IIIrd International Winter Meeting on Fundamental Physics
Sierra Nevada (Granada, Spain)
February 15, 1975#

TEST OF MODELS AND POLARIZATION EFFECTS : THE QUARK MODEL

by

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This conference is a part of a more complete publication in preparation by the three authors. Meanwhile, we hope to receive more data on reactions $0^- + \frac{1}{2}^+ \to 1^- + \frac{3}{2}^+$ from our experimental colleagues.

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This conference was to be given by Louis MICHEL. The French "Relations culturelles" were asked by the organizing Committee, through the channel of the French Embassy in Madrid, for travel support of the French lecturers. His was the only one refused.

0. INTRODUCTION

Strictly speaking, models can never be proved by experiments : they are only disproved whenever their predictions disagree with experimental data. However, the confidence in a model will grow with the number of its independent experimental verifications. The differential cross section of a reaction between particles is only one of many other observables (see 4. Table 1). Those are obtained by measuring polarization effects. The intrinsic interest of such measurements is evident; and models should always be tested by polarization data. However, it must be said that presently, the methods generally used for these tests are not very satisfactory.

We outline here the strategy we suggest for such tests. Of course we shall illustrate it by an example : we have chosen the quark model and a family of reactions: pseudo scalar meson on an unpolarized nucleon target.

We will freely use our previous work on polarization (Ref. [1], [2]). A nut-shell summary has been made in a conference by one of us [3]. Its text has been distributed in advance to the meeting participants.

1. The Predictive Value of a Model for a given Experiment.

There are no crude data. Experiments must be interpreted by means of admitted first principles, which can be sometimes checked, but are never discussed in "normal science" [4]. These first principles may impose restrictions on the possible data. Data inconsistent with these restrictions cannot be used. To have a predictive power in a given experiment, the model must impose stronger restrictions on the data.

The k particles involved in the reaction have spin j_i . Let(*) $n' = \prod_{i=1}^{k} (2j_i+1)$. For particles with fixed energy-momenta, the k-parti-

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(*) For photons and neutrinos replace respectively $2j_i+1$ by 2 and 1. When one of the particles is a resonnance interfering with the background one can still use a density matrix, with larger n (see e.g. ref. [1.a], Part III).

cle polarization can be described by a $n' \times n'$. Hermitean positive, trace one matrix, the density matrix ρ. Such a matrix can be represented as a point in a n'^2-dimensional (real)vector space $\mathcal{E}'$ with the Euclidean norm $(\mathrm{tr}\,\rho^2)^{1/2}$. The conservation of angular momentum imposes to ρ to be in a domain(*) $\mathcal{D}' \subset \mathcal{E}'$. Parity conservation may eventually decrease the polarization domain $\mathcal{D}'$ to its intersection by a linear subspace. For instance, when the polarization of the beam and the target is fixed (most often they are unpolarized) this restricts ρ to a linear subspace $\mathcal{E}''$ of $\mathcal{E}'$. Generally the polarization of the final particles is only partially measured (with the present techniques a complete observation might often be impossible) : what one observes is the projection ρ_{ob} of ρ on an Euclidean vector space $\mathcal{E}$ (which can be identified as a subspace of $\mathcal{E}''$). Angular momentum and parity conservation imposes to ρ_{ob} to be in the polarization domain $\mathcal{D} \subset \mathcal{E}$, the projection of $\mathcal{D}''$ on $\mathcal{E}$.

The energy momenta of the particles are not known with an absolute precision. Some of them might even be ignored : inclusive experiment. Generally, to obtain a sufficient statistics in high energy experiments, one makes a partial integration over phase space (e.g. bins in t, see table 3). This corresponds to an averaging in the polarization space $\mathcal{E}$ and the observed polarization domain $\mathcal{D}$ is a part, and often the totality of the convex hull of $\mathcal{D}$.

For these experimental conditions the model predicts the polarization ρ_{ob} to be in a subdomain $\mathcal{M} \subset \mathcal{D}$. To have a predictive value the domain $\mathcal{M}$ must be strictly smaller than $\mathcal{D}$. Roughly, ℓ, the dimension of $\mathcal{D}$, is the number of independent polarization parameters measured in the experiments; if m is the dimension of $\mathcal{M}$, then

$$\ell - m = \dim \mathcal{D} - \dim \mathcal{M} \qquad (1)$$

is the number of constraints imposed by the model and is a rough numerical evaluation of its predicting power.

What we have said for models is also valid for general theories;

(*) Here $\mathcal{D}'$ is the manifold of equation $\rho^2 = \rho$; its dimension is $2(n-1)$.

for instance we can study the consequences of isospin conservation (or of any internal symmetry). It is of course necessary to know the consequence of this general theory (e.g. isospin conservation - see [1 d,e] and references there -)in order to show that a particular model makes more specific predictions !

2. How much the Polarization Measurement agrees with the Model ?

In most of the papers of table 3 the quark model is "tested" by giving the left hand side and the right hand side experimental values (and their errors) of 6, 10, even 14 relations predicted by the quark model. Generally this list of numbers is not very inspiring and the conclusion depends on the mood of the authors. This is very unsatisfactory.

To see a plot with the experimental points and their errors and among them the point or the curve predicted by the model is more exciting. However, in the present physics literature, these plots are generally made for the value of the density matrix elements, so they depend on the choice of the reference frame and are not very intrinsic; nearly never the bounds imposed by angular momentum are actually plotted; and even so, this would be far from sufficient if ℓ , the dimension of $\mathcal{D}$, is not small.

Since any experiment is affected with errors there is a well established method in statistics for testing the compatibility of experiments with theoretical predictions : the χ-square test. But it does not apply in our case because the domain $\mathcal{D}$ of value of the measured polarization parameters is bounded. For instance, if the size of the errors is comparable to that of $\mathcal{D}$, the χ-square test will be very good. It just means that if one measures practically nothing, the measurement will be in good agreement with most theories !

The strategy we suggest is very obvious. First, verify that the point representing ρ_{ob} is inside the polarization domain $\mathcal{D}$ and that the errors are reasonably small(*) compared to the size of $\mathcal{D}$. Second, since the polarization space $\mathcal{E}$ has a natural, and physically meaningful

(*) For example, in the reference [13] we compare the volume of the error ellipsoid ΔE corresponding to a given level of confidence (for instance $\frac{1}{2}$) to the volume of $\mathcal{D}$.

metric, give the shortest distance ET (and its errors ΔET) from the experimental point E to the model predicted domain $\mathcal{D}$. (i.e. T is the point of $\mathcal{D}$ nearest to E , ET = 0 $\Leftrightarrow$ E $\in \mathcal{D}$). If the distance ET is of the magnitude of the error Δ(ET) , the experiment is in agreement with the model.

A plot can be very inspiring. For each point of the data the simplest plot will often be made in the 2-plane defined by E (experimental), T (given by the model) and O the origin of polarization, representing the unpolarized state. Of course, on the same 2-dimensional figure we will draw the projection of the error-ellipsoid and the intersection of $\mathcal{D}$ with the 2-plane ETO . This gives the bounds imposed by the consevation laws and allows an explicit comparison between the size of the errors and that of the allowed polarization domain. In the same case, a three dimensional (or even higher dimensional) plot might be still much more interesting as we shall see in the examples. Indeed, in the most usual experiments we have:

$$\left\{\begin{array}{l}\text{"unpolarized initial particles, two final particles or resonan-}\\ \text{ces, or inclusive experiment; only the even multipoles of the}\\ \text{polarization are measured".}\end{array}\right\} \quad (2)$$

The polarization domain $\mathcal{D}$ of a spin $\frac{3}{2}$ particle is the inside of a sphere in a 3-space and for a spin 1 particle it is a truncated cone in a 3-space (*) when conditions (2) are met.

Different models favour different parts of the polarization domain $\mathcal{D}$. The role of $\mathcal{D}$ for the polarization is similar to that of the Dalitz plot for the energy. $\mathcal{D}$ is an intrinsic object of physics (while the choice of quantization frame for expressing the matrix elements of ρ is a question of convenience) and each part of it has a meaning in terms of models. As example, Fig. 1 and 2 represent the Minnaert cone and the Doncel sphere. For forward or backward reaction the polarization domain shrinks to a line segment which is indicated. The physical significance of the different parts of $\mathcal{D}$ is recalled.

In ref. [2] we have discussed the constraints on polarizations

(*) Results respectively obtained by Doncel (1967) and Minnaert (1966) in their thesis.

predicted by Regge-pole models and also for broad hypotheses such as factorization of vertices, parity conservation at the vertex, etc... Here we have chosen the quark model for illustration.

3. The Quark Model.

As you know baryons are made of three quarks and mesons of one quark q and one antiquark $\bar{q}$. In a scattering process between hadrons one assumes that the scattering amplitude is expressed as a coherent sum of amplitudes for q-q or q-$\bar{q}$ scattering and that only one quark from each particle interacts (see references [5a,b]). This model has no predictive power for elastic processes, e.g. $pp \to pp$ since we replace our ignorance on nucleon-nucleon scattering by that on quark-quark scattering. The quark model implies the SU(3) and also the SU(6) symmetries, so we can relate different scattering processes to each other. Predictions of the model, more specific than those symmetries, were made by Białas and Zalewski [7] using the lengthy computations of ref. [6a,b]. (See also ref. [8], lectures by A. Kotanski, for a more complete bibliography). We shall explain these predictions and compute them again with the pocket table of Clebsh-Gordan coefficients.

Consider a reaction meson nucleon :

$$0^-\tfrac{1}{2} \to 0^-\ \tfrac{3}{2} \tag{3}$$

such as $\pi^+p^+ \to \pi^0\Delta^{++}$, $K^-p^+ \to \bar{K}^-Y^{*+}$, $\pi^-p^+ \to K^0Y^{*0}$..., in which the meson remains pseudoscalar while the nucleon is transformed into a member of the decuplet. The transition matrix is a 2×4 matrix which we denoted in [1f], table 5f, for transversity quantization(*).

$$T = \begin{array}{c} \\ 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{array} \begin{array}{c} \begin{array}{cc} 1/2 & -1/2 \end{array} \\ \begin{pmatrix} 0 & b' \\ a & 0 \\ 0 & a' \\ b & 0 \end{pmatrix} \end{array} \tag{4}$$

(*) We do not need to precise the polarization frame (s,t or u channel, or Donohue-Högaasen [9]), we only assume that the frame for the final particle is obtained from that of the initial particle by a boost.

The zeros are due to parity conservation. By "rotations" (in the center of mass) the elements of T are transformed under the representation $D_{\frac{3}{2}} \otimes D_{\frac{1}{2}}$. The complex conjugate $\bar{D}_{\frac{1}{2}}$ is equivalent to $D_{\frac{1}{2}}$:

$$\bar{D}_{\frac{1}{2}} = K \, D_{\frac{1}{2}} \, K^{-1} \quad \text{with} \quad K = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{5}$$

So the elements of the matrix of

$$T = T \, K^{-1} \tag{6}$$

are transformed under the representation :

$$D_{\frac{3}{2}} \otimes D_{\frac{1}{2}} \sim D_2 \oplus D_1 \tag{7}$$

as the vectors :

$$-b' \, |2,2> \; , \; b \, |2,-2> \; , \tag{8}$$

$$a \, |\tfrac{3}{2},\tfrac{1}{2}> \otimes \, |\tfrac{1}{2},-\tfrac{1}{2}> \; = \frac{a}{\sqrt{2}}(\, |2,0> + \, |1,0> \,) \tag{8'}$$

$$-a' \, |\tfrac{3}{2},-\tfrac{1}{2}> \otimes \, |\tfrac{1}{2},\tfrac{1}{2}> \; = \frac{-a'}{\sqrt{2}}(\, |2,0> - \, |1,0>) \tag{8''}$$

Since in the lowest octet and decuplet, the quarks have no orbital angular momentum, in the $\bar{q}$-q or q-q scattering one of the quarks of the nucleon must have a spin flip in order to change the total spin from $\frac{1}{2}$ to $\frac{3}{2}$: indeed, T has no scalar elements. Since quarks have spin $\frac{1}{2}$, a quark spin flip means a change of angular momentum of 1 and not 2 units. So the quark model requires that the $j=2$ part of T vanishes; from (8), (8'), (8") this means

$$b = b' = 0 \quad , \quad a' = a \tag{9}$$

Hence the density matrix of the decuplet baryon is, in transversity :

$$\frac{1}{2|a|^2} \, T \, T^* = \rho = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

This is the south pole of the Doncel sphere (Fig. 2)(*).

(*) Conditions (2) requires that $\rho_{33} = \rho_{-3-3}$, $\rho_{11} = \rho_{-1-1}$ (we write 3 for $\frac{3}{2}$, 1 for $\frac{1}{2}$, etc.) $\mathrm{tr}\, \rho = 1 = 2(\rho_{33} + \rho_{11})$. In (10), $\rho_{33} = 0$.

This is a very strong result (independent of s and t !) which was also predicted by other models e.g. that of Stodolsky-Sakurai [10] ; indeed it means a vector meson exchange. As is well known, this prediction is rather good for medium beam energy (2 to 8 GeV). See, for instance, ref. [11]. (This CERN and Saclay group uses abundantly the sphere and the cone!). As Doncel [12] emphasized at last year meeting, this prediction cannot be true at small $t' = |t-t_{min}|$ i.e. in the forward (or backward) direction since the polarization at $t'=0$ has to be on the segment $P_o P_2$ of Fig. 2 (see [11] again). From the Δ decay mode we can measure only the even, $L = 2$ polarization multipole; however, from the sequential decays of Y^* ($\rightarrow \pi$, $\Lambda^o \rightarrow \pi N$, see Pierre Minnaert lectures) one can measure ρ completely (then, $\dim \mathcal{E} = 7$). One must then verify that it is of rank two and that the odd multipoles ($L = 1$ and $L = 3$) vanish. In [13], some data had been analyzed critically. We would like to see a compilation of the world data.

What does the quark model predict if the meson vertex is replaced by anything(*) ? e.g. multi-meson production and the reaction is analyzed inclusively :

$$0^- + \tfrac{1}{2} \rightarrow Y^* + X \qquad (11)$$

For unpolarized target, the Y^* polarization is given by :

$$\rho = \left(\int T\ T^* d\mu\right) / \left(\mathrm{tr} \int T\ T^* d\mu\right) \qquad (12)$$

where the Stieljes measure $d\mu$ is a spin summation and some phase space integration. The integrand is transformed by rotations under the representation $D_1 \otimes \bar{D}_1 \sim D_2 \oplus D_1 \oplus D_o$. So the model predicts that (see e.g. [14])

"The $L = 3$ multipole of ρ vanishes" (13)

In ref. [13] for two experimental results the OPE diagram with its contour is drawn. Since the ρ of an inclusive reaction is the barycenter, with measure $d\mu$, of exclusive ρ , it is remarkable that the experimental

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(*) It can also be a simple baryon vertex, e.g. $N \rightarrow N$ or $N \rightarrow$ decuplet. What is said here is also true in that case, and the model adds other specific predictions : see [7,8] points are near the boundary of the domain, and strong physical conclusions could be drawn if these data are confirmed; on the other hand(*) these two analyzed points have a large $L = 3$ multipole and are not in favour of the selection rule (13) predicted by the quark model. We would like to see an analysis of all present data.

The quark model has no predicting power for reactions such as

$$0^- + \tfrac{1}{2}^+ \to 1^- + \tfrac{1}{2}^+ \qquad (14)$$

where $\frac{1}{2}^+$ is a baryon of the lowest octet and 0^- and 1^- mesons belong to the lowest SU(6) representation. We leave out the production of two resonances by nucleon nucleon reactions, to study in detail the corresponding meson-nucleon reactions.

4. Test of the Quark Model by $0^- \frac{1}{2}^+ \to 1^- \frac{3}{2}^+$ Reactions.

The mesons and baryons of these reactions are in the lowest SU(6) representation. The initial nucleon is unpolarized. The polarization density matrix ρ of the two final resonances is 12×12 (since $12 = (2 \times 1 + 1) \times (2 \times \frac{3}{2} + 1))$, and it is of rank two $(= 2 \times \frac{1}{2} + 1)$. If parity were not conserved the number of amplitudes would be $n' = 24$; however, parity conservation reduces the number of amplitudes to 12 and imposes 71 linear relations on the elements of ρ. The density matrix can be expanded in bi-multipoles $T^{L_1 L_2}_{M_1 M_2}$ with $-L_i \le M_i \le L_i$, $0 \le L_1 \le 2$ (for the meson), $0 \le L_2 \le 3$ (for the baryon). Those with $L_1 = 1$ cannot be observed; furthermore, if $\frac{3}{2}^+$ is a Δ also those with $L_2 = 1$ and 3 cannot be observed from the Δ decay. In table 1, column (c) we give the number of the observable parameters including the cross section. As we see from column (d) respectively 48 (for Y^* or Ξ^*) and 16 (for Δ) polarization measurements just check parity conservation (in transversity $M_1 + M_2$ odd $\Rightarrow T^{L_1 L_2}_{M_1 M_2} = 0$). If this check fails, one cannot continue the analysis explained here; very likely one has to include the interferences between the resonances and with the background. The comparison between column (e) and (a) shows that for a Y^* the 48 observables satisfy 26 algebraic relations; we refer to our table 13 of

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(*) Though the experimental authors claim the contrary!

[1f] (we even consider a polarized target). For a Δ, both columns (e) and (a) are equal to 20. The algebraic inequalities which define the 19 dimensional $\mathcal{D}$ in the 19-dimensional polarization space $\mathcal{E}$ are very uninspiring. ($\mathcal{D}$ is not convex). Note also that there are four parameter ambiguities for amplitude reconstruction.

TABLE 1

Reaction $0^- \frac{1}{2}^+ \to 1^- \frac{3}{2}^+$ at fixed s,t

(a) Number of amplitude real parameters which could be reconstructed.

(b) arbitrariness in amplitude reconstruction.

(c) number of measurable observables (1 for cross section, others = polarization).

(d) observables which verify parity conservation.

(f) number of the other polarization observables (-1 is to substract cross section which is contained in e).

	(a)	(b)	(c)	(d)	(e)	(f)
$\frac{3}{2}^+$ is a Y*	22	$2\epsilon(U_1 \times U_1)$	96	48		48 - 1 = 47
$\frac{3}{2}$ is a Δ	20	$4\epsilon(U_2)$	36	16		20 - 1 = 19

In [1f] table 13 e, we write the 2 x 12 transition matrix, in transversity

$$T = \begin{array}{rc} 1 & \\ 0 & \frac{3}{2} \\ -1 & \\ 1 & \\ 0 & \frac{1}{2} \\ -1 & \\ 1 & \\ 0 & -\frac{1}{2} \\ -1 & \\ 1 & \\ 0 & -\frac{3}{2} \\ -1 & \end{array} \overset{\begin{array}{cc} \frac{1}{2} & -\frac{1}{2} \end{array}}{\begin{pmatrix} e & 0 \\ 0 & d' \\ f & 0 \\ 0 & c' \\ a & 0 \\ 0 & b' \\ b & 0 \\ 0 & a' \\ c & 0 \\ 0 & f' \\ d & 0 \\ 0 & e \end{pmatrix}} \qquad (15)$$

A quark or anti-quark of the meson must have a spin flip $\Delta J = 1$; but $D_o \otimes \overline{D}_1 \sim D_1$ shows that this implies no restriction on (15). A quark of the baryon must also have a spin flip $\Delta J = 1$; the cancellation of the terms $\Delta J = 2$ for the baryon leads to calculations similar to that of (8) and (9) and yields the results :

Conditions A.

$$d = d' = 0 \ , \ a' = a \ , \ e = \sqrt{3}c' \ , \ f = \sqrt{3}b' \ , \ f' = \sqrt{3}b \ , \ e' = \sqrt{3}c \qquad (16)$$

(These conditions (16) are usually called Conditions A.) There are only 5 independent (complex) amplitudes a,b,b',c,c' . The cross section is

$$\sigma = |a|^2 + 2|b|^2 + 2|b'|^2 + 2|c|^2 + 2|c'|^2 \qquad (17)$$

An overall phase is irrelevant. In the 19 dimension polarization space $\mathcal{E}$, the domain $\mathcal{D}$ predicted by the quark domain is a 8 dimensional manifold. The predictivity of the model is therefore 19-8 = 11, which is quite strong. Moreover, among the 11 equations which define $\mathcal{D}$, 6 are linear (in other words $\mathcal{D}$ is in a 13 dimensional linear manifold of $\mathcal{E}$)

In nucleon-nucleon and in nucleon-antinucleon scattering, one must take in account the fact that, up to isospin, the particles are identical or charge conjugated. The same applies to q-q or $\overline{q}$-q scattering : this leads to condition B.

For every elastic scattering, time reversal imposes some relations; applied to q-q or q-$\overline{q}$, it leads to condition C. The problem with these two types B and C of conditions is to transform them from the quark-quark frame to the hadron frame. This is ambiguous. The most natural conventions lead to

Condition B $\qquad b = b' \qquad (18)$

Condition C $\qquad c = c' \qquad (19)$

However, this is quite unsatisfactory because these conditions are not covariant. Their verification depends on the choice of frame for discribing polarization, e.g. s,t,u-channel, or Donohue-Høgaasen (the choice between helicity and transversity is immaterial : one passes from one to the other by a rotation of $\frac{\pi}{2}$).

We summarize in table 2, the number of different types of conditions.

TABLE 2.

Predictivity of the Quark model for the reactions $0^- \frac{1}{2} \to 1^- \frac{3}{2}^+$ in the 19 dimensional polarization space.

Conditions :	A	A + B	A + B + C
Total :	11	13	15
Linear :	6	10	14

There are no physical ground to consider conditions C independently of B, and those conditions B independently of A.

In present experimental conditions, the number of events at given s and t is not large enough for a good determination of the 19 polarization parameters; actually large bins in t are used (see table 3). This partial integration over phase space blurs the non linear conditions of the quark model; however, it does preserve the linear conditions. The data of table 3 can be only used for analyzing the linear conditions (second line of table 2).

These linear conditions were first given in [7]. In [1a] we have already explained these predictions of the quark model for the separate polarization of the final resonances : make a dilation by a factor $\sqrt{3}$ of the spin $\frac{3}{2}$ polarization sphere, so the spin 1 polarization cone can be exactly inscribed in the sphere. Then the quark model predictions are for the separated polarization :

$$\left\{ \begin{array}{ll} A & \text{The two points have same vertical coordinate} \\ A + B & \text{The two points coincide} \\ A + B + C & \text{Their azimuth is zero} \end{array} \right\} \qquad (20)$$

In this form (20) we verify again that condition A is covariant. Condition B depends on the frame but is invariant by simultaneous rotation around the normal to the scattering plane of the quantization frame of each particle. Condition C can be verified only in one frame : by its own definition, it is the Donohue-Høgaasen frame (see our complete publication for the explanation).

We have listed in table 3 all the data we have analyzed up to now.

There is also a tactical problem for the present tests of the quark model : should we test many predictions with a poor statistics or few of the better conditions with a very good statistics. For instance, in transversity, if one measures only the polar angles θ (i.e. one integrates over the azimutal angles φ), only the diagonal ρ is measured (i.e. T^{22}_{00}, T^{20}_{00}, T^{02}_{00}), but with a good precision. This was proposed by L. Lyons et al [15]. We have already studied [1e] the polarization space : a 3 dimensional tetrahedron ABCD; The prediction of the quark model is the line AQ within the face ACD of the tetrahedron. We present the data of table 3 in 4 Figures 3a, 3b, 3c, 3d. The test is remarkably good. Of course this procedure tests only two A conditions‡ (which may come from broader hypothesis than the quark model); they seem well verified.

That the domain predicted by the quark model is projected on this 3-plane in the boundary of the projected polarization domain, with the fact that the 3-plane is an equatorial plane (see [1e]) proves that the quark model domain is in the boundary of the 19-dimensional polarization domain $\mathcal{D}$. In table 4 we present the ETO test for this data. We do not want to draw a conclusion. We just make some obvious comments. We also present as an example a two dimensional ETO plot for the three reference frames of point D, Fig. 4a, 4b, 4c. The plotted domain is slightly larger than the actual intersection of $\mathcal{D}$ by the 2 plane ETO. We could draw the boundary of the true intersection by using a large computer(*), but we have no analytic expression for it. Note that the theoretical point T is on the boundary.

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‡ When one also takes into account angular momentum and parity conservation, one can deduce two other conditions A from the two studied here. This is essentially due to the fact, proven in next paragraph, that the intersection of the polarization domain $\mathcal{D}$ with the linear manifold predicted by the conditions A is in $\partial\mathcal{D}$, the boundary of $\mathcal{D}$.

(*) This paper was prepared with the IHES desk computer (Hewlett Packard 9830).

<u>Conclusion</u> : The amount written for this conference is much more than what can be reasonably said in the schedule time of fifty minutes. In our complete publication we will add several appendices. Indeed we must give enough details on our computations, so that they can be easily reproduced by the interested readers (e.g. the experimentalists who want to analyze their data). For the sake of completeness we will also explain how the quark model predictions work in the forward and backward direction. Since conditions A have a firmer physical basis, five of them can be tested in a 9 dimensional polarization space : this is again very predictive. Furthermore this 9-plane is an equatorial plane, so the domain $\mathcal{D}$ is obtained by the condition that $\rho_{ob} \geq 0$.

Finally, in our complete publication, we hope to analyze more data. We are grateful in advance to our experimental colleagues who wish to communicate their results to us.

TABLE 3 EXPERIMENTAL DATA

Ref.	Reaction	P_{lab}	$t' = \|t-t_{min}\|/(GeV)^2$	Reference frame
A1	$\pi^+ p^+ \rightarrow \rho^o \Delta^{++}$	5	$t' \leq .2$	Tt, Ts, Td
A2	$\pi^+ p^+ \rightarrow \omega^o \Delta^{++}$	id	$t' \leq .5$	Tt, Ts
B1	$K^- p^+ \rightarrow \rho^- Y^{*+}$	3,9-4,6	$t' \leq .5$	Tt, Ts
B2	$K^- p^+ \rightarrow \Phi Y^{*o}$	id	$t' \leq 1.0$	Tt, Ts
C	$K^+ p^+ \rightarrow K^{*o} \Delta^{++}$	4,3-5	$t' \leq .25$	Tt, Ts, Td
D	$K^+ p^+ \rightarrow K^{*+} \Delta^+$	5	$t' \leq .25$	Tt, Ts, Td
F1	$\pi^+ p^+ \rightarrow \rho^o \Delta^{++}$	8	$t' \leq .2$	Tt, Ts
F2	$\pi^+ p^+ \rightarrow \omega^o \Delta^{++}$	8	$t' \leq .6$	Tt, Ts
G	$K^- n^o \rightarrow K^{*o} \Delta^-$	3	$t' \leq .25$	Tt, Ts

Resonances : K^* (892) Δ (1236) Y^* (1385)

Reference frame : T = transversity, t or s channel, d = Donohue-Högaasen

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Commentary to Table 4

This table is just a sample of what we hope to give with the world data.

One proves easily (once the observed density matrix has at least rank two) that the polarization degree must be smaller than $(5/11)^{\frac{1}{2}} = .674$. This makes rather dubious point B2. Of course the observed polarization degree decreases by projection so it must be shorter on the 3-plane. The polarization degree in the 19 dimensional polarization space should be independent of the choice of frame.

On the other hand, the more conditions to be satisfied (A (3 parameters), A, A+B, A+B+C) the longer is in general ET which measures directly the discrepancy between the model prediction and the data.

The 2 (and even 4) conditions A tested by the 3-plane are well verified for all points. Conditions A are frame independent. Conditions B are not very sensitive to the choice of frame. They are nearly as well verified as conditions A for $A_1F_1(\rightarrow \rho\Delta)$, $A_2F_2(\rightarrow \omega\Delta)$, and even for CDG $(\rightarrow K^*\Delta)$. Conditions C are slightly worse in Donohue-Högaasen frame; they are very poor in t frame and they are definitly bad in s frame. Furthermore we are also making a more detailed study for recognizing how much each of the 14 linear relations is satisfied by the data.

Table 4. TEST OF QUARK MODEL

E = Experimental point (see table 3 for reference of data and symbols for reference frame)

P = Nearest point from E predicted by the quark model

O = Unpolarized state, OE = observed degree of polarization

PE measures the disagreement with the model.

Data frame		19 polarization parameters				3 parameters	
		Conditions	A+B+C	A+B	A	Cond. A	
		OE	PE	PE	PE	PE	OE
A1	Tt	.50 ± .03	.22 ± .03	.07 ± .03	.06 ± .03	.02 ± .03	.22 ± .02
	Ts	id	.44 ± .03	.07 ± .03	.06 ± .03	id	id
	Td	id	.12 ± .03	.07 ± .03	.06 ± .02	id	id
F1	Tt	.56 ± .03	.36 ± .02	.08 ± .02	.06 ± .02	.02 ± .03	.30 ± .07
	Ts	id	.48 ± .02	.10 ± .02	.09 ± .02	id	id
A2	Tt	.40 ± .05	.25 ± .05	.12 ± .04	.10 ± .04	.06 ± .03	.18 ± .03
	Ts	.37 ± .05	.26 ± .05	.12 ± .04	.11 ± .04	id	id
F2	Tt	.35 ± .06	.30 ± .06	.21 ± .04	.17 ± .06	.12 ± .04	.26 ± .04
	Ts	.38 ± .06	.32 ± .06	.21 ± .06	.18 ± .06	id	id
C	Tt	.51 ± .02	.25 ± .03	.09 ± .02	.08 ± .02	.07 ± .02	.21 ± .01
	Ts	.46 ± .02	.41 ± .03	.09 ± .02	.07 ± .02	id	id
	Td	.51 ± .02	.12 ± .03	.09 ± .02	.08 ± .02	id	id
D	Tt	.51 ± .04	.27 ± .04	.09 ± .04	.05 ± .04	.03 ± .03	.22 ± .03
	Ts	id	.23 ± .04	.11 ± .04	.07 ± .04	id	id
	Td	id	.18 ± .04	.11 ± .04	.07 ± .04	id	id
G	Tt	.33 ± .02	.29 ± .02	.14 ± .02	.08 ± .02	.04 ± .02	.17 ± .03
	Ts	.30 ± .03	.35 ± .02	.13 ± .02	.09 ± .02	id	id
B1	Tt	.44 ± .06	.40 ± .05	.24 ± .06	.18 ± .06	.08 ± .05	.27 ± .05
	Ts	.49 ± .06	.46 ± .06	.28 ± .06	.20 ± .06	id	id
B2	Tt	.83 ± .15	.64 ± .14	.60 ± .14	.20 ± .12	.04 ± .09	.40 ± .12
	Ts	id	.73 ± .14	.58 ± .14	.20 ± .12	id	id

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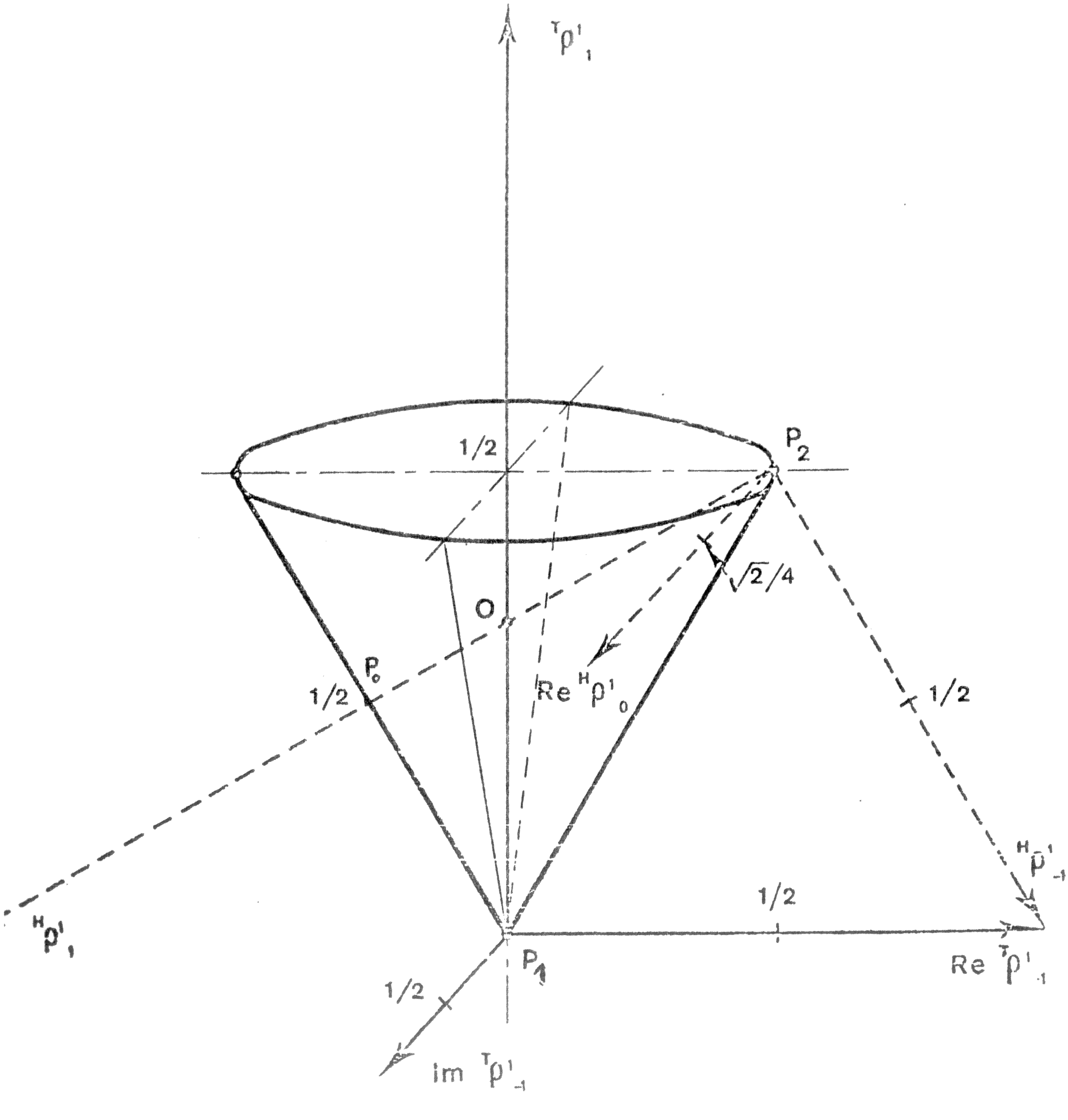


Fig. 1. This cone is the three dimensional even-polarization domain for a spin 1^{ϵ} particle with condition (2) for parity conservation (two body collision or one particle inclusive reaction). The matrix elements in helicity (${}^{H}\rho$) or transversity (${}^{T}\rho$) quantization form a basis in this polarization space. The physical meaning of each point of the cone is well defined once P_o and P_2 have been fixed : the polarization for forward or backward reaction is on the segment P_oP_2 . For spin 1^{ϵ} (ϵ = parity) production by a one meson exchange or a single Regge trajectory exchange, the basis circle corresponds to an exchange for $\epsilon = 1$ of natural parity, for $\epsilon = -1$ of unnatural parity (then P_2 corresponds to one π exchange). The vertex P_1 corresponds to the opposite naturality (for $\epsilon = -1$ e.g. ρ exchange, which has natural parity)

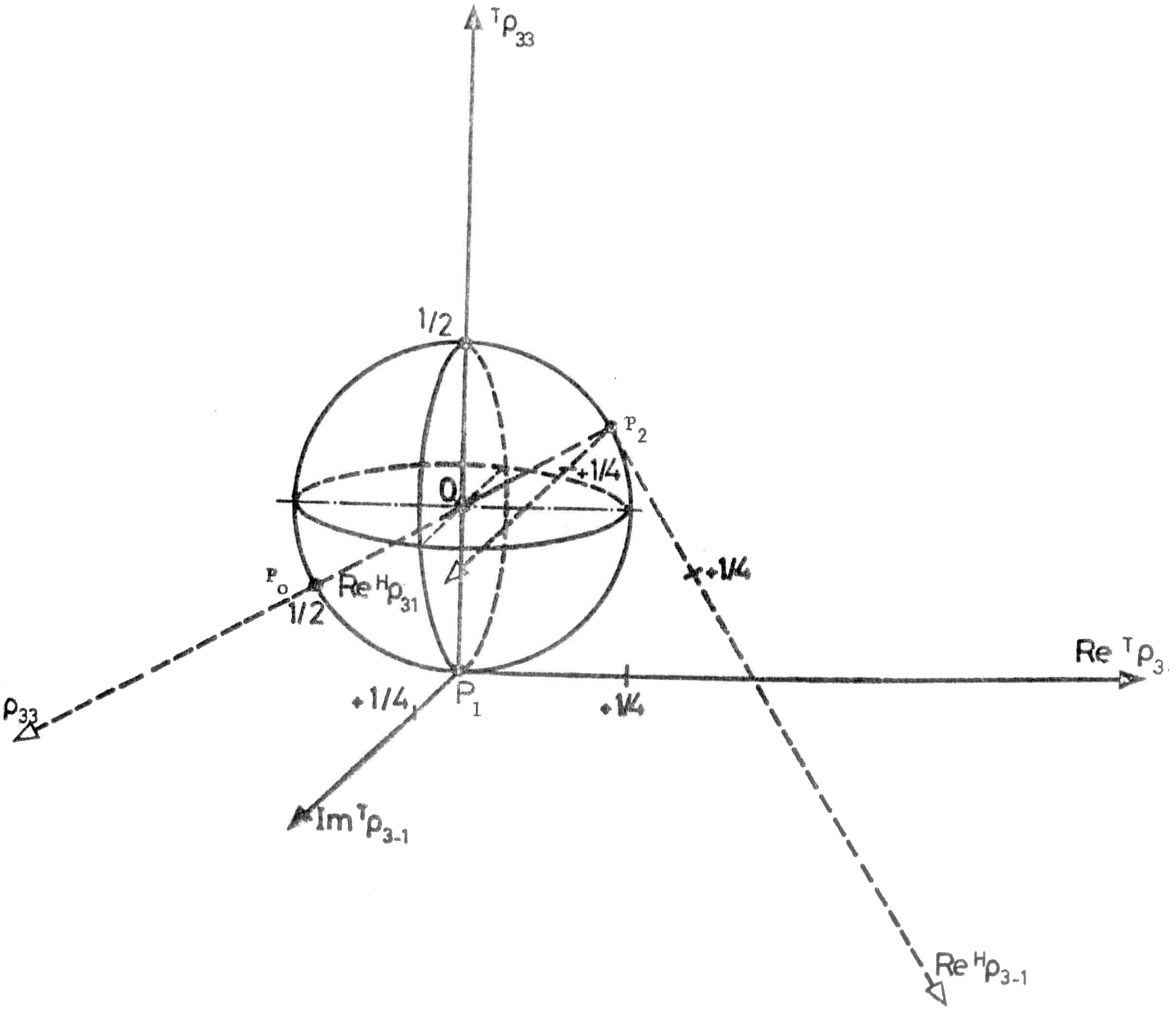


Fig. 2. This solid sphere is the three dimensional even-polarization domain for a spin $\frac{3}{2}$ particle with condition (2) for parity conservation (two body collision or one particle inclusive reaction). The matrix elements in helicity (${}^{H}\rho$) or transversity (${}^{T}\rho$) quantization form a basis in this polarization space. The physical meaning of each point of this ball is well defined once $P_oP_1P_2$ have been fixed. The polarization for forward or backward reaction is on the segment P_oP_2 and the south pole P_1 corresponds to vector exchange (Stodolsky Sakurai [10]).

Caption of Figures 3

The 3 dimensional domain $\mathcal{D}$ is a tetrahedron ABCD whose opposite edges are orthogonal. The quark model predicts the line AQ which is in the ACD face. We have plotted three orthogonal projection of ABCD : the directions of projection are respectively CD, AB and AQ. So, in the projection on the upper right of the figure, one sees directly the distance from the quark model prediction.

If the experimental point is inside the two projection triangle ABC and ACD, it is inside the tetrahedron. We have also drawn the projections of the tetrahedron A'BC'D. A point outside this tetrahedron come from non positive angular distribution (we reject such data!).

The agreement with the model is quite remarkable. The physical meaning of the position of the data along the line AQ is the following : A corresponds to vector meson exchange, i.e. to the vertex of the cone (lowest point) and the south pole of the sphere (Stodolsky Sakurai point). As one goes from A to Q the vertical coordinates increase linearly. Q corresponds to unnatural parity exchange (e.g. π meson). This is the case of $\pi p \rightarrow \rho\Delta$ as shown by Fig. 3A, point 1,2. This is not possible for $\pi p \rightarrow \omega\Delta$ and the points 3 and 4 of Fig. 3A are away from Q .

Code for the data

Points	1	2	3	4	
Fig. A	A1	F1	A2	F2	$\pi^+ p^+ \rightarrow \rho^o$ or ω^o, Δ^{++}
Fig. B	C	D	G		$K\ N \rightarrow K^* \Delta$
Fig. C	B1	B2			$K^- p^+ \rightarrow \rho$ or Φ

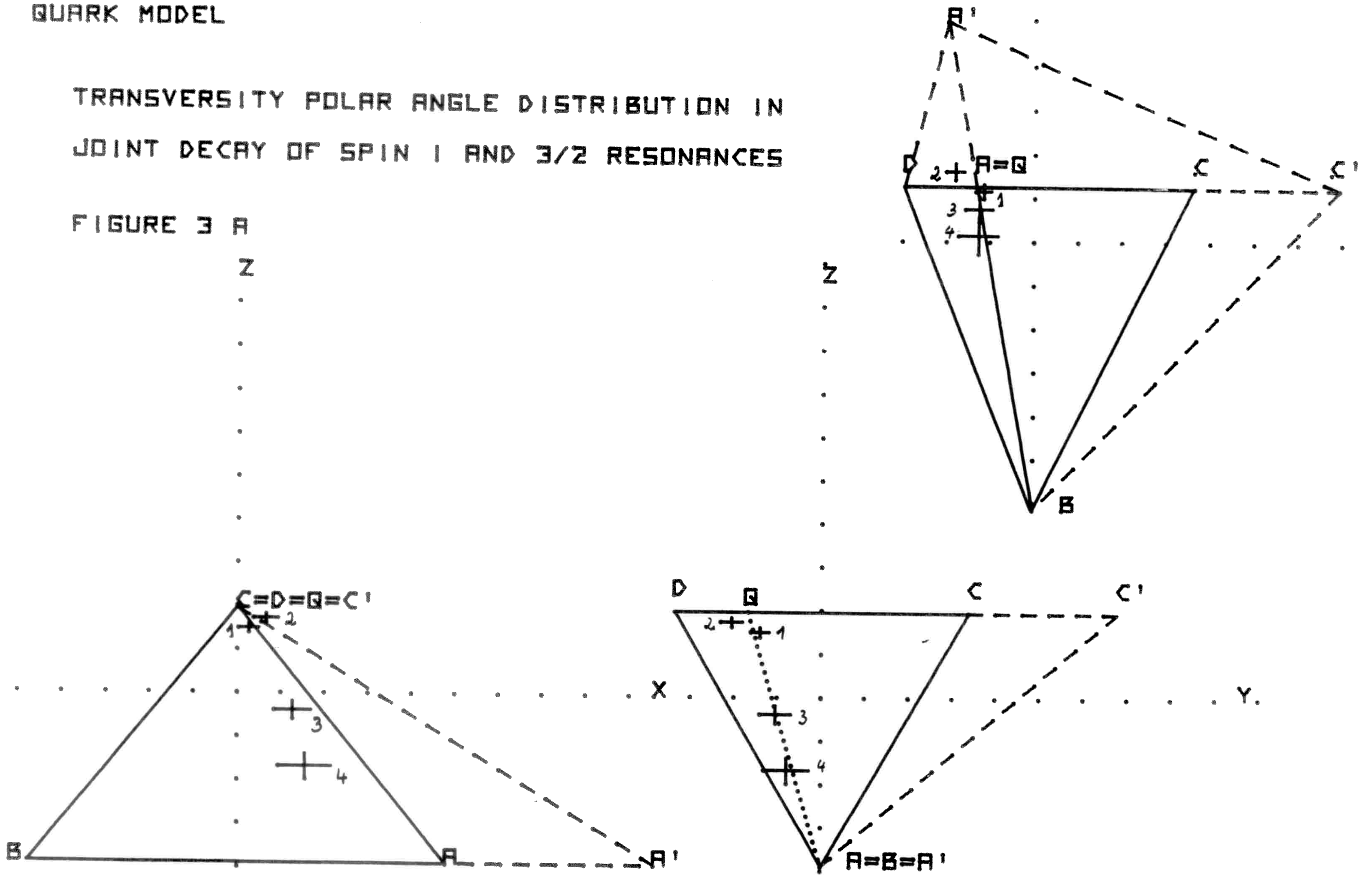
Caption of Figures 4

This is the plot ETO for the point D.
$K^+ p^+ \rightarrow K^{*+} \Delta^+$ at 5 GeV , $t' \leq 25$ for the t-channel frame, and the three types of conditions. The boundary drawn is only an upper bound of the actual boundary (which contains the theoretical point T).

QUARK MODEL

TRANSVERSITY POLAR ANGLE DISTRIBUTION IN
JOINT DECAY OF SPIN 1 AND 3/2 RESONANCES

FIGURE 3 A

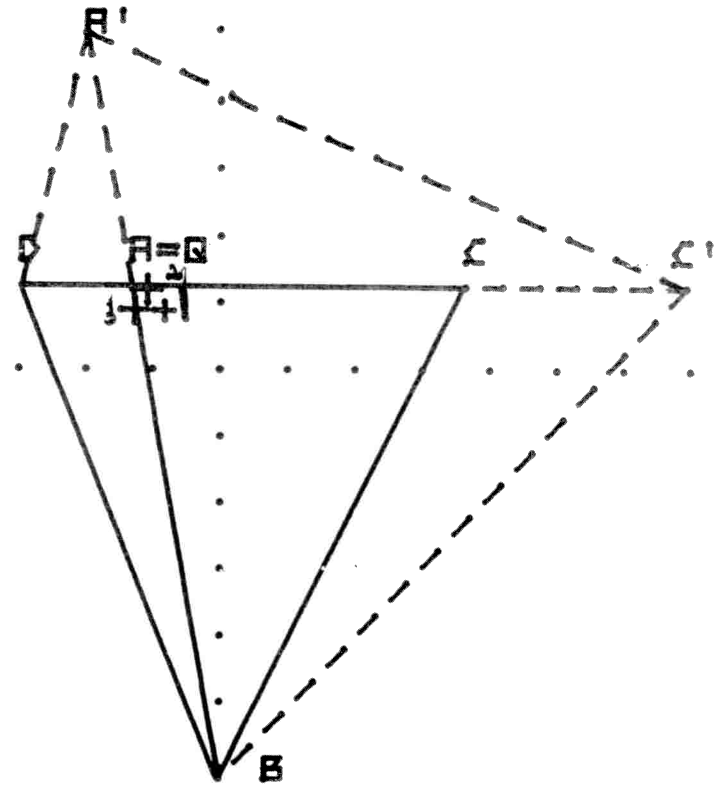


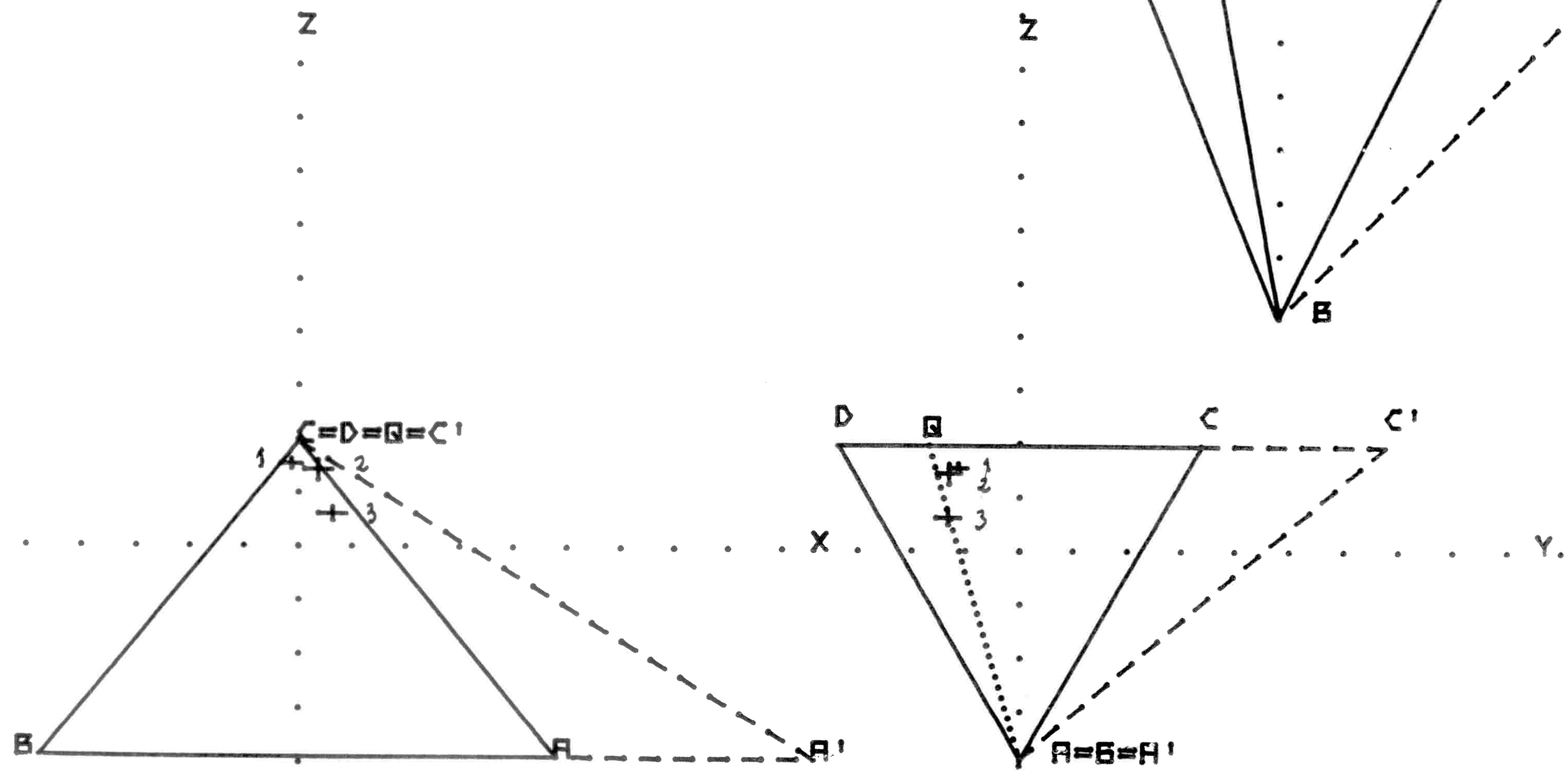
MODEL

TRANSVERSITY POLAR ANGLE DISTRIBUTION IN

JOINT DECAY OF SPIN 1 AND 3/2 RESONANCES

FIGURE 3 B

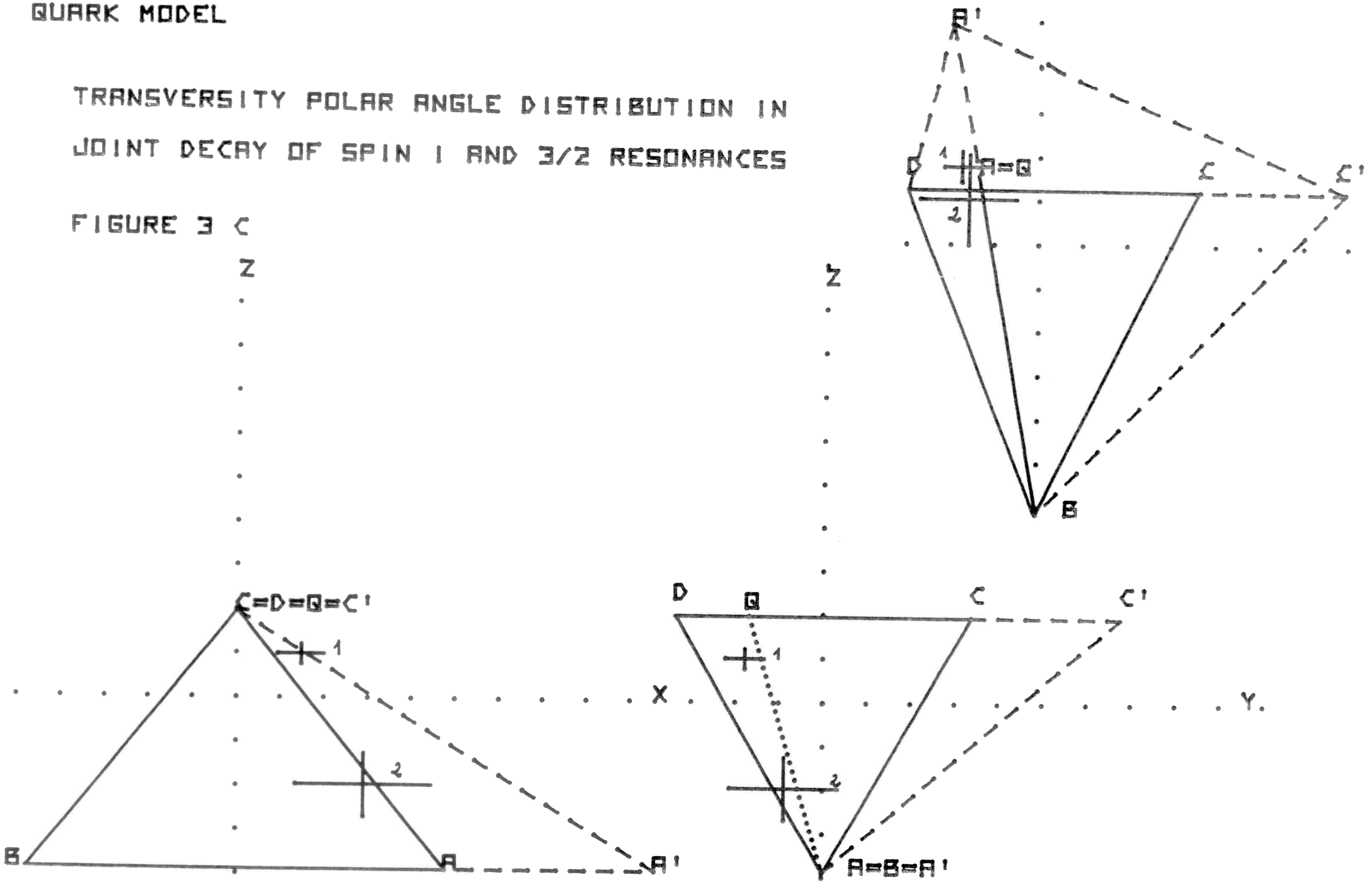




QUARK MODEL

TRANSVERSITY POLAR ANGLE DISTRIBUTION IN
JOINT DECAY OF SPIN 1 AND 3/2 RESONANCES

FIGURE 3 C



QUARK MODEL 0+1/2 -> 1+3/2
19 PARAMETERS
CONDITION A

POINT D

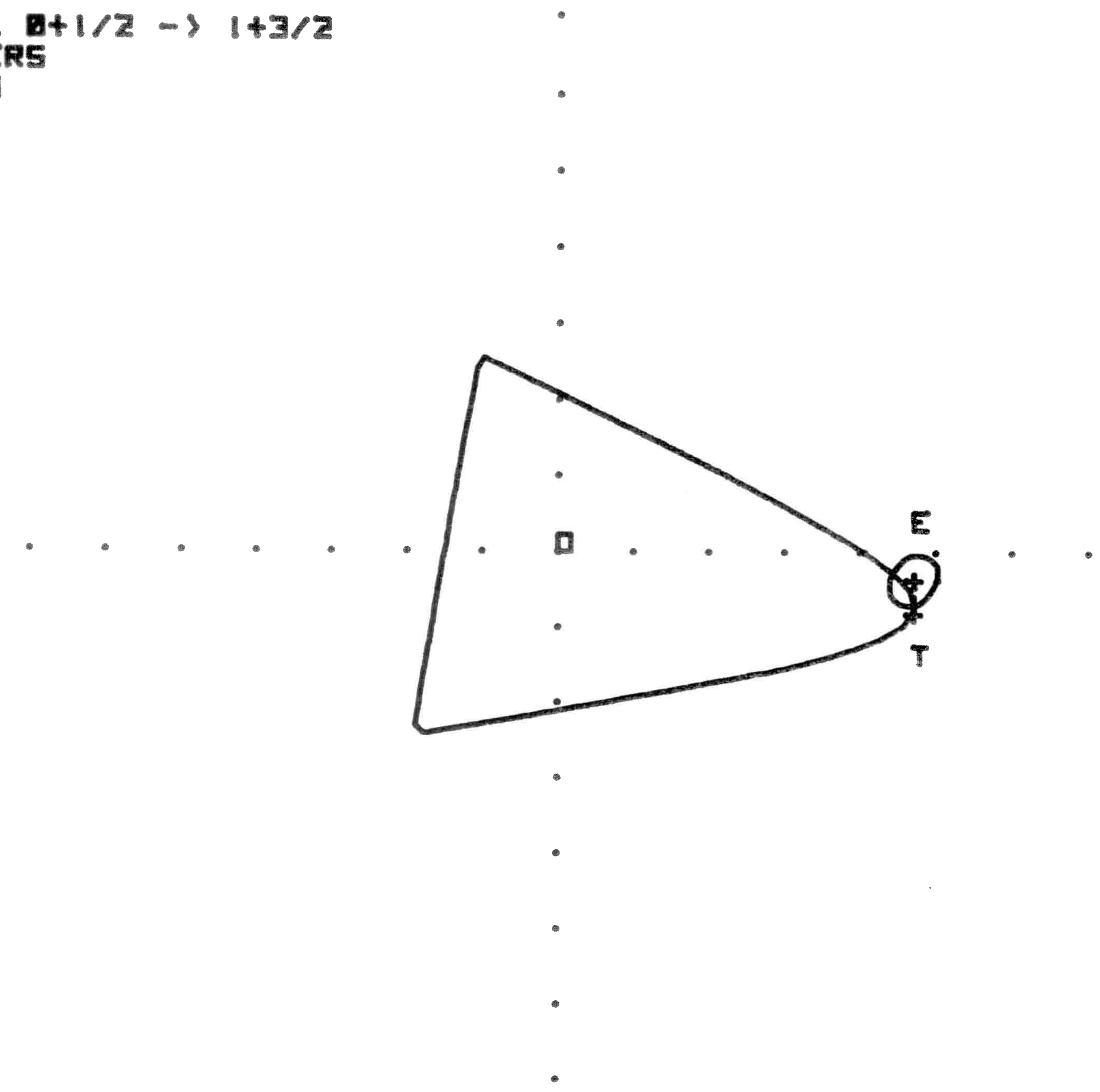


FIGURE 4 A

QUARK MODEL 0+1/2 -> 1+3/2
19 PARAMETERS
CONDITIONS AB

POINT D

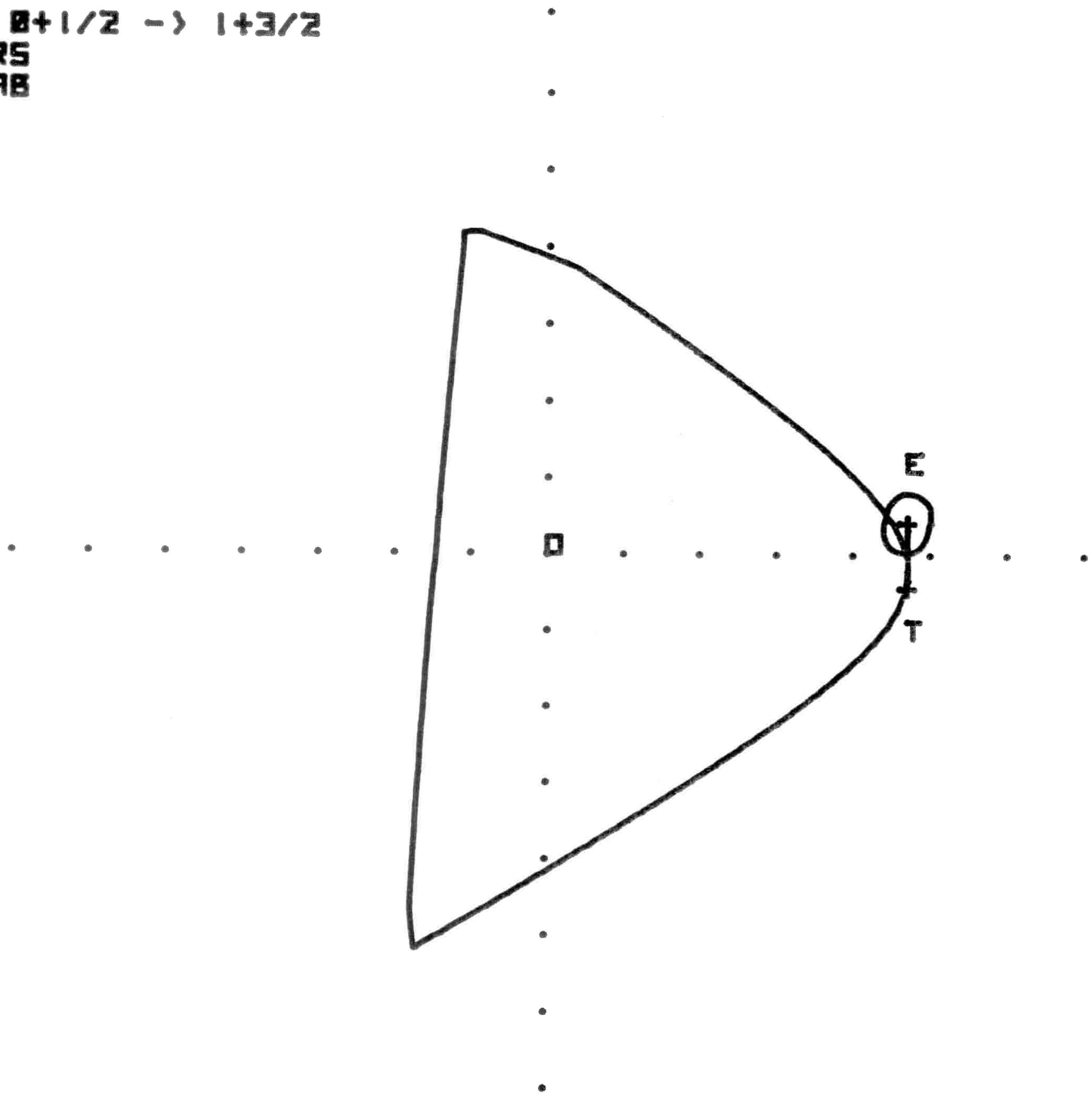


FIGURE 4 B

QUARK MODEL 0+1/2 -> 1+3/2
19 PARAMETERS
CONDITIONS ABC

POINT D

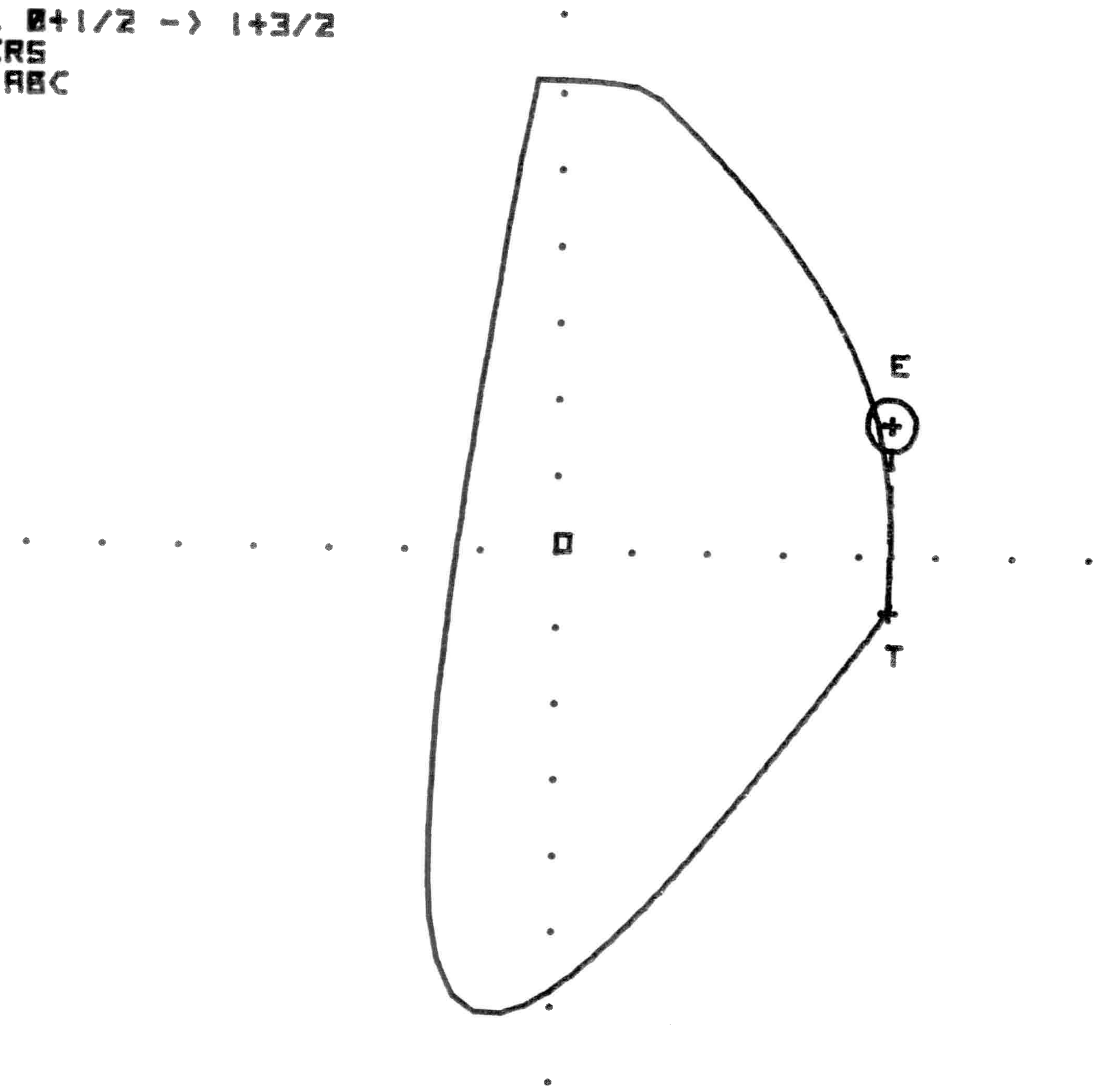


FIGURE 4 C