

S T A N D I N G   P R O B L E M S   I N   W E A K   I N T E R A C T I O N S  
A N D   C P - V I O L A T I O N

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The main report on weak interactions at the Berkeley Conference was the excellent Cabibbo's report. On the theoretical side, many papers were presented for several applications of current algebras. As you know, one year ago, Adler and Weissberger, independently, related the value of the renormalized axial vector coupling to a sum rule for  $\pi - N$  cross section at  $m_\pi = 0$ . Commutation relations for integral or density of weak currents have been applied by a hundred of authors to every possible decay mode, mainly, non leptonic hyperon decays - good for S waves, less good for P waves - and the quite new relations for K decay into a lepton pair plus 0, 1, 2 pions as an application of low energy theorems (Callan and Treiman, Bouchiat and Meyer, Weinberg; there is some difficulty because the  $\pi$  or  $\pi$ 's are not always soft).

On the experimental side, many more refined experiments; they favor more and more the selection rule  $\Delta I = \frac{1}{2}$ : same asymmetry parameter  $\alpha$  for  $\Xi^-$  and  $\Xi^0$  decays ( $\beta$  and  $\gamma$  not measured yet for  $\Xi^0$ ); the triangle for  $\Sigma^+ \rightarrow n + \pi^+$ ,  $\Sigma^+ \rightarrow p^+ + \pi^0$ ,  $\Sigma^- \rightarrow n + \pi^-$  decays closes well. Indeed the new value of  $\alpha$  ( $\Sigma^+ \rightarrow p^+ + \pi^0$ ) is  $-0.986 \pm 0.072$ . The parameter  $\gamma$  has also been measured  $\gamma(\Sigma^+ \rightarrow n + \pi^+) = \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2} = -1$  so this process is via a pure P wave.

Although the experimental determination of the two form factors in  $K \rightarrow \mu + \nu + \pi$  decay is still ambiguous, the rule  $\Delta I = \frac{1}{2}$  stands

well for all K decay processes ( $K^+ \rightarrow \pi^+ + \pi^0$  is forbidden but  $(K^+ \rightarrow \pi^+ + \pi^0)/(K_1^0 \rightarrow \pi^+ + \pi^-) = 1.5 \times 10^{-3}$  which is not too fast for a radiative electromagnetic transition, remarking that  $SU_3$ , with reasonable assumptions, forbids  $K \rightarrow 2\pi$ ), if we decide to wait and see for the very confusing experimental situation concerning the validity of the  $\Delta S/\Delta Q = 1$  rule for  $K^0 \rightarrow \pi + \mu + \nu$  decay (vector coupling). The absence of  $K^+ \rightarrow \pi^+ + \pi^+ + \ell^- + \bar{\nu}$  ( $\ell = \mu$  or  $e$ ) for more than one hundred  $K^+ \rightarrow \pi^+ + \pi^0 + \ell^+ + \nu$  seen, shows that  $\Delta S/\Delta Q = 1$  is quite good for axial vector coupling; and the absence of  $\Sigma^+ \rightarrow n + e^+ + \nu$  while more than ten cases  $\Sigma^- \rightarrow n + e^- + \nu$  have been seen, <sup>shows</sup> that it cannot be too bad also for vector coupling.

A cartoon shown by Cabibbo gave the mood of his report: two ostriches, head in sand, saying "We understand well the weak interactions" while the cloud of the explosion "CP violation" was growing nearby! But CP violation belonged to another report: that of Fitch.

The experiment of Christenson, Cronin, Fitch and Turlay which discovered CP violation in 1964 has been well confirmed by other groups. All agree on the value  $|\eta_{+-}| = (1.83 \pm 0.12) \times 10^{-3}$  where

$$\eta_{+-} = \frac{\text{amplitude } K_2^0 \rightarrow \pi^+ + \pi^-}{\text{amplitude } K_1^0 \rightarrow \pi^+ + \pi^-}$$

The recent determination of sign of the mass difference

$$\Delta m = m_{K_2^0} - m_{K_1^0} = .5 \hbar / \tau_1 \quad \text{allows a measurement of the phase of } \eta_{+-}.$$

Within  $15^\circ$  it is found equal to that of  $\epsilon$ , i.e.  $\tan^{-1} \left( \frac{M_2 - M_1}{\frac{1}{2} \Gamma_1} \right) = 42^\circ$

where  $\epsilon$  is defined by

$$K^0 = \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \left( (1+\epsilon) K^0 \pm (1-\epsilon) \bar{K}^0 \right) \text{ where } \bar{K}^0 = (\text{CPT}) K^0$$

and CPT is assumed to be an exact symmetry.

The ratio  $K_2^0 \rightarrow 2 \pi^0 / K_2^0 \rightarrow \pi^+ + \pi^-$  has not yet been measured. What is most significant is that no CP or T (or CPT!) violation has yet been found in any other phenomenon. So we are in the uneasy situation that many CP violating theories have been made to explain a unique experiment! The reporter, T.D. Lee, did not report in detail on those theories. He mainly presented a modified version of his previous theory for electromagnetic C violation, proposing now a much smaller asymmetry in  $\eta \rightarrow 3 \pi$  decay which could not be seen in the presently most precise experiment: CERN, asymmetry  $0.3 \pm 1.0\%$  for more than  $10^4 \eta$  decays.

From my short report on those three Berkeley reports, do not conclude that it is not worthwhile to try to explain CP violation! Also, please, do not forget that there are other challenging problems to be explained, at the same time, in weak interaction. I made a tentative list of them in the table 1.

T A B L E 1

Problems of the Weak Interaction

Weak Interaction is defined by its universal constant (Fermi)

$$G = 0.72 \times 10^{-5} \times m_p^{-2}$$

- I Structure at short distance : W's or what ?
- II Cabibbo angle  $\theta = \sin^{-1} 0.21$
- III  $\Delta I = 1/2$
- IV no neutral leptonic currents?
- V why two kinds of leptons separately conserved?
- VI  $\mu - e$  symmetry, but  $\mu - e$  mass difference?
- VII CP violation

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V - A nature of the interaction, P and C violation and two component neutrinos seem well understood now.

I would like now to discuss rapidly these problems and, for some of them, to present speculations most of them due to M.L. Good, E. de Rafael and myself (cf. the I.H.E.S., Bures-sur-Yvette, preprint " A Theory of Neutral Leptonic Currents" to appear soon in Physical Review).

The Weak Interaction is characterized by the Fermi constant

$$G = \frac{1}{\sqrt{2}} \times 1.02 \times 10^{-5} \times m_p^{-2} = 0.72 \times 10^{-5} \times m_p^{-2}$$

All known particles, except photons, seem to be source of weak interaction. The "V - A" nature of the interaction, the two component nature

of the neutrinos (at least  $\nu_e$ ) seems well established and P and C violations well understood.

The remarkable hypothesis of Feynman and Gell-Mann (proposed earlier by Gerstein and Zeldovich) of conservation (up to electromagnetic and weak interaction) of the  $\Delta S = 0$ , vector current relates the corresponding weak interaction hadronic form factor to the isovector part of the electromagnetic form factor. This hypothesis is well verified experimentally (weak magnetism) and it can be generalized approximately to an octet of vector and axial vector currents.

I.

But no other effects on the momentum dependence of the weak interaction are known. This implies a short range, smaller than  $\hbar c (2 \text{ GeV})^{-1}$ . In other words, it is still possible to admit that weak interaction, (similarly to nuclear and electromagnetic ones) is mediated through a boson field W which must have spin one and (from high energy neutrino experiments) a mass  $M_W \gg 2 \text{ GeV}$ . The hadronic and leptonic current would be coupled to the W fields by the universal dimensionless constant

$$g = G^{\frac{1}{2}} M_W = 2.7 \times 10^{-3} \times \frac{M_W}{m_p}$$

(Note that G and  $m_p$  are known so g increases with the W mass; e.g.  $g \sim \alpha = 137^{-1}$  for  $M_W \sim 3 \text{ GeV}$ ; so g is not so small; it is "semi-weak").

II

Cabibbo's angle  $\theta$  is problem II of our table. This angle gives both ratios of amplitudes:  $a(\Delta S = 1) / a(\Delta S = 0) \sim \text{tg } \theta$  and  $a(\Delta S = 0) / a(\text{lept}) \sim \cos \theta$  when  $a(\Delta S = 0 \text{ or } 1)$  is for hadron-lepton transitions and  $a(\text{lept})$  for pure leptonic processes (as  $\mu \rightarrow e + \nu + \bar{\nu}$ ). It can be expressed in term of  $m_\pi / m_K$ . Its value is  $\theta = \sin^{-1}(0.21)$ . To compute it is a challenging problem. Let us study its meaning. Electromagnetic interaction singles out an axis in the three dimensional space on which acts the fundamental representation of  $SU_3$ . By convention this axis is called 1. The  $SU_2$  subgroup of  $SU_3$  which leaves this axis invariant and therefore acts only on the 2, 3 space is called U-spin group and it is a group of invariance of the electromagnetic interaction.

Both the semi strong interaction and the weak interaction have a preferred direction in this 2-3 space. The semi-strong interaction are invariant by the  $SU_2$  subgroup isospin group, which leaves fixed axis 3 ("strangeness"), while the preferred direction of weak interaction is

$$\text{"axis 2'"} = \text{"2"} \cos \theta + \text{"3"} \sin \theta .$$

The  $SU_2$  subgroup of  $SU_3$  which leaves fixed this axis 2' is called  $\mathcal{Q}$ -spin by Cabibbo (Phys.Rev.Letters 10-531.1963).

It is therefore likely that one can explain the Cabibbo angle by a dynamical relation between weak interactiong and the  $SU_3$  breaking strong, (one also says "semi-strong") interaction. One possibility for such a relation is that those two interactions are both due to the same W bosons; these are coupled singly to hadron (and lepton) currents for weak

interaction and they might be coupled by pairs to the hadronic sources for the semi strong interactions. (Such a scheme has been suggested by Marshak and Okubo.)

III

As I reported, the  $\Delta I = \frac{1}{2}$  selection rule seems now to be very well satisfied for both semi-leptonic and non-leptonic processes. However, when one couples the octet of hadronic currents with themselves, one obtains the following  $SU_3$  representations:

$$8 \times 8 = 27 + 10 + \bar{10} + 8 + 8 + 1$$

and  $\Delta I = 3/2$  transitions do appear. A frequently proposed explanation for  $\Delta I = 1/2$  is then an "octet enhancement" due to a dynamical effect which suppresses the unwanted 27, 10,  $\bar{10}$  representations of  $SU_3$ . However it seems better (or at least as good) to have a Lagrangian which picks up the octet in  $8 \times 8$  so the  $\Delta I = \frac{1}{2}$  rule is built in the interactions. The most economical and elegant solution has been proposed by d'Espagnat (Phys. Lett. 7, 209, 1963):

There are three W's whose fields  $W^i$  form the "3" representation of  $SU_3$ , while their hermitian conjugate form the " $\bar{3}$ " representation. The Lagrangian itself, for W-hadron interaction, has the variance of a triplet, pointing in the "2" direction

$$\mathcal{L}_W = g J^2_i W^i + h.c \quad (1)$$

Or, explicitly with the Cabibbo angle.

$$\mathcal{L}_W = g(J^2_i \cos\theta + J^3_i \sin\theta) W^i + h.c \quad (1')$$

where  $J^i_j$  ( $i, j = 1, 2, 3$ ) are a nonet of hadronic currents (vector



- axial vector); it becomes an octet if  $\sum_i J_i^i = 0$  .

In our paper (referred below as "G.M.R." ), Good, de Rafael and I tried to attack simultaneously the remaining four problems. Why are they neutral currents? Of course an interaction can be universal (e.g. the electric charge) and not be carried by every particle. But it must be emphasized that, up to now there are no experimental evidence for or against the existence of neutral leptonic currents ( $e^+e^-$ ,  $\mu^+\mu^-$ ,  $\nu_e\bar{\nu}_e$ ,  $\nu_\mu\bar{\nu}_\mu$ ) in purely leptonic process (see GMR for a review and also the T.T. Wu preprint, to appear in Phys. Rev.). The main experimental evidence against neutral leptonic current are in  $\Delta S = 1$  hadron-lepton transition (comparable  $\Delta S=0$  transitions loose the competition against real or virtual  $\gamma$  transition). G M R proposed the simplest extension of Lagrangian (1) to leptons:

$$\mathcal{L}_W = g (J^{2'}_{i'} + i L^{2'}_{i'}) W^{i'} + h.c. \quad (2)$$

where  $L^{i'}_{j'} = \bar{\psi}_{i'} \gamma^\mu (1 - \gamma^5) \psi_{j'}$ , are leptonic currents, whose indices are determined by the assignments:

$$\begin{array}{ll} 1' & e \quad \nu_e \\ 2' & \nu_\mu \quad \mu \end{array} \quad 3' \text{ has no value for leptons}$$

(see G M R for details)

The terms containing the neutral leptonic currents in (2) are explicitly:

$$(\nu_e \bar{\nu}_e + \mu^+ \mu^-) i (W^{2'} - W_{2'}) \quad (2')$$

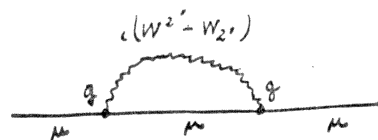
The hermitian field  $i(W^{2'} - W_{2'})$  is just the one not coupled in (1) , (only  $W^{1'}$ ,  $W_{1'}$ ,  $W^{3'}$ ,  $W_{3'}$ , and  $W^{2'} + W_{2'}$  are) so (2) does not induce

hadron-neutral leptonic current transitions.

To summarize,  $\mathcal{L}_W$  in (2) has the following properties:

V Lepton conservation is assumed and with electric charge conservation it implies (because of the charge flip in 3) the two separate conservation laws for  $\mu$  and  $e$  leptons. The  $\mu - e$  symmetry is complete in charged leptonic currents but is broken in neutral ones. So the  $\mu$  has self energy diagrams not shared by electrons e.g. diagram 1

VI (Although in  $g^2$ , the contribution of such a diagram might be large since  $\mu$ -form factor for  $W$  coupling is very extended in  $k$ -space  $\sim$  very point like in  $x$ -space).



VII The  $i$  in front of the leptonic currents in  $\mathcal{L}_W$  (equation 2) does not imply CP violation. We leave to the reader to write the CP operator. Do remember that CP is defined up to a phase for each field and one can find a choice of phases such that

$$\mathcal{L}^{(4)} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{SU}_3} \text{ (very strong)} + \mathcal{L}_{\text{e.m.}} \text{ (minimal)} + \mathcal{L}_W \text{ (equ.(2))} \quad (4)$$

is invariant. This requires that the both hermitian fields  $W^{2'} + W_2$ , and  $i(W^{2'} - W_2)$  have same CP-transformations that the electromagnetic field. One may add to  $\mathcal{L}^{(4)}$  a CP-invariant  $\mathcal{L}_{\text{SS}}$  (semi-strong)  $\text{SU}_3$ -breaking interaction such that

$$\mathcal{L}_{\text{total}} = \mathcal{L}^{(4)} + \mathcal{L}_{\text{SS}} \quad (4')$$

is either CP invariant or not CP invariant.

The latter case would be obtained most naturally if W fields enter (by pairs) in  $\mathcal{L}_{SS}$ . (We have already suggested it for an explanation of the Cabibbo angle.)

In any case, when there is an interaction which is based on the fact that  $W^{2'} + W_2$ , and  $i(W^{2'} - W_2)$  are the real and imaginary part of the same non-hermitian field, (this requires them to have opposite CP transformation) there will be CP violation.

In G M R, in order to compute, CP violation has been introduced more phenomenologically according to two possible models: the " $\alpha$  model", a non sophisticated one: a non-minimal  $\mathcal{L}_{em}$  is added in the form of an anomalous magnetic moments of  $W^1, W^2, W^3$  of the order of one W-Bohr magneton;

the " $\beta$  model" is in spirit of what has been said above on the semi-strong interaction due to the coupling of W pairs with hadrons. Such a coupling introduces a mass splitting:  $M_{W1} = M_{W2} = M_{W3} + \beta M_{W1}$  where  $\beta$  is a parameter characteristic of  $SU_3$  breaking. We introduced this splitting phenomenologically in the free Lagrangian part of (4) and could predict precisely neutral leptonic current effects (in  $\beta^2$  compared to charged leptonic ones) for hadron lepton transitions and also CP violation related to those neutral leptonic currents. Such currents do enter in the  $K^0$  mass matrix giving a CP violation in  $\alpha\beta$  for  $K_1^0 \rightarrow 2\pi$ . All our predictions are in agreement with present experimental data. Our predictions for other CP violating effects are very specific.

Let me conclude by a few remarks on possible theories of CP

violation. What is nasty about them is that in order to explain CP violation, you have to introduce it. So the theory is even more ad hoc if you have to introduce a small number ( $\sim 10^{-3}$ ) in order to have a small CP violation. Of course  $\alpha = (137)^{-1}$  is not too far from it, but this does not seem to me a sufficient reason to blame the electromagnetic interaction for CP violation. However, an  $\alpha$  factor can be incorporated in CP violation very naturally and the G M R paper is one possible concrete example of the following general idea.

The total Lagrangian  $\mathcal{L}_{\text{tot}}$  is CP violating, but if any term  $\mathcal{L}'$  of  $\mathcal{L}_{\text{tot}}$  is removed ( $\mathcal{L}'$  is either  $\mathcal{L}_{\text{SS}}$  or  $\mathcal{L}_{\text{em}}$  or  $\mathcal{L}_{\text{W}}$  for instance) one can find a CP operator leaving ( $\mathcal{L}_{\text{tot}} - \mathcal{L}'$ ) invariant. (The choice of phase for the CP transformation of the different fields in  $\mathcal{L}$  depends on  $\mathcal{L}'$  and they are in conflict for the total  $\mathcal{L}$ ). So, no single type of interaction is responsible for CP violation, but all of them must cooperate to produce CP violation. So no CP violation occurs for non-weak transitions as well as for pure leptonic transitions (since leptons have no strong interaction). CP violation will appear in radiative corrections, both electromagnetic ( $\sim \alpha$ ) and semi-strong ( $\sim \beta$ ) of weak interaction processes. It is a small effect in those processes where  $\alpha$  and  $\beta$  occurs in a product (e.g. in G M R, the CP - violating element of the mass matrix of  $K^0$  is in  $G^2 \alpha \beta$ , a double radiative correction to the  $K_1^0 - K_2^0$  mass difference which is in  $G^2$ ) or it can be nearly maximal when it is due to the competition between two radiative corrections of the same order of magnitude ( $\alpha \sim \beta$ ) for the same weak process. For instance G M R also predicts a large CP violation in

$K^+ \rightarrow \pi^+ + \mu^- + \mu^+$  decay (the branching ratio is  $10^{-6}$  to  $10^{-7}$ ,  
experimental limit today :  $\lesssim 10^{-6}$ ).

(See also the discussion of the neutron electric dipole ; its measurement is crucial to many theories of CP violation).