coupling terms between four fermions we have at least to pair them. The natural pairing established in § 3. 2, i.e. (p, n), (ε, ν) and (μ, ν) is now confirmed by the interaction (p, n)(μ, ν). The situation is illustrated by Fig. 8: between each two of the three pairs of fermions exists the same direct coupling **g** (**g** represents the set of the constants g_i of this coupling). This introduces the concept of a correspondence between the different known fermions. Such a correspondence between (μ, ν) and (ε, ν) suggests that μ and ε are two states of the same particle, but the comparison with nucleons is less inspiring!

Another point of view is to admit this interaction as general between any set of four fermions. Since we cannot any longer fix the order of the four wave functions in the coupling terms, we have to choose the only interaction invariant by any permutation of this order, the CRITCHFIELD and WIGNER [1941] interaction $\mathbf{g}_0$ defined in (41). Moreover, it is of course necessary to give some

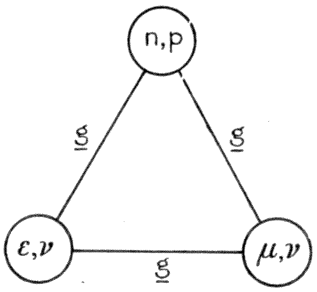

Fig. 8.

selection rules forbidding non-observed couplings. Such a proposal has been made by YANG and TIOMNO [1950c] by introducing for the different fermion wave functions different transformation laws under space reflexions R (see § 1. 3), ψ is transformed into ψ' with

$$\psi' = \eta\beta\psi. \tag{64}$$

In the commonly used formalism, $\eta = \mathrm{i}$ (or $-\mathrm{i}$) and it is the same for all fermions. Here the possible values of: L, $-$L; i, $-$i($\mathrm{L} = \pm 1$, according to the charge conjugate state of the particle, has been defined in § 1. 2) are distributed among the different kinds of fermions and then the authors can lay down a fundamental principle: "Between any four fermions, not all of the same kind, there exists a direct coupling $\mathbf{g}_0$, if the product of their four η is 1 and the K (= LM defined in § 1. 2) of fermions of the same kind are equal". (Of course, electric charge must be conserved, i.e. from equation (1) the sum of the K's of charged particles must be = 0.) But as we pointed out in § 1. 6, the law (64) instead of the usual one is equivalent to the introduction of pseudoscalar coupling constants, and since it is not possible for the YANG and TIOMNO formulation to have the equivalent of the use of both kinds (scalar and pseudoscalar) of coupling constants for the same coupling, we already know that their formulation does not differ physically (provided the neutrino mass = 0) from the ordinary formalism. The authors have expressed their choice of couplings in a very

elegant way (one sentence only) but for that they have sacrificed the natural elegance of the commonly used formalism (for them $\psi^*\beta\psi$ for nucleons can be a scalar, but then $\psi\beta\psi$ is a pseudoscalar, and the formulation of parity conservation becomes more inelegant); and, this being the main point, their interesting proposal does not give a deeper insight into the present theory. From their choice of coupling the authors implicitly assume that the two neutrinos emitted in μ-meson decay are identical, and that the direct coupling responsible for the class of reaction:

$$\mu^+ + \varepsilon^- \rightarrow \mu^- + \varepsilon^+ \tag{65}$$

exists; but neither assumption can be tested by actual experiments (the cross-section of (65) is 10^{-40} cm.^{2} per electron for the observed μ-mesons).

The basic physical test for the existence of a universal interaction between different groups of four fermions is the validity of $\mathbf{g_0}$ (Critchfield–Wigner) interaction. We have indicated (§ 3. 2) the resulting neutron mean life, and have also noted experimental evidence against it in § 3. 4. Further, the experimental energy spectrum of electrons from μ-meson decay (see MICHEL, [1950a], p. 1371, for the comparison of the results of LEIGHTON *et al.* [1949] with the theoretical curve, and the excellent discussion by LAGARRIGUE and PEYROU [1951] of their own data and those of LEIGHTON *et al.*) is not in favour of the $\mathbf{g_0}$ interaction.

4. Possible coupling schemes

4. 1 Coupling scheme (1)

The fact that a single direct coupling between any two of the following pair of fermions (p, n), (μ, ν), (ε, ν) is consistent with all data considered so far is very attractive and its consequences must be studied. Besides this coupling we have the π-meson-nucleon coupling and the not yet studied consequences of the total scheme, the indirect couplings π-(μ, ν) and π-(ε, ν), (see Fig. 9). In this scheme, there is a complete symmetry of coupling when μ and ε are exchanged and it seems that $\pi \rightarrow \varepsilon + \nu$ will in any case be more rapid than $\pi \rightarrow \mu + \nu$. However, calculations have been carried out by STEINBERGER [1949b] (some selection rules are wrong), RUDERMAN and FINKELSTEIN [1949], SASAKI *et al.* [1949] (V and

P π-mesons only) and by NAKAMURA *et al.* [1950]. Furry's Theorem (T4 in § 1. 9) applies and forbids many possibilities and the equi-

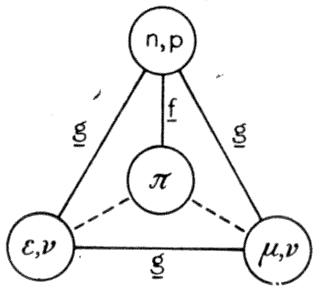

Fig. 9. Coupling scheme (1)
---- indirect coupling.

valence (E) theorem (§ 1. 10) also applies; seven other cases (marked X in Table 4) have matrix elements = 0 (at least for the lowest order). For non-forbidden cases, the results are divergent (logarith-

TABLE 4

g \ π	*Ss*	*Sv*	*Vv*	*Vt*	*At*	*Aa*	*Pa*	*Pp*
g_1	5.7	T4	T4	T4	T4	X	X	X
g_2	T4	E	4.4	4.4	X	T4	T4	T4
g_3	T4	E	2.7	2.7	2.4	T4	T4	T4
g_4	X	T4	T4	T4	T4	4.4	1.3×10^{-4}	1.3×10^{-4}
g_5	X	T4	T4	T4	T4	X	5.7	5.7

mically with the new methods). However, if we consider only one coupling (either f or f', see § 1. 5) for spin 1 mesons and only one term g_i of the two direct coupling terms giving allowed decay (this is not necessary for A π-mesons); the ratio $\tau_{\pi\mu}/\tau_{\pi\varepsilon}$ is of the form

$$\tau_{\pi\mu}/\tau_{\pi\varepsilon} = \zeta\, \mathfrak{A}/\mathfrak{A}, \tag{66}$$

where both $\mathfrak{A}$ represent the same divergent integral. Mathematically (66) is not defined, and, on account of the imperfect state of the theory, theoretical physicists have to apply a procedure to $\mathfrak{A}$ (regularization, cut-off) in order to get a finite result. Then $\tau_{\pi\mu}/\tau_{\pi\varepsilon} = \zeta$ and this is independent of the procedure. Table 4 gives the value of ζ; for a pseudoscalar π-meson and g_4 coupling ζ is very small (10^{-4}). The symmetry of the scheme for μ, ε is therefore no longer a difficulty for a P $\pi^{\pm}$-meson and for any combination of terms containing g_4 (but not g_5) for the direct coupling. But this result is not really quantitative since the calculations give rise to

divergences; regularization gives a much too long lifetime (10^{-2} sec.) for $\tau_{\pi\mu}$, and although $\mathcal{A}$ is only logarithmically divergent, its value is very sensitive to the choice of a cut-off value. A "normal" cut-off value gives the right order of lifetime of about 10^{-8} sec. [17].

We can also see if this coupling scheme is coherent with our conclusions on $V^{\pm}$-mesons (if these heavy mesons are nuclear mesons; no question arises here about V°-mesons). $V^{\pm}$ can decay by the same indirect coupling as $\pi^{\pm}$, into $\mu^{\pm} + \nu$ or $\varepsilon^{\pm} + \nu$. We were left in § 2. 5 with only two possibilities for V:
(a) $V^{\mp}$ is pseudoscalar (Pp or Pa); $\tau_{\nu\mu}/\tau_{\nu\varepsilon} \sim 10^{-4}$; and (see NAKAMURA *et al.* [1950]),

$$\tau_{V\mu}/\tau_{\pi\mu} = (f^2/F^2)(\varkappa_\pi/\varkappa_V)^3; \tag{67}$$

inserting the estimated values of §§ 2. 4 and 2. 5, $\tau_{V\mu}/\tau_{\pi\mu} = (100/3) \cdot (1/4)^3 \approx \frac{1}{2}$, $\tau_{V\mu}$ is of the order 10^{-8} sec., and the reaction $V^{\pm} \to \mu^{\pm} + \nu$ cannot be observed instead of reaction (29) which is more rapid.
(b) $V^{\pm}$ is pseudovector (Aa coupling); $\tau_{V\mu}$ and $\tau_{V\varepsilon}$ are of the same order of magnitude, but in this case the lifetimes are much longer than for S V-mesons, according to Nakamura *et al.*, and from the formula of these authors it is easy to see that there are no difficulties ($\tau_{V\mu} \sim 10^{-8}$ sec.).

Summing up, the coupling scheme of Fig. 9 is quite satisfactory for the present state of the theory and in agreement with actual experimental data. The only trouble is that the value of $\tau_{\pi\mu}$, although reasonable, is ambiguous since it is given by divergent calculations, but this cannot be an argument against the existence of the scheme. Equally, while this scheme is very elegant, since it introduces only two different couplings, this elegance is not an argument for its existence. However, it is natural to adopt and test first the simplest hypothesis, i.e. this scheme (1) for coupling properties of particles. It requires a pseudoscalar $\pi^{\pm}$-meson, and this is in agreement with the discussion in § 1. 3. It requires also the g_4 term for the direct coupling, and $g_5 = 0$. Then we are led to a direct coupling with two terms in g_2 and g_4 according to the

[17] Ruderman and Finkelstein have to take a very large cut-off value because they choose f^2 (Pp coupling of π-meson) about ten times too small (see value in § 2. 1). Nakamura *et al.* say that Pv coupling is better because they choose $f = f'$ instead of applying the equivalence theorem, valid here, or taking the values of f and f' given in § 2. 1.

discussion in § 3. 4. These qualitative consequences will probably soon be tested by experiments.

4. 2 Coupling scheme (2), an alternative to scheme (1)

We noted in § 3. 5 the agreement between the calculated and observed μ-meson capture when it is described by an indirect coupling (Fig. 10), f, $\check{f}$, the value of $\check{f}$ being determined from π-decay to $\mu + \nu$. We can therefore replace scheme (1) by scheme (2) and then we are no longer obliged to compare $\tau_{\pi\mu}$ and $\tau_{\pi e}$ directly and to restrict π to be a P-meson. However, since the calculation of $\tau_{\pi e}$ in this scheme leads to divergent results, we cannot draw

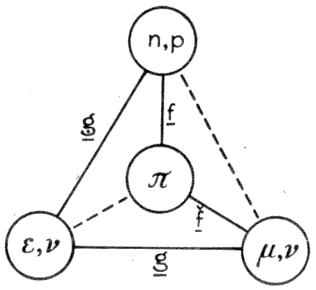

Fig. 10. Coupling scheme (2)
---- indirect coupling.

final conclusions, and from a reasonable cut-off it seems doubtful if charged π-mesons could have any other coupling than Aa or P, their decay into electrons being otherwise too rapid. From other experimental results we have seen that there is very strong presumption that π-mesons are indeed P mesons. Then the coupling scheme (2) introduces one more different coupling (i.e. at least one more coupling constant) without giving (with the present state of the theory) the possibility of a better fit of experimental data. At present, there seems to be no reason to reject scheme (1) in favour of scheme (2).

4. 3 Is direct coupling necessary for β-radioactivity?

β-radioactivity through an intermediate virtual nuclear meson was studied by YUKAWA *et al.* [1938b], SAKATA [1941b] and ROZENTAL [1941a, 1945]. As we saw in § 3. 4, the actual experimental accuracy (by drawing the linear plots of spectra) rejects all terms which are not of the form $k_i J_i$ where k_i are constants. When there is only one meson coupling term, there are no interference terms